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Provincial Department of Education - NWP

10 E I

Second Term Test - Grade 13 - 2019

Index No : ....

Combined Mathematics I

Three hours only

Instructions:

* *This question paper consists of two parts.*
  Part A (Question 1 - 10) and **Part B** (Question 11 - 17)
* **Part A**
  Answer all questions. Write your answers to each question in the space provided. you may use additional sheets if more space is needed.
* **Part B**
  Answer five questions only. Write your answers on the sheets provided.
* *At the end of the time allocated, tie the answers of the two parts together so that* **Part A** *is on top of* **part B** *before handing them over to the supervisor.*
* *You are permitted to remove only* **Part B** *of the question paper from the Examination Hall.*

For Examiner's Use only

(10) Combined Mathematics I

Part	Question No	Marks Awarded
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
	Total	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	
Paper 1 total		
Percentage		

Paper I	
Paper II	
Total	
Final Marks	

Final Marks

In Numbers	
In Words	

Marking Examiner	
Marks Checked by 1	
2	
Supervised by	

Combined Mathematics (Part A)

01. Using the principle of mathematical induction, prove that $13^n - 4^n$ is divisible by 9 for all $n \in Z^+$.

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02. Find the set of solution of x satisfying the inequality $3 - x \leq |3x - 1|$.

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03. Evaluate $\lim_{x \to \pi} \dfrac{1-\sin\left(\dfrac{x}{2}\right)}{(\pi - x)^2}$

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04. Find number of possible ways of selecting 12 members including at least 5 women out of 7 women and 8 men. Out of these committees.

i. How many teams consists majority as women?

ii. How many teams consists majority as men?

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05. Write down r^{th} term Ur of the series $1^3 + 2^3 + 3^3 + \ldots\ldots\ldots\ldots$.

If $f(r) = (r-1)^2 r^2$, show that $f(r+1) - f(r) = 4\,Ur$. Hence evaluate $\sum_{r=1}^{n} Ur$.

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06. Find the volume generated by resolving the area enclosed by the curves $y = 3x$, $y = 0$ and $y = -x^2 + 4$ in the xy coordinate plane, through the x axis.

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07. A curve C is defined parametrically as $x=\sqrt{t}$ and $y-t=\frac{1}{\sqrt{t}}$. Prove that $\frac{dy}{dx}=2\sqrt{t}-\frac{1}{t}$.

Find the gradient of the tangent drawn to the curve at $t=4$. Hence find the equation of the tangent drawn to the curve at $t=4$.

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08. Straight line passes through $(1,1)$ meets the X axis at A and Y axis at B. Prove that the locus of the midpoint of AB is, $x+y-2xy=0$

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09. Find the equations of the circles which touches both x and y axes and the line $4x + 3y = 6$.

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10. Prove that $\sin 2\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$. Hence or otherwise solve the equation $\sin 2\theta - \tan\theta = 0$ for $0 \leq \theta \leq 2\pi$.

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Combined Mathematics 13 - I (Part B)

11. *a.* Find the value of λ such that both roots of the equation $\lambda(x^2+x+1)=2x+1;$ to be real and positive. $\lambda \in R$

If α and β are the roots of the above equation when $\lambda = 1\frac{1}{9}$, find the equation whose roots are α^2 and β^2.

Also find the equation whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$.

b. If α and β are the solutions of the equation $m^2(x^2-x)+2mx+3=0$ and if m_1 and m_2 are the values of m related to the relation $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{4}{3}$, show that the value of $\frac{m_1{}^2}{m_2}+\frac{m_2{}^2}{m_1}$ is $-\frac{68}{3}$

c. Let $f(x)=2x^3+rx^2-12x-7;\quad x\in R$, If $(x-\alpha)^2$ is a factor of $f(x)$ show that $\alpha=-1$. $\alpha \in R$. Find the value of r and factorized $f(x)$.

12. *a.* Prove that $1+r+r^2+..........+r^n=\frac{(1-r^n)}{1-r}$. Hence prove that sum of n terms of the series $(1+x)+(1+x+x^2)+(1+x+x^2+x^3)+..........$ is given by $\left(\frac{1}{1-x}\right)\left\{n-\frac{x^2(1-x^n)}{1-x}\right\}$.

b. r^{th} term of a series is given by $U_r=\frac{r^2-r-1}{(r+1)!}$.

Find the values of λ, μ, δ such that $U_r=\frac{\lambda}{(r-1)!}+\frac{\mu}{r!}+\frac{\delta}{(r+1)!}$, by substituting $r = 1,2,3$.

Verify results for $r = 4$.

Hence find, $\sum_{r=1}^{n} Ur$.

Is this series convergent? Justify your answer.

13. *a.* Find the number of permutations that can be prepared using all the letters of the word ANURADHAPURA.
Out of all these permutations, how many words contains all the four letters of A near by. How many of them are at the beginnings?

b. In a programme of school annual variety entertainment, there were scheduled to perform 3 dramas, 6 songs, and 4 dancing items. However the chief guest had taken more time than the scheduled. So the organizers were decided to perform 2 dramas, 4 songs and 3 dancing items. Find the possible ways of arranging the scheduled,
i. If it is possible to arrange any items in any way.
ii. If it is possible to arrange any items in the same type in any way.
iii. If they can arrange same type of programs according to a scheduled and any order.

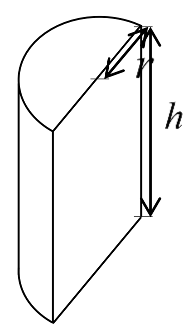
c. Sketch the graph of the functions $y = x^2 + 5$ and $y = |5x+1|$ on a same coordinate plane.
Hence find the set of solution of x satisfying the inequality $|5x+1| < x^2 + 5$.

14. *a.* $f(x)$ is given as $f(x) = \dfrac{1}{(x-2)(x^2+1)}$ for $x \neq 2$

Show that $f'(x) = -\left\{\dfrac{(3x-1)(x-1)}{(x-2)^2(x^2+1)^2}\right\}$

Hence find the turning points. Find the intercept on Y axis. Sketch the graph of the function $f(x) = \dfrac{1}{(x-2)(x^2+1)}$ indicating turning points and asymptotes.

b. Using given amount of steal it is expected to prepare a solid semicircular cylinder as shown in the diagram. Taking radius of the semicircular part as r and height as h, write down expression volume (v) of the solid. Also write down expression for area (s) of the solid using given dimensions. Represent surface area (s) in terms of r and v and show that the ratio between r and h such that surface area is minimum is given by $\pi : \pi + 2$.



15. *a.* Represent $\dfrac{1}{(t-4)(t-9)}$ as partial fraction and evaluate $\displaystyle\int \frac{1}{(t-4)(t-9)}\,dt$

Using substitution $x^2 = t$ evaluate $\displaystyle\int \frac{x}{(x^2-4)(x^2-9)}\,dx$

b. Show that $\displaystyle\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$ Where a is a constant.

Let, $I = \displaystyle\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x}\,dx$ and $J = \displaystyle\int_{-\pi}^{\pi} \frac{a^x \cos^2 x}{1+a^x}\,dx$. Show that $I + J = \pi$

Using above result and considering another linear relationship between I and J find I and J.

Using above type method evaluate $\displaystyle\int_{-\pi}^{\pi} \frac{a^x \sin^2 x}{1+a^x}\,dx$. Deduce the value of $\displaystyle\int_{-\pi}^{\pi} \frac{a^x}{1+a^x}\,dx$.

c. Using integration by parts evaluate $\int x^2 \cos x \, dx$

Hence find the area enclosed by the curves $y = x^2 \cos x$, $x = 0$, $x = \dfrac{\pi}{2}$ and $y = 0$.

16. *a*. Show that ant point on a line which is perpendicular to the line $l \equiv ax + by + c = 0$ and passing through point $P(h, k)$ can be written in the form of $(h + at, k + bt)$.

Equations of perpendicular bisectors of the sides AB and AC of triangle ABC are $x - y + 5 = 0$ and $x + 2y = 0$ respectively. Coordinate at A is $(1, -2)$ Find the equation of side BC.

If O is the circumcenter of triangle ABC, find coordinates at O.
Find the equation of the circle passing through mid-points of sides AB, AC, and points A and O.

b. Find the equations of the external tangents drawn to the two circles $x^2 + y^2 + 10y + 21 = 0$ and $x^2 + y^2 - 10y - 39 = 0$. Find the coordinates of the two intersecting points of these tangents and the second circle.

17. *a*. State and prove the sine rule for any triangle in the standard notation. Also state the cosine rule.
Let D is the midpoint of side BC of the triangle ABC and $AD \perp AC$.

Applying sine rule for the triangles ABD and ADC respectively, show that, $AD = -\dfrac{a \sin B}{2 \cos A}$ and

$AD = \dfrac{a}{2} \sin C$

Hence prove that $3b^2 = a^2 - c^2$

Also show that $\cos A \cos C = \dfrac{2(c^2 - a^2)}{3ac}$

b. Prove that, $\tan 3\theta = \dfrac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}$

Hence, solve the equation $\tan \theta + \tan\left(\theta + \dfrac{\pi}{3}\right) + \tan\left(\theta + \dfrac{2\pi}{3}\right) = 3$

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Provincial Department of Education - NWP

10	E	II

Second Term Test - Grade 13 - 2019

Index No : ....

Combined Mathematics II

Three hours only

Instructions:

* *This question paper consists of two parts.*
  Part A (Question 1 - 10) and **Part B** (Question 11 - 17)
* **Part A**
  *Answer **all** questions. Write your answers to each question in the space provided. you may use additional sheets if more space is needed.*
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For Examiner's Use only

(10) Combined Mathematics II		
Part	Question No	Marks Awarded
A	1	
	2	
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	4	
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	6	
	7	
	8	
	9	
	10	
	Total	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	
Paper 1 total		
Percentage		

Paper I	
Paper II	
Total	
Final Marks	

Final Marks

In Numbers	
In Words	

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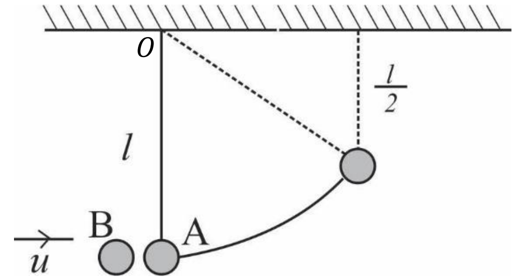
(Part A)

01) A body A of mass m is attached to one end of a light inextensible string of length l and the other end of the string is attached to a fixed point O on the ceiling and the system is kept in equilibrium such that A is vertically below O. A body B of mass m with horizontal velocity u collides directly with A, If the coefficient of restitution between A and B is $e\ (0 < e < 1)$, show that the velocity of A after the collision is $\frac{u}{2}(1+e)$. After the collision, if the mass A moves a maximum distance $\frac{l}{2}$ vertically below the ceiling, show that the velocity of A after the collision is $\sqrt{gl}$.
If $u = \frac{4}{3}\sqrt{gl}$ show that $e = \frac{1}{2}$.



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02) A particle is projected with a velocity u and angle of elevation 60^0 from a point O, at a vertical height H from the ground. The particle hits the ground at a point A which is at a horizontal distance a from O. Show that $u^2 = \frac{2\,g\,a^2}{\sqrt{3}\ a+H}$ and the maximum height reached by the particle from the horizontal level O is $\frac{3\,a^2}{4\,(\sqrt{3}\,a+H)}$.

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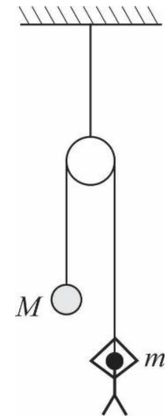
03) A light inextensible string passing around a small smooth pulley, fixed to a horizontal ceiling at O is shown in the figure. A mass M is attached to one end of the string and a child of mass m is hung in the other end of the string and he moves along the string. If the child moves up with an uniform acceleration f relative to the string Write down equations necessary to calculate acceleration of the string. Hence find the acceleration of the string.



04) A motor car of mass $M\ kg$ travels along a level road with constant speed $V\ ms^{-1}$ with power of the engine $H\ kw$. After that the motor car travels with the same power with constant velocity $\frac{v}{3}\ ms^{-1}$ along a road with inclination α to the horizontal. Then the resistance of the motor car is twice as the resistance of the level road. Show that $1000H = Mgv\sin\alpha$. Where g is the acceleration due to gravity.

05) $\underline{a}$ and $\underline{b}$ are two non-zero vectors. If $|\underline{a} + \underline{b}| = |\underline{a}| = |\underline{b}|$ find the angle between $|\underline{a}|$ and $|\underline{b}|$. Describe how two vectors $\underline{a}$ and $\underline{b}$ should be exist such that $|\underline{a} - \underline{b}| = |\underline{a}| = |\underline{b}|$.

06) A peg is fixed at a point O, which is at a horizontal distance $3a$ from a smooth vertical wall An uniform rod AB of length $4l$ and weight $2W$ is in equilibrium on the peg such that the end A is in contact with the wall. If the rod makes an angle θ $\left(O < \theta < \frac{\pi}{2}\right)$ with the wall, show that $\sin\theta = \left(\frac{3a}{2l}\right)^{\frac{1}{3}}$. $(3a < 2l)$

07) Velocities of the particle which experience simple harmony motion, at a distance $\boldsymbol{a}$ from one end and the center are $\sqrt{6g\boldsymbol{a}}$ and $\sqrt{8g\boldsymbol{a}}$ respectively. Find the amplitude and period of time of the motion.

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08) Let A and B are any two events in the sample space Ω. If $P(A \cap B') = \frac{5}{12}$, $P(A') = \frac{1}{3}$, $P(B) = \frac{1}{2}$, find $P(A \cap B)$ and $P(A \cup B)$.

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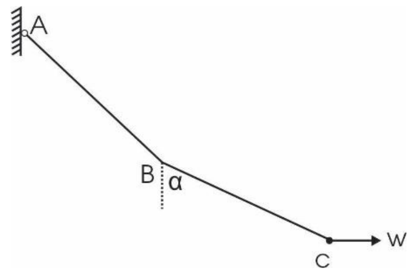
09) Two uniform rods AB, BC each of weight W are freely jointed at B and end A is smoothly and freely hinged to a fixed point. A horizontal force of W is applied at the end C, such that it is in the same plane ABC. If α is the inclination of the rod BC to the downward vertical at the position of equilibrium, show that $\alpha = \tan^{-1}(2)$, Also show that the magnitude of the reaction at B on the rod BC is $\sqrt{2}\,W$.



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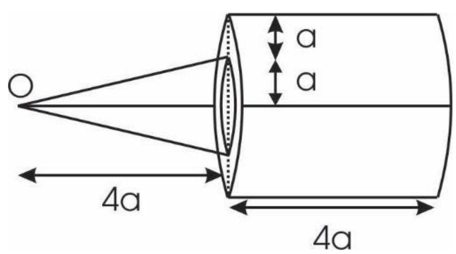
10) A composite solid body composed of a solid cone of radius a and height $4a$ and uniform cylinder of radius $2a$ and heeight $4a$ as shown in the figure. Show that the centre of mass of the body lies at a distance $\frac{75\,a}{13}$ from the vertex O of the cone on its axis of symmetry.



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Combined Mathematics 13 - II (Part B)

11) (a) In a motor car race, two motor cars A and B start their motion with velocities $u, 2u$ and maintain uniform accelerations $2a$ and a respectively, through the race until finished the race without wining or loss. If the maximum possible distance between two vehicles occurs in the race after time T from the beginning, draw corresponding velocity time graphs for the motion of two motor cars A and B on the same coordinate plane. Hence show that,

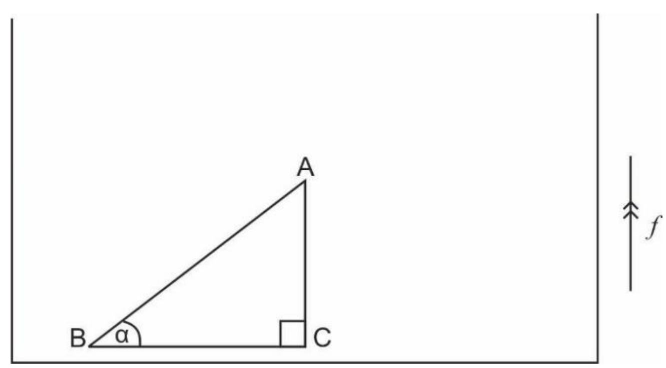
i. $T = \frac{u}{a}$

ii. If the time taken to finish the race is t, $t = 2T$

iii. The distance travelled by two cars (the length of the track) is $\frac{6u^2}{a}$.

(b) Two motor vehicles P and Q are moving along two straight roads towards point O, where two roads meet each other. Motor vehicles P is travelling due East from West with velocity $v\ ms^{-1}$ and motor vehicles Q is travelling with velocity $14\ ms^{-1}(< v)$, in the direction of angle 30^0 West of South. If the motor vehicle Q travels a distance $12\sqrt{79}$ during 6 seconds relative to the motor car P, using a velocity triangle show that v, satisfies the equation $v^2 - 14v - 120 = 0$. Hence find the value of v. Show that the direction which the motor car Q should travels relative to the motor car P is at an angle of $tan^{-1}\left(\frac{7\sqrt{3}}{13}\right)$ North of West.

12) (a) The triangle ABC in the figure represents a vertical cross section through the centre of gravity of an uniform smooth wedge of mass M. The line AB is a line of greatest slope and $A\hat{B}C = \alpha, A\hat{C}B = \frac{\pi}{2}$. The wedge is kept on due smooth horizontal wide stage of a lift such that the side BC is on the bottom of the lift. The lift travels upwards with uniform acceleration f, and a particle of mass $2m$ is kept at the point A on the line AB and released gently. Obtain equations which are sufficient to determine acceleration of the particle with respect to the wedge until particle leaves the wedge. Using the above equations, show that,

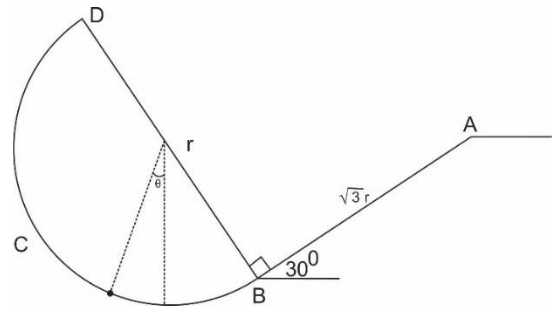


i. acceleration of the wedge is, $F = \frac{2m\,(g+f)\sin\alpha\cos\beta}{(M+2m\,sin^2\alpha)}$ and

ii. The acceleration of the particle relative to the wedge is, $\frac{(g+f)\,sin\alpha(m+2m)}{(M+2m\,sin^2\,\alpha)}$

(Consider that the wedge can moves horizontally on the bottom of the stage.)

(b) A smooth circular road which is connected with a smooth straight road made for an adventure game is shown in the figure. The length of the straight part AB is and $\sqrt{3}\, r$ and it is inclined 30^0 to the horizontal and the radius of the semicircular surface BCD, connected to it is r. The parts AB and BCD are fixed in the same vertical plane such that the diameter of the circular part is perpendicular to the straight line part. A child with mass m starts to travel downwards from the stage A and travels in the same vertical plane through the parth ABCD. If V is the velocity of the boy when he is in a position on the circular surface making an angle θ with the vertical at the centre,



i. Show that $V^2 = 2grcos\,\theta$.

ii. Show that the total distance travelled along the path before reversing his motion is,

$\frac{r}{3}\left[2\pi + 3\sqrt{3}\right]$.

Show that this motion is protective along the circular path.

13) (a) One end of an light elastic string of natural length l is attached to a fixed point O on a ceiling and a bin of mass m is attached to the other end. A monkey with mass m is inside the bin and the system is in equilibrium at a vertical height $\frac{3l}{2}$ from O.

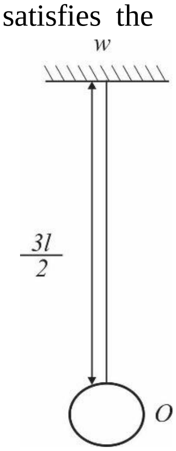
i. Show that the modules of elasticity of the string is $4mg$.

ii. After that the monkey gets down from the bin gently. In the subsequent motion when the bin is at a distance $x > (l)$ below O, show that the distance x satisfies the equation $\ddot{x} + \frac{4g}{l}\left[x - \frac{5l}{4}\right] = 0$

Now, let $x - \frac{5l}{4} = X$

Then show that the above motion is in the form $\ddot{X} + \omega^2 X = O$.

Hence show that the motion is simple harmonic.



iii. Find the centre and the amplitude of the motion and show that this is a complete simple harmonic motion.

iv. Show that the period of the motion is $\pi\sqrt{\frac{l}{g}}$.

v. The monkey takes the bin and comes to the point O and jump vertically downwards with the bin with velocity $\sqrt{2gl}$ such that it moves under gravity. When the monkey is at a distance y (y > l) below O, show that the distance y satisfies the equation $l\dot{y}^2 + 2g\,(y - l)^2 - 2gyl - 2gl^2 = 0$

vi. Using the above equation, show that the maximum distance reached by the monkey with the bin from O is $3l$.

14) (a) Let the position vectors of the points A and B relative to a point O are $\underline{a}$ and $\underline{b}$ respectively. The point D is on AB such that $AD : DB = 1:3$. The line OD is produced to E such that $OD : DE = 1:2$.

Show that $\overrightarrow{DE} = \frac{3\underline{a} + \underline{b}}{2}$ and $\overrightarrow{AE} = \frac{5\underline{a} + 3\underline{b}}{4}$.

Now a line is drawn through B parallel to OE. It meets the produced line AE at F.

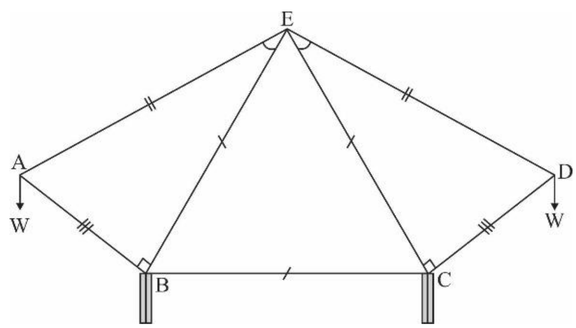
Taking $\overrightarrow{BF} = \lambda\, \overrightarrow{OE}$ and $\overrightarrow{AF} = \mu\, \overrightarrow{AE}$, show that $\lambda = \frac{8}{3}$ and $\mu = 4$.

Hence find the ratio $AE : EF$. Also find $\overrightarrow{BF}$.

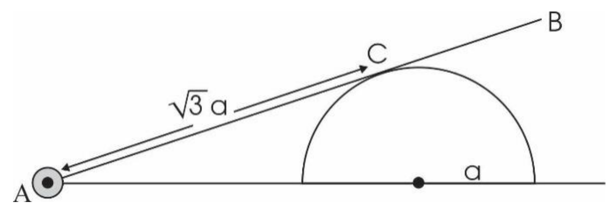
(b) $A \equiv (8,0), B \equiv (8,6), C \equiv (0,6)$ are three points on the OXY plane. O is the origin. Force of magnitudes P, Q, R and S newton's act along the sides OA, AB, CB and OC in the order of the letters respectively. All the distances here are measured in meters. The line of action of this system of forces reduces to a single force along the line. $2x - 3y - 6 = 0$. The anticlockwise moment of the system of forces about the origin O is $42Nm$. If $S = 2$, find the magnitude of the forces P, Q and .

Now by adding an additional force to the system of forces, the new system reduces to a couple of anticlockwise moment $70\ Nm$. Find the magnitude, direction and the line of action of the additional force.

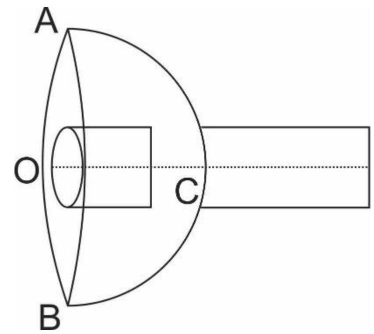
15) (a) Frame work ABCDE consisting of 7 light roads is given in the figure. The weights w are attached at the joints A and D. B and C are two smooth supports and the frame work is kept in equilibrium in a vertical plane.$BC = BE = EC$ and $AB = CD$ and $EA = ED$. $A\hat{E}B = C\hat{E}D = 30^0$ and $A\hat{C}D = A\hat{B}E = 90^0$. Find the reactions at B and C and find the stress on the rods classifying them in to tensions or thrusts.



(b) The length of an uniform rod AB is $2a$ and it is hinged smoothy at A. The rod is kept such that the rod touch the circular surface of an uniform semicircular disc of radius a and weight w at C and $AC = \sqrt{3}\,a$. The system is kept in equilibrium such that the base of the semicircular disc is on a rough horizontal floor and the surface at C is smooth. If the coefficient of friction between the floor and the plane base is μ show that $\mu \geq \frac{4-\sqrt{3}}{13}$.



16) Using integration show that the centre of mass of an uniform solid hemisphere of radius a lies at a distance $\frac{3}{8}a$ from its centre.



A solid right circular cylinder part of radius a and height $2a$ is removed from an uniform solid hemisphere of radius $3a$ and density σ. Now a cylinder of radius a, height $3a$ and density $k\sigma$ is attached at point C of the above remaining part such that their symmetic axis coinside with each other. Now show that the centre of mass of the solid body R is at a distance $\frac{(73+54K)\,a}{4(16+3k)}$ from O, on the axis of symmetry.

Then the solid body R is kept in equilibrium by touching surface containing AOB on a rough plane, inclined at an angle α $(0 < \alpha < \frac{\pi}{2})$ to horizontal with line AOB is along the line of greatest slope.

If the coefficient of friction between the body R and the plane is μ.

Show that $\mu \geq \tan\alpha$ and show that $\tan\alpha < \frac{12\,(16\,+3k)}{73\,+\,54\,k}$.

17) (a) A and B are any two events in the sample space S. Define what is meant by each of the followings given below.

i. A and B are exhaustive events.

ii. A are B are mutually exclusive events.

(b) Let A and B are any two events in a sample space. A' and B' are the complementary events of A and B respectively. Show that,

i. $P(A') = 1 - P(A)$

ii. $P(A) = P(A \cap B) + P(A \cap B')$

iii. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

If $P(A) = \frac{2}{5}$, $P(B') = \frac{3}{10}$, $P(A' \cup B') = \frac{17}{20}$ find, $P(A' \cap B'), P(A \cup B)$ and $P(A' \cap B)$.

(c) In a box there are 3 blue, 4 red and 5 green identical balls. Three balls are taken randomly, one by one , one at a time from the box without replacement. Find the probabilities of,

i. All the 3 balls are in the same colour.

ii. Getting blue, red and green balls respectively.

iii. Not getting any red ball.

iv. Getting at least one red ball.

Part A.

1. When $n=1$, $L.H.S = 13^n - 4^n$

$= 9$

Therefore, it is true for $n=1$. 5

Take any $p \in Z^+$, assuming that the result is true for $n=p$.

i.e. $13^p - 4^p = 9\alpha$; $\alpha \in N$. 5

when $n=p+1$, $13^{p+1} - 4^{p+1}$

$= 13^p \times 13 - 4^{p+1}$ 5

$= (4^p + 9\alpha) \times 13 - 4^{p+1}$

$= 4^p \times 13 - 4^p \times 4 + 13 \times 9\alpha$

$= 4^p \times 9 + 13 \times 9\alpha$

$= 9\underbrace{(4^p + 13\alpha)}_{r \in N}$ 5

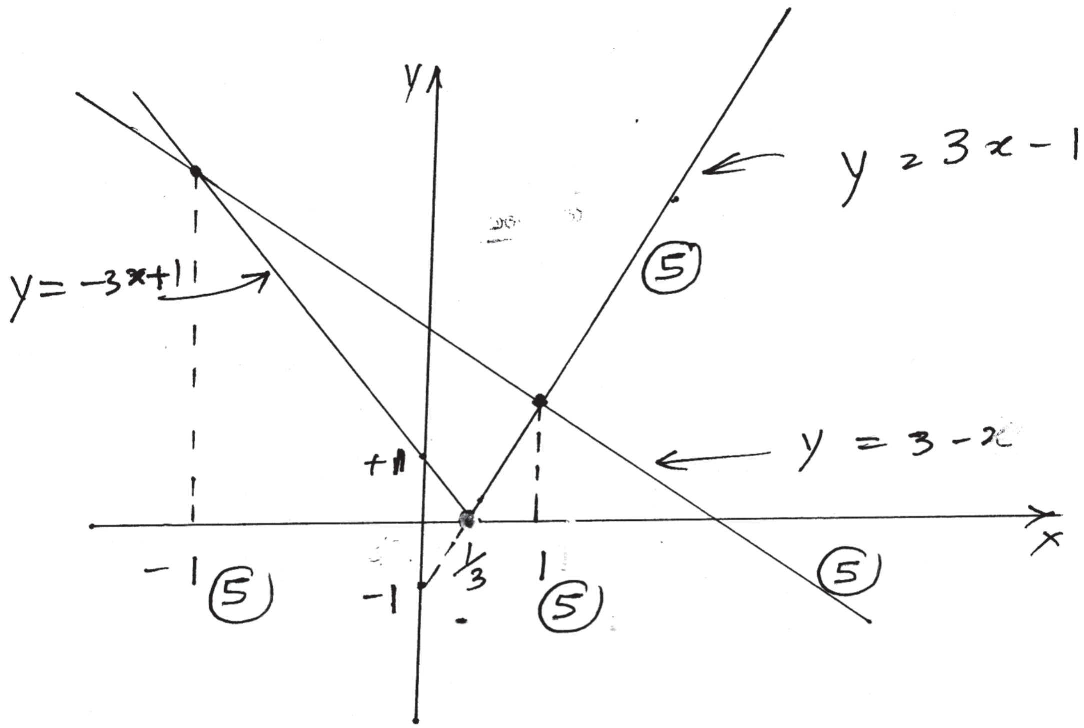
$= 9r$; $r \in Z$

Hence, if the result is true for $n=p$, then it is also true for $n=p+1$. There fore by using the principle of Mathematical Induction, the result is true for all $n \in Z^+$. 5

25

02. $3 - x \leq |3x-1|$

$$|3x-1| = \begin{cases} 3x-1 & ; \; x \geq \frac{1}{3} \\ -3x+1 & ; \; x < \frac{1}{3}. \end{cases}$$



$-3x+1 = +3-x$

$-2x = 2.$

$\therefore \; x = -1$

$3x - 1 = 3 - x$

$4x = 4$

$x = 1$

$x \in [-1, 1]$ 5

25

3. $\lim_{x \to \pi} \dfrac{1 - \sin\frac{x}{2}}{(\pi - x)^2}$

Let $x - \pi = \theta$. Then, 5

$$\lim_{x \to \pi} \frac{1 - \sin\frac{x}{2}}{(\pi - x)^2} = \lim_{\theta \to 0} \frac{1 - \sin\left(\frac{\pi}{2} + \frac{\theta}{2}\right)}{(-\theta)^2}$$

$$= \lim_{\theta \to 0} \frac{1 - \cos\frac{\theta}{2}}{\theta^2} \quad 5$$

$$= \lim_{\theta \to 0} \frac{\sin^2\frac{\theta}{2}}{\theta^2\left(1 + \cos\frac{\theta}{2}\right)}$$

5

$$= \lim_{\theta \to 0} \left[\frac{\sin\theta/2}{\theta/2}\right]^2 \times \frac{1}{4} \times \frac{1}{1 + \cos\frac{\theta}{2}}$$

$$= \left\{\lim_{\theta \to} \frac{\sin\theta/2}{\theta/2}\right\}^2 \times \lim_{\theta \to 0} \frac{1}{4\left(1 + \cos\frac{\theta}{2}\right)}$$

5

$$= 1^2 \times \frac{1}{4(1+1)}$$

$$= \underline{\underline{\frac{1}{8}}} \quad 5$$

25

04. Women -7 $\left.\right\}$ $\longrightarrow 12$
men -8

Women	men	
5	7	$\longrightarrow {}^7C_5 \times {}^8C_7$
6	6	$\longrightarrow {}^7C_6 \times {}^8C_6$
7	5	$\longrightarrow {}^7C_7 \times {}^8C_5$

$${}^7C_5 \times {}^8C_7 = \frac{7!}{2!\,5!} \times \frac{8!}{1!\,7!} = \frac{7\times \cancel{6}^{3}}{\cancel{2}} \times 8$$
$$= 21 \times 8$$
$$= 168$$

$${}^7C_6 \times {}^8C_6 = \frac{7!}{1!\,6!} \times \frac{8!}{2!\,6!} = 7 \times \frac{\cancel{8}^{4} \times 7}{\cancel{2}}$$
$$= 49 \times 4$$
$$= 196 \qquad (10)$$

$${}^7C_7 \times {}^8C_5 = \frac{7!}{0!\,7!} \times \frac{8!}{3! \times 5!} = 1 \times \frac{8 \times 7 \times \cancel{6}}{\cancel{3} \times \cancel{2}}$$
$$= 56$$

$\therefore$ The number of permutations of committees with 12 members $\left.\right\}$ $= 168 + 196 + 56$
$= \underline{\underline{420}}$ (5)

(i). ${}^7C_7 \times {}^8C_5 = \underline{\underline{56}}$ (5)

(ii) ${}^7C_5 \times {}^8C_7 = \underline{\underline{168}}$. (5)

25

5. $U_r = r^3$ 5

$$f(r+1) - f(r) = r^2(r+1)^2 - (r-1)^2 r^2$$
$$= r^2\{r^2 + 2r + 1 - r^2 + 2r - 1\}$$
$$= r^2(4r)$$
$$= 4r^3$$
$$= 4U_r \quad 5$$

$$\therefore \quad f(r+1) - f(r) = 4U_r$$

when $r = 1$ $\quad f(2) - f(1) = 4U_1$

$r = 2$ $\quad f(3) - f(2) = 4U_2$ 5

$r = 3$ $\quad f(4) - f(3) = 4U_3$

$\vdots \quad \vdots \quad \vdots \quad \vdots$

$r = n-1$ $\quad f(n) - f(n-1) = 4U_{n-1}$ $\cancel{5}$

$r = n$ $\quad f(n+1) - f(n) = 4U_n$

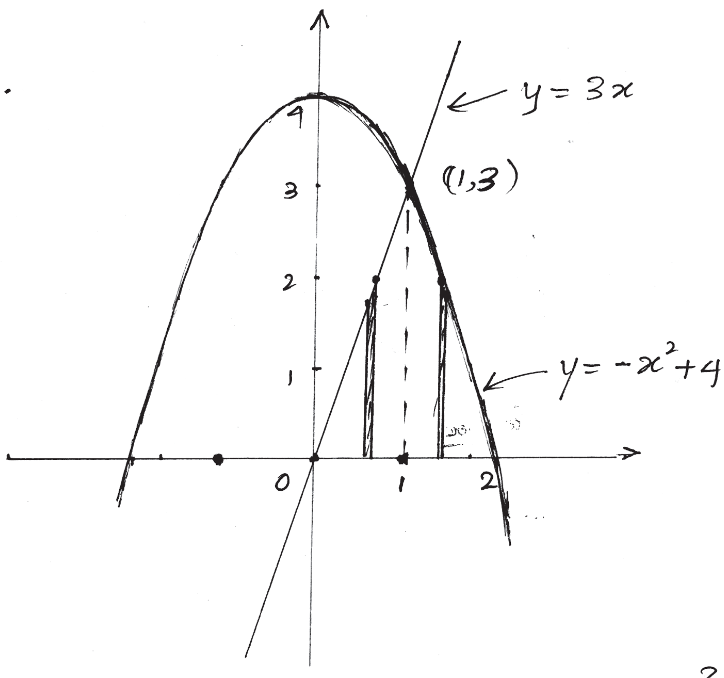
$$f(n+1) - f(1) = 4\sum_{r=1}^{n} U_r \quad 5$$

$$n^2(n+1)^2 - 0 = 4\sum_{r=1}^{n} U_r$$

$$\therefore \quad \sum_{r=1}^{n} U_r = \frac{n^2(n+1)^2}{4.} \quad 5$$

25

06.



$$\text{Volume generated} = \int_0^1 \pi(3x)^2\,dx + \int_1^2 \pi\,(-x^2+4)^2\,dx \quad 5$$

$$= 9\pi\left[\frac{x^3}{3}\right]_0^1 \; (5) + \pi\left[\frac{x^5}{5} - 8\frac{x^3}{3} + 16x\right]_1^2 \; (5)$$

$$= 9\pi\frac{1}{3} + \pi\left(\frac{32}{5} - \frac{64}{3} + 32 - \frac{1}{5} + \frac{8}{3} - 16\right) \quad 5$$

$$= 3\pi + \pi\left(\frac{31}{5} - \frac{56}{3} + 16\right)$$

$$= \pi\left(\frac{93 - 280 + 19\times15}{15}\right)$$

$$= \pi\,\frac{93 - 280 + 285}{15}$$

$$= \frac{98}{15}\pi \quad 5$$

25

07. $x = \sqrt{t} \Rightarrow \frac{dx}{dt} = \frac{1}{2} t^{-1/2} = \frac{1}{2\sqrt{t}}$

$y = t + \frac{1}{\sqrt{t}} \Rightarrow \frac{dy}{dt} = 1 + -\frac{1}{2} t^{-3/2} = 1 - \frac{1}{2t\sqrt{t}}$ (5)

$\frac{dy}{dx} = \frac{2t\sqrt{t} - 1}{2t\sqrt{t}} \times \frac{2\sqrt{t}}{1} = \underline{\underline{2\sqrt{t} - \frac{1}{t}}}$ (5)

When $t = 4$; $\frac{dy}{dx} = 2 \times 2 - \frac{1}{4} = 3\frac{3}{4}$ (5)

Also the point relevent to $t = 4$ is

$(2, 4\frac{1}{2})$ (5)

Then the equation of tangent drawn at the point $(2, 4\frac{1}{2})$

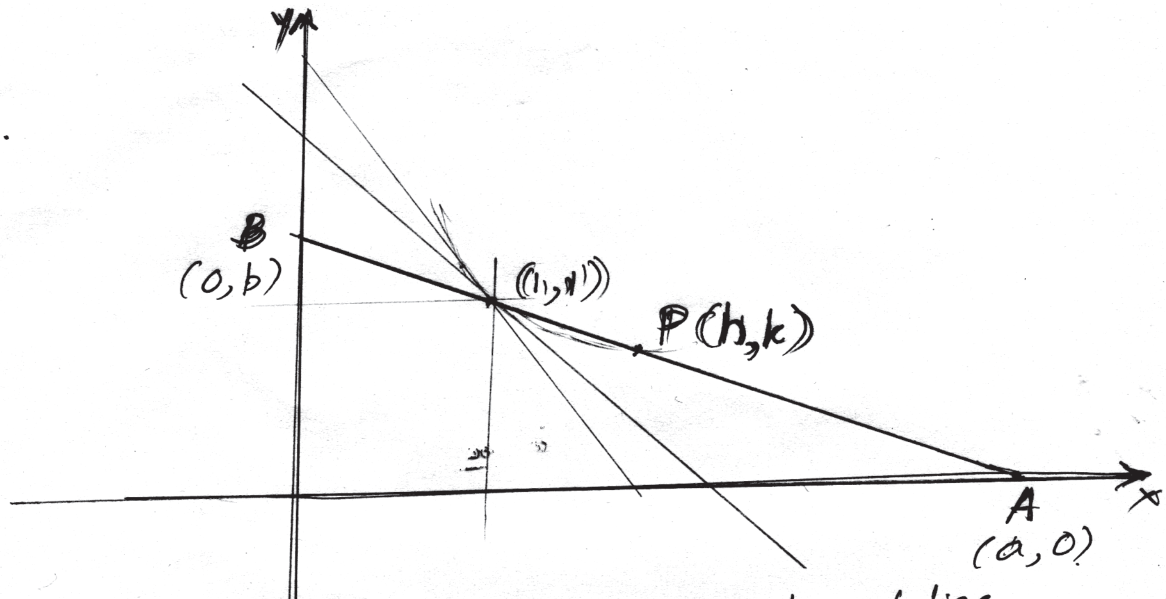
$\frac{y - 4\frac{1}{2}}{x - 2} = 3\frac{3}{4} \Rightarrow \frac{2y - 9}{\cancel{2}} = (x-2) \times \frac{15}{\cancel{4}\,2}$

$4y - 18 = 15x - 30$

$\underline{\underline{15x - 4y - 12}} = 0$ (5)

25

08.



Let P(h,k) be the mid-point of line segment AB passing through the point (1,1).
Let the line segment AB meets the coordinate axes at the points A (a,0) and B (0,b) respectively.

$m = -\frac{b}{a}$ 5

$\frac{y-0}{x-a} = \frac{-b}{a}$ 5 $\Rightarrow ay = -bx + ab$

$bx + ay = ab \Rightarrow \frac{x}{a} + \frac{y}{b} = 1$

Since it passes through (1,1)

$\frac{1}{a} + \frac{1}{b} = 1 \Rightarrow a + b = ab$ 5 — ①

But $P(h,k) = P\left(\frac{a}{2}, \frac{b}{2}\right)$

$\therefore a = 2h$ and $b = 2k$ 5

Substituting these values in ①

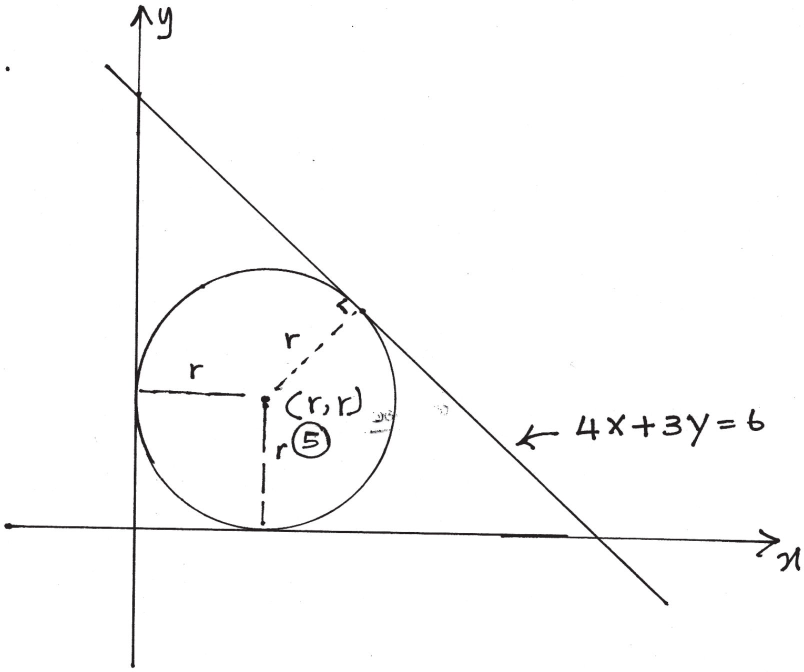
25

$2h + 2k = 4hk$

Since h, k are variables, let $h = x$ and $k = y$.
5

Then $2x + 2y = 4xy \Rightarrow x + y - 2xy = 0$

09.



$$r = \left| \frac{4r+3r-6}{\sqrt{4^2+3^2}} \right|$$

$$\pm r = \frac{7r-6}{5}$$

$\oplus \rightarrow \underline{\underline{r=3}}$ $\quad$ $\ominus \rightarrow \underline{\underline{r=\frac{1}{2}}}$ 10

The eq^n of circles are.;

$\underline{\underline{(x-3)^2+(y-3)^2=3^2}}$ and

10

$\underline{\underline{\left(x-\frac{1}{2}\right)^2+\left(y-\frac{1}{2}\right)^2=\left(\frac{1}{2}\right)^2}}$

25

10. $\sin 2\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$. (5)

$L:H:S = \sin 2\theta = \dfrac{2\sin\theta.\cos\theta}{\cos^2\theta+\sin^2\theta}$

$= \dfrac{\dfrac{2\sin\theta\cos\theta}{\cos^2\theta}}{\dfrac{\cos^2\theta}{\cos^2\theta}+\dfrac{\sin^2\theta}{\cos^2\theta}}$

$= \dfrac{2\tan\theta}{1+\tan^2\theta} = R:H:S$

$\sin 2\theta - \tan\theta = \dfrac{2\tan\theta}{1+\tan^2\theta} - \tan\theta$

$= \tan\theta\left\{\dfrac{2-1-\tan^2\theta}{1+\tan^2\theta}\right\}$

$= \tan\theta\left\{\dfrac{1-\tan^2\theta}{1+\tan^2\theta}\right\} = 0$

$1+\tan^2\theta > 0.$

$\therefore \tan\theta(1-\tan\theta)(1+\tan\theta) = 0$

$\Rightarrow \tan\theta = 0, \quad \tan\theta = 1 \quad$ or $\tan\theta = -1$ (5)

$\therefore \tan\theta = \tan 0, \quad \tan\theta = \tan\frac{\pi}{4} \quad$ or $\tan\theta = \tan\left(-\frac{\pi}{4}\right)$

$\therefore \theta = n\pi; n\in\mathbb{Z}, \quad \theta = n\pi+\frac{\pi}{4}; n\in\mathbb{Z} \quad$ or $\theta = n\pi-\frac{\pi}{4}; n\in\mathbb{Z}.$

When $n=0$, $\theta=0$; $n=1$, $\theta=\pi$; $n=2$, $\theta=2\pi$

when $n=0$; $\theta=0$; $n=1$; $\theta=5\pi/4$ (10)

when $n=0$; $\theta=-\pi/4$; $n=1$, $\theta=\frac{3\pi}{4}$; $n=2$, $\theta=\frac{7\pi}{4}$

$\therefore \theta = \left\{0, \pi, \frac{5\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}, 2\pi\right\}$. (5)

25

(11) a) $\lambda(x^2+x+1)=2x+1$

$\lambda x^2+(\lambda-2)x+\lambda-1=0$

When the roots are real and positive,

$\Delta \geq 0$, $\alpha+\beta>0$, $\alpha\beta>0$ (10)

$(\lambda-2)^2-4\lambda(\lambda-1)\geq 0$

$\lambda^2-4\lambda+4-4\lambda^2+4\lambda \geq 0$

$-3\lambda^2+4\geq 0$

$3\lambda^2-4\leq 0$

$(\sqrt{3}\lambda-2)(\sqrt{3}\lambda+2)\leq 0$

	$\left(-\infty,-\frac{2}{\sqrt{3}}\right)$	$\left(-\frac{2}{\sqrt{3}},\frac{2}{\sqrt{3}}\right)$	$\left(\frac{2}{\sqrt{3}},\infty\right)$
$(\sqrt{3}\lambda-2)(\sqrt{3}\lambda+2)$	$+$	$-$	$+$

$\lambda \in \left[-\frac{2}{\sqrt{3}},\frac{2}{\sqrt{3}}\right] \rightsquigarrow$ (A). (10)

$-\frac{(\lambda-2)}{\lambda}>0$

$\frac{(\lambda-2)\lambda}{\lambda^2}<0$

	$(-\infty,0)$	$(0,2)$	$(2,\infty)$
$\lambda(\lambda-2)$	$+$	$-$	$+$

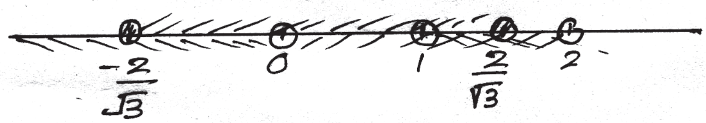
$\lambda \in (0,2) \rightsquigarrow$ (B) (5)

$$\frac{x-1}{x} > 0$$

$$\frac{x(x-1)}{x^2} > 0.$$

	$(-\infty,0)$	$(0,1)$	$(1,\infty)$
	$+$	$-$	$+$

$x \in (-\infty,0) \cup (1,\infty) \rightarrow$ ⓒ. ⑤



$x \in \left(1, \frac{2}{\sqrt{3}}\right]$ ⑤

When $x = 1\frac{1}{9} = \frac{10}{9}.$

$$\alpha+\beta = \frac{x-2}{x} = \left(\frac{10}{9}-2\right) \div \frac{10}{9}$$

$$= \frac{-8}{9} \times \frac{9}{10}$$

$$= \frac{-4}{5}$$

$$\alpha\beta = \frac{x-1}{x} = \left(\frac{10}{9}-1\right) \div \frac{10}{9}$$

$$= \frac{1}{9} \times \frac{9}{10}$$

$$= \frac{1}{10}.$$ ⑤

$$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$$

$$=\left(\frac{-4}{5}\right)^2-2\times\frac{1}{10}$$

$$=\frac{16}{25}-\frac{1}{5}$$

$$=\frac{16-5}{25}=\underline{\underline{\frac{21}{25}}}$$

$$\alpha^2\beta^2=(\alpha\beta)^2$$

$$=\frac{1}{100}$$

$\therefore$ Eq^n is.

$$x^2-(\alpha^2+\beta^2)x+(\alpha\beta)^2=0$$

$$x^2-\frac{21}{25}x+\frac{1}{100}=0$$

$$\underline{\underline{100x^2-84x+1=0}} \quad 10$$

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{\alpha^2+\beta^2}{(\alpha\beta)^2}=\frac{21}{25}\times\frac{100}{1}=84.$$

$$\frac{1}{(\alpha\beta)^2}=100$$

$\therefore$ The Eq^n is

$$x^2-\left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}\right)x+\frac{1}{(\alpha\beta)^2}=0$$

$$\underline{\underline{x^2-84x+100=0.}} \quad 10 \qquad 60$$

b) $m^2(x^2 - x) + 2mx + 3 = 0$

$m^2x^2 - m^2x + 2mx + 3 = 0$

$m^2x^2 + (2m - m^2)x + 3 = 0$

$\alpha + \beta = \frac{-(2m - m^2)}{m^2} = \frac{m-2}{m}.$ 5

$\alpha\beta = \frac{3}{m^2}$ 5

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{4}{3} \Longrightarrow \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{4}{3}$

$3(\alpha^2 + \beta^2) = 4\alpha\beta.$

$3(\alpha + \beta)^2 - 6\alpha\beta = 4\alpha\beta$

$3(\alpha + \beta)^2 - 10\alpha\beta = 0$ 5

$3\left\{\left(\frac{m-2}{m}\right)^2 - 10 \cdot \frac{1}{m^2}\right\} = 0$ 5

$m^2 - 4m - 6 = 0$ 5

$m_1 + m_2 = 4$ and $m_1 m_2 = -6.$ 5

$\frac{m_1^2}{m_2} + \frac{m_2^2}{m_1} = \frac{m_1^3 + m_2^3}{m_1 m_2}.$

$= \frac{(m_1 + m_2)^3 - 3m_1 m_2 (m_1 + m_2)}{m_1 m_2}$ 5

$$= \frac{4^3 - 3(-6)4}{-6} \quad (5)$$

$$= \frac{-68}{3}$$

40

c) $f(x) = 2x^3 + rx^2 - 12x - 7$

$$= (x-\alpha)^2(2x+A). \quad (10)$$

$$= (x^2 - 2\alpha x + \alpha^2)(2x+A).$$

$$= 2x^3 + Ax^2 - 4\alpha x^2 - 2A\alpha x + 2\alpha^2 x + \alpha^2 A$$

$$= 2x^3 + (A-4\alpha)x^2 + (2\alpha^2 - 2A\alpha)x + \alpha^2 A$$

x^2 as $\quad A - 4\alpha = r$ — (1)

(15)

x as. $\quad 2\alpha^2 - 2A\alpha = -12$ — (2)

constant as $\quad \alpha^2 A = -7$ — (3)

(2) $\times \alpha$ + (3) $\times 2$

$$2\alpha^3 = -12\alpha - 14$$

$$\alpha^3 + 6\alpha + 7 = 0$$

$$(\alpha - 1)(\alpha^2 - \alpha - 7) = 0 \quad (5)$$

$\alpha = 1 \quad$ or $\quad \alpha^2 - \alpha - 7 = 0$

$\Delta = (-1)^2 - 4 \times 1(-7)$

$\# \quad \alpha \in \mathbb{Z}.$

$\therefore \alpha = 1 \quad (5)$

(3) $\Rightarrow$

$A = -7$

(1) $\Rightarrow$

$-7 - 4 = r$

$r = -11$ (5)

$\therefore$

$f(x) =$

$(x-1)^2(2x-7)$

(10)

50

12. a. Let $S_n = 1+r+r^2+\cdots+r^n$

$$rS_n = r+r^2+r^3+\cdots+r^{n+1}$$

$$S_n(r-1) = r^{n+1}-1$$

$$S_n = \frac{r^{n+1}-1}{r-1}$$

$$= \frac{1-r^{n+1}}{1-r}$$ 10

$$(1+x)+(1+x+x^2)+(1+x+x^2+x^3)+\cdots$$

$$= \frac{1-x^2}{1-x}+\frac{1-x^3}{1-x}+\frac{1-x^4}{1-x}+\cdots$$ 10

$$= \left(\frac{1}{1-x}\right)\left\{n-(x^2+x^3+x^4+\cdots x^{n+1})\right\}$$

$$= \left(\frac{1}{1-x}\right)\left\{n-x^2(1+x+x^2+\cdots x^{n-1})\right\}$$

$$= \left(\frac{1}{1-x}\right)\left\{n-\frac{x^2(1-x^n)}{1-x}\right\}$$ 25 35

b. $U_1 = \dfrac{1^2-1-1}{(1+1)!} = -\dfrac{1}{2}$

$$U_2 = \frac{2^2-2-1}{(2+1)!} = \frac{1}{6}$$

$$U_3 = \frac{3^2-3-1}{(3+1)!} = \frac{5}{4\times3\times2} = \frac{5}{24}$$

$$U_4 = \frac{4^2-4-1}{(4+1)!} = \frac{11}{5\times24} = \frac{11}{120}$$ 20

Also; $U_1 = \dfrac{\lambda}{0!} + \dfrac{\mu}{1!} + \dfrac{\delta}{2!}$

$-\dfrac{1}{2} = \dfrac{\lambda}{1} + \dfrac{\mu}{1} + \dfrac{\delta}{2}$

$-1 = 2\lambda + 2\mu + \delta$ —— (1) (5)

$U_2 = \dfrac{\lambda}{1!} + \dfrac{\mu}{2!} + \dfrac{\delta}{3!}$

$\dfrac{1}{6} = \dfrac{\lambda}{1} + \dfrac{\mu}{2} + \dfrac{\delta}{6}$

$1 = 6\lambda + 3\mu + \delta$ —— (2) (5)

$U_3 = \dfrac{\lambda}{2!} + \dfrac{\mu}{3!} + \dfrac{\delta}{4!}$

$\dfrac{5}{24} = \dfrac{\lambda}{2} + \dfrac{\mu}{6} + \dfrac{\delta}{24}$

$5 = 12\lambda + 4\mu + \delta$ —— (3) (5)

(2) $-$ (1) $\Rightarrow$ $2 = 4\lambda + \mu$ —— (A)

(3) $-$ (2) $\Rightarrow$ $4 = 6\lambda + \mu$ —— (B)

(B) $-$ (A) $\Rightarrow$ $2 = 2\lambda$ $\Rightarrow$ $\lambda = 1$ (5)

$\therefore$ $\mu = -2$ (5)

$\therefore$ $\delta = 5 - 12 + 8$

$\delta = 1$ (5)

$\therefore$ $\lambda = 1,\ \mu = -2,\ \delta = 1$

Now $U_r = \dfrac{1}{(r-1)!} - \dfrac{2}{r!} + \dfrac{1}{(r+1)!}$ (5)

when $r = 4$, $U_4 = \dfrac{1}{3!} - \dfrac{2}{4!} + \dfrac{1}{5!}$

$$u_4 = \frac{1}{6} - \frac{2}{24} + \frac{1}{120}$$

$$= \frac{20 - 10 + 1}{120}$$

$$= \frac{11}{120} \quad (5)$$

$\therefore$ it is true for $n = 4$.

$$U_r = \frac{1}{(r-1)!} - \frac{2}{r!} + \frac{1}{(r+1)!}$$

for $r=1$, $\quad u_1 = \frac{1}{0!} - \frac{2}{1!} + \frac{1}{2!}$

for $r=2$, $\quad U_2 = \frac{1}{1!} - \frac{2}{2!} + \frac{1}{3!} \quad (10)$

$r=3$, $\quad U_3 = \frac{1}{2!} - \frac{2}{3!} + \frac{1}{4!}$

$\vdots$

$r=n-2 \quad U_{n-2} = \frac{1}{(n-3)!} - \frac{2}{(n-2)!} + \frac{1}{(n-1)!}$

$r=n-1 \quad u_{n-1} = \frac{1}{(n-2)!} - \frac{2}{(n-1)!} + \frac{1}{n!} \quad (5)$

$r=n \quad u_n = \frac{1}{(n-1)!} - \frac{2}{n!} + \frac{1}{(n+1)!}$

$$\sum_{r=1}^{n} U_r = 1 - 2 + 1 + \frac{1}{n!} - \frac{2}{n!} + \frac{1}{(n+1)!} \quad (5)$$

$$= \frac{1}{(n+1)!} - \frac{1}{n!}$$

$$= \frac{1 - (n+1)}{(n+1)!}$$

$$= \frac{-n}{(n+1)!} \quad (10)$$

$\frac{70}{70}$

$$\sum_{r=1}^{n} u_r = \frac{-n}{n!\,(n+1)} = \frac{-1}{n!}\left(\frac{n}{n+1}\right)$$

$$= \frac{-1}{n!}\left(\frac{1}{1+\frac{1}{n}}\right)$$

$$\lim_{n\to\infty} \sum_{r=1}^{n} u_r = \lim_{n\to\infty} \frac{-1}{n!}\left(\frac{1}{1+\frac{1}{n}}\right)$$

$$= \frac{-1}{\infty}\left(\frac{1}{1+\frac{1}{\infty}}\right)$$

$$= 0 \times 1$$

$$= 0 \quad (10)$$

$\therefore$ the series is convergent. (5)

15

13. (a) ANURADHAPURA

A	N	U	R	D	H	P	
4	1	2	2	1	1	1	$\rightarrow 12$

$$\frac{12!}{4!\,2!\,2!} = \frac{\cancel{12}^{3}\times 11\times 10\times 9\times 8\times 7\times 6\times 5\times \cancel{4!}}{\cancel{4!}\times \cancel{2}\times 2}$$

$= \underline{\underline{4\ 989\ 600}}$ ← No. of all permutations. (15)

$$\frac{9!}{2!\,2!} = \frac{9\times 8\times 7\times 6\times 5\times \cancel{4}^{2}\times 3\times \cancel{2!}}{\cancel{2!}\times 2}$$

$= \underline{\underline{90\ 720}}$ ← No. of Ps taking 4 As as 1 letter. (20)

$$\frac{8!}{2!\cdot 2!} = \underline{\underline{10\ 080}}$$ ← No. of Ps when 4 As are at the beginning. (20)

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(b) Dramas − 3 ⟶ 2
Songs − 6 ⟶ 4
Dances − 4 ⟶ $\frac{3}{9}$

(i). $9! = \underline{\underline{362\ 880}}$ (10)

(ii) $3! \times 2! \times 4! \times 3! = 36 \times 48$
$= \underline{\underline{1728}}$ (15)

(iii) $3! = \underline{\underline{6}}$ (15)

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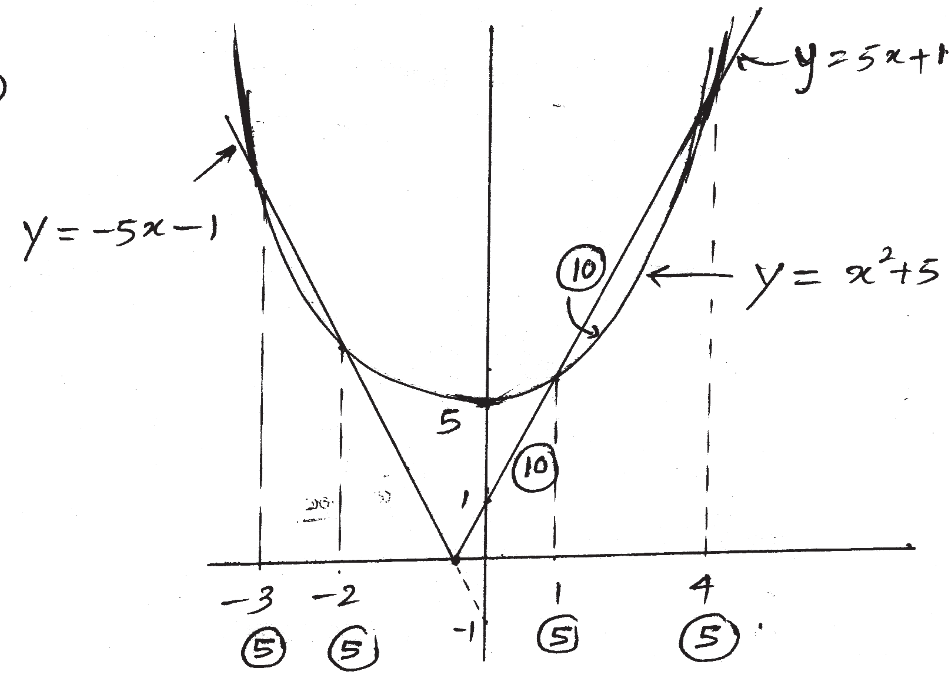
(13) (c)



$$-5x-1 = x^2+5$$
$$x^2+5x+6 = 0$$
$$x = -3\ ,\ x = -2$$

$$5x+1 = x^2+5$$
$$x^2-5x+4 = 0$$
$$(x-4)\ (x-1) = 0$$
$$x = 4\ ,\ x = 1$$

$$\underline{(-\infty, -3) \cup (-2, 1) \cup (4, \infty)}\ \ 15$$

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14) a) for $x \neq 2$; $f(x) = \dfrac{1}{(x-2)(x^2+1)}$

$$\frac{dy}{dx} = \frac{(x-2)(x^2+1)\times 0 - 1\{(x-2)2x + (x^2+1)\}}{(x-2)^2(x^2+1)^2}$$

$$\frac{dy}{dx} = \frac{-2x^2+4x - x^2-1}{(x-2)^2(x^2+1)^2}$$

$$\frac{dy}{dx} = \frac{-3x^2+4x-1}{(x-2)^2(x^2+1)^2}$$

$$= -\left\{\frac{(3x-1)(x-1)}{(x-2)^2(x^2+1)^2}\right\} \quad 25$$

When $x = \frac{1}{3}$ and $x = 1$; $f'(x) = 0$ 10

Then, $f(x) = \dfrac{1}{(x-2)(x^2+1)}$

$$f\left(\tfrac{1}{3}\right) = \frac{-27}{30}$$

$$f(1) = -\frac{1}{2}$$

$\therefore \left(\frac{1}{3}, -\frac{27}{30}\right)$ and $\left(1, -\frac{1}{2}\right)$ are turning points.

There is a verticle asymptote ; $x = 2$. 5

	$(-\infty, \frac{1}{3})$	$x=\frac{1}{3}$	$(\frac{1}{3}, 1)$	$x=1$	$(1,2)$	$(2,\infty)$
$f'(x)$.	$-$	0	$+$	0	$-$	$-$

(20)

$\therefore \left(\frac{1}{3}, -\frac{27}{30}\right) \leftarrow$ Local minimum. (10)

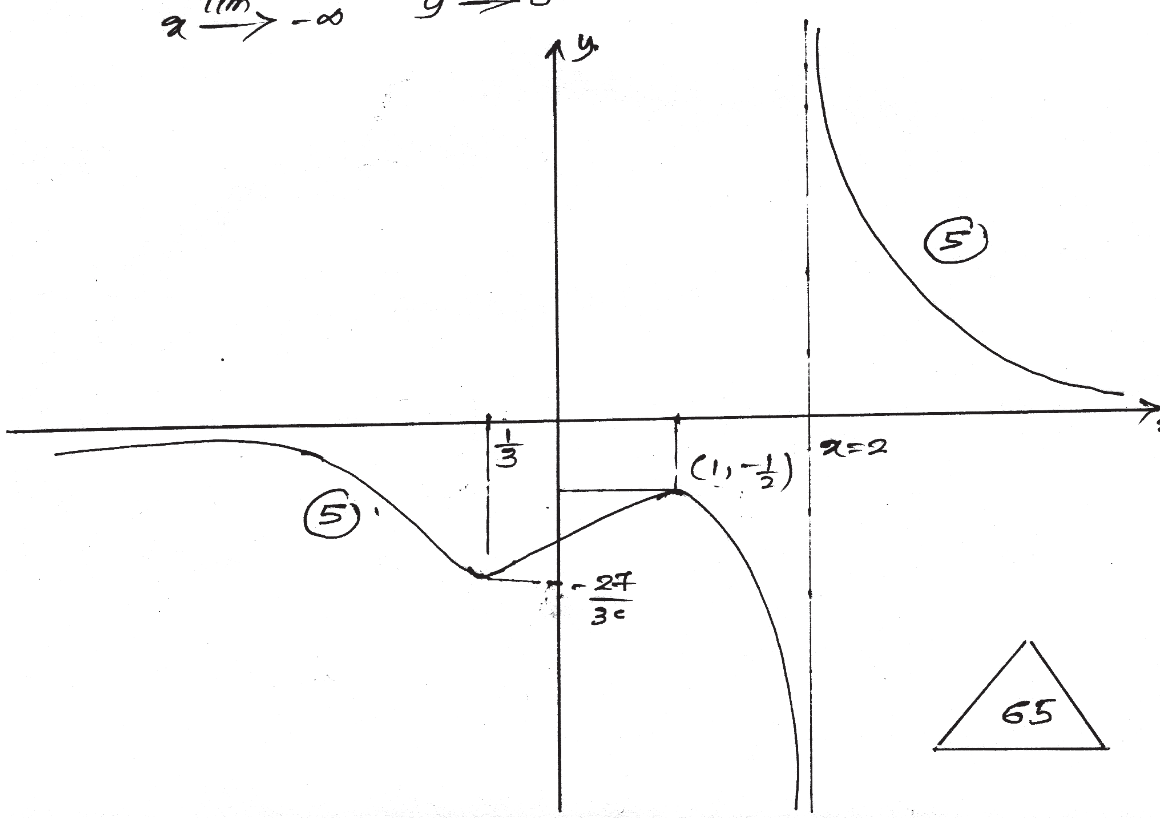
$\left(1, -\frac{1}{2}\right) \leftarrow$ Local maximum.

$\lim_{x \to 2^+} y \to +\infty$ (5)

$\lim_{x \to 2^-} y \to -\infty$.

$\lim_{x \to \infty} y \to 0$ (5)

$\lim_{x \to -\infty} y \to 0$.



65

b) $V = \frac{1}{2}\pi r^2 h$

$$S = \pi r^2 + 2rh + \frac{2\pi r h}{2}$$

$$= \pi r^2 + 2rh + \pi r h.$$

$$= \pi r^2 + (2+\pi) r \cdot \frac{2V}{\pi r^2}$$

$$= \pi r^2 + (2+\pi) \frac{2V}{\pi r}.$$

$$\frac{dS}{dr} = 2\pi r + \frac{2(2+\pi)V}{\pi}(-1)r^{-2}$$

$$= 2\pi r + \frac{2V(2+\pi)}{\pi} \cdot \left(\frac{-1}{r^2}\right)$$

For maximum or minimum S

$$\frac{dS}{dr} = 0$$

$$2\pi r - \frac{2V(2+\pi)}{\pi r^2} = 0.$$

$$\cancel{2}\pi r = \frac{\cancel{2}V(2+\pi)}{\pi r^2}$$

$$r^3 = \frac{V(2+\pi)}{\pi^2}$$

$$V = \frac{\pi^2 r^3}{(2+\pi)}$$

$$\frac{\pi^2 r^3}{(2+\pi)} = \frac{\pi r^2 h}{2}.$$

$$\frac{\pi r}{(2+\pi)} = \frac{h}{2}$$

$$\frac{r}{h} = \frac{2+\pi}{2\pi}$$

$$r : h = (2+\pi) : 2\pi$$

60

15) a) $$\frac{1}{(t-9)(t-4)} = \frac{A}{t-9} + \frac{B}{t-4}. \quad 5$$

$$1 \equiv A(t-4) + B(t-9)$$

When $t=4$.

$$1 = B(-5)$$

$$B = \frac{-1}{5}.$$

When $t=9$

$$1 = A(9-4)$$

$$A = \frac{1}{5} \quad 10$$

$$\frac{1}{(t-9)(t-4)} = \frac{1}{5(t-9)} - \frac{1}{5(t-4)} \quad 15$$

$$\int \frac{1}{(t-9)(t-4)}\,dt = \int \frac{1}{5(t-9)}\,dt - \int \frac{1}{5(t-4)}\,dt$$

$$= \frac{1}{5}\ln|t-9| \; 5 \; - \frac{1}{5}\ln|t-4| \; 5$$

$$= \frac{1}{5}\ln\left|\frac{t-9}{t-4}\right| + c \quad 5. \qquad 15$$

When $t=x^2$

$$\frac{dt}{dx} = 2x.$$

$$\int \frac{1}{(x^2-9)(x^2-4)} \cdot 2x\,dx = \frac{1}{5}\ln\left|\frac{x^2-9}{x^2-4}\right| + c$$

$$2\int \frac{x}{(x^2-9)(x^2-4)}\,dx = \frac{1}{5}\ln\left|\frac{x^2-9}{x^2-4}\right|.$$

$$\int \frac{x}{(x^2-9)(x^2-4)}\,dx = \frac{1}{10}\ln\left|\frac{x^2-9}{x^2-4}\right| + c \quad 15$$

b) $$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx.$$ 10

$$I = \int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x}\,dx \qquad J = \int_{-\pi}^{\pi} \frac{a^x \cos^2 x}{1+a^x}\,dx$$

$$I + J = \int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x}\,dx + \int_{-\pi}^{\pi} \frac{a^x \cos^2 x}{1+a^x}\,dx$$

$$= \int_{-\pi}^{\pi} \frac{(1+a^x)\cos^2 x}{(1+a^x)}\,dx \quad (5)$$

$$= \int_{-\pi}^{\pi} \cos^2 x\,dx.$$

$$= \int_{-\pi}^{\pi} \frac{1+\cos 2x}{2}\,dx \quad (5)$$

$$= \frac{1}{2}\left\{x + \frac{\sin 2x}{2}\right\}_{-\pi}^{\pi}$$

$$= \frac{1}{2}\left\{\pi - (-\pi) + \frac{1}{2}(\sin 2\pi + \sin 2\pi)\right\}$$

$$= \frac{1}{2} \cdot 2\pi$$

$$= \underline{\underline{\pi}} \quad (5)$$

$$I = \int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x}\,dx$$

$$= \int_{-\pi}^{\pi} \frac{\cos^2(-x)}{1+a^{-x}}\,dx \quad (5)$$

$$= \int_{-\pi}^{\pi} \frac{a^x \cos^2 x}{1+a^2}\,dx. = J. \quad (5)$$

$$I + J = \pi \rightsquigarrow (1)$$

$$I = J \rightsquigarrow (2)$$

(1) and (2) $\quad I = J = \dfrac{\pi}{2}$ (5)

$$I_1 = \int_{-\pi}^{\pi} \frac{\sin^2 x}{1+a^x}\,dx \qquad J_1 = \int_{-\pi}^{\pi} \frac{a^x \sin^2 x}{1+a^x}\,dx.$$

$$I_1 + J_1 = \int_{-\pi}^{\pi} \sin^2 x \, dx.$$

$$= \int_{-\pi}^{\pi} \frac{1-\cos 2x}{2}\,dx$$

$$= \frac{1}{2}\left\{ x - \frac{\sin 2x}{2} \right\}_{-\pi}^{\pi}$$

$$= \frac{1}{2}\left\{ \left(\pi - \frac{\sin 2\pi}{2}\right) - \left[(-\pi) - \frac{\sin 2\pi}{2}\right] \right\}$$

$$= \pi \rightsquigarrow (3) \quad (5)$$

$$I_1 = \int_{-\pi}^{\pi} \frac{\sin^2 x}{1+a^x}\,dx$$

$$= \int_{-\pi}^{\pi} \frac{\sin^2(-x)}{1+a^{-x}}\,dx$$

$$= \int_{-\pi}^{\pi} \frac{a^x \sin^2 x}{1+a^x}\,dx$$

$$= J_1 \rightsquigarrow (4) \quad (5)$$

③ and ④

$$I_1 = J_1 = \frac{\pi}{2} \quad ⑤$$

$$\int_{-\pi}^{\pi} \frac{a^x \cos^2 x}{1+a^x}\,dx = \pi/2 \longrightarrow Ⓐ$$

$$\int_{-\pi}^{\pi} \frac{a^x \sin^2 x}{1+a^x}\,dx = \pi/2 — Ⓑ$$

$$Ⓐ + Ⓑ \quad \int_{-\pi}^{\pi} \frac{a^2(\sin^2 x + \cos^2 x)}{1+a^x}\,dx = \pi \quad ⑤$$

$$\int_{-\pi}^{\pi} \frac{a^x}{1+a^x}\,dx = \pi \quad ⑤$$

55

c) $$\int x^2 \cos x\,dx = \int x^2 \frac{d}{dx}\sin x\,dx \quad ⑤$$

$$= x^2 \sin x - \int \sin x \cdot 2x\,dx \quad ⑤$$

$$= x^2 \sin x + 2\int x \cdot \frac{d}{dx}\cos x\,dx \quad ⑤$$

$$= x^2 \sin x + 2\left\{x\cos x - \int \cos x\,dx\right\} \quad ⑤$$

$$= x^2 \sin x + 2x\cos x - 2\sin x + C \quad ⑤$$

25

$$\int_0^{\pi/2} x^2 \cos x\,dx = \left\{x^2 \sin x + 2x\cos x - 2\sin x\right\}_0^{\pi/2}$$

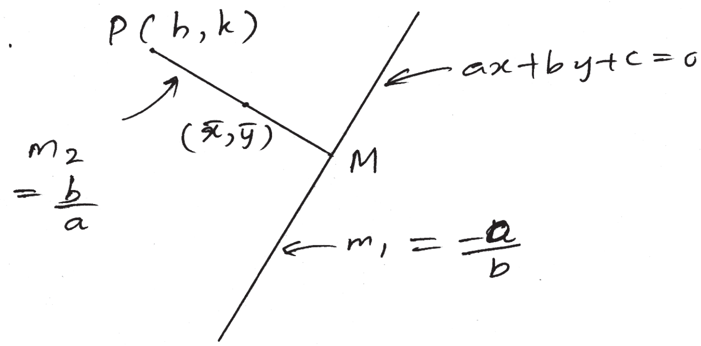
$$= \left\{\frac{\pi^2}{4}\sin\frac{\pi}{2} + 2\cdot\frac{\pi}{2}\cos\frac{\pi}{2} - 2\sin\frac{\pi}{2}\right\} - \{-2\sin 0\}$$

$$= \frac{\pi^2}{4} - 2 + 0 = \frac{\pi^2 - 8}{4}$$

$$\int_0^{\pi/2} x^2 \cos x\,dx = \frac{\pi^2 - 8}{4}$$

15

(16) (a).



$$\frac{\bar{y}-k}{\bar{x}-h} = \frac{b}{a} \Rightarrow \frac{\bar{y}-k}{b} = \frac{\bar{x}-h}{a} = t$$

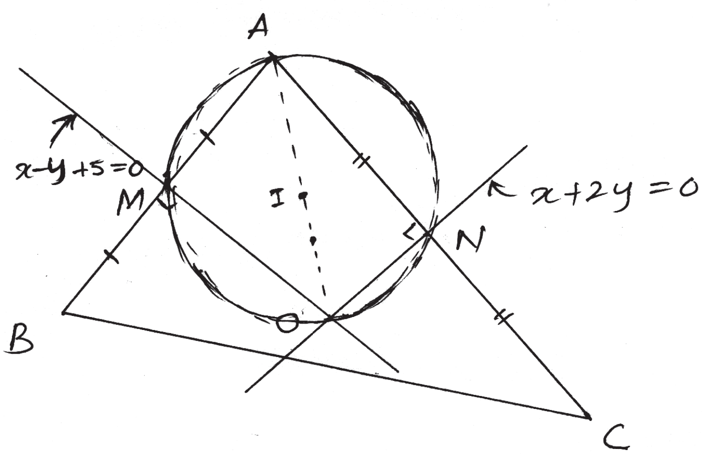
where t is a parametre.

Then $\bar{x} = at+h$ and $\bar{y} = bt+k$

$\therefore (\bar{x}, \bar{y}) = (at+h, bt+k)$

$\therefore$ The coordinates of any point on PM is given as $\underline{(at+h, bt+k)}$

20



Any point on AB can be written as $(t+1, -t-2)$. When it is the mid-point M of AB, it satisfies the equation $x-y+5=0$.

Then $t+1+t+2+5=0 \Rightarrow 2t=-8$

$\Rightarrow t=-4$.

$\therefore M \equiv (-3, 2)$

In the same manner, any point on AC can be written as $(t+1,\ 2t-2)$. When it is the mid-point (N) of AC, it satisfies the equation $x+2y=0$.

$$\therefore\ t+1+2(2t-2)=0$$

$$\Rightarrow\ 5t-3=0\ \Rightarrow\ t=3/5.$$

$$\therefore\ N=\left(\frac{3}{5}+1,\ 2\times\frac{3}{5}-2\right)$$

$$=\left(\frac{8}{5},\ \frac{-4}{5}\right)$$

Now, let's find B. Let $B=(x_B, y_B)$

Then, $(-3,2)=\left(\frac{1+x_B}{2},\ \frac{-2+y_B}{2}\right)$

$$x_B=-6-1 \qquad y_B=4+2$$

$$=-7 \qquad\qquad =6$$

$$\therefore\ B\equiv(-7,6)$$

Let's find C; let $C\equiv(x_c, y_c)$.

Then $\left(\frac{8}{5},\frac{-4}{5}\right)=\left(\frac{1+x_c}{2},\ \frac{-2+y_c}{2}\right)$

$$\Rightarrow\ 16=5+5x_c \qquad -8=-10+5y_c$$

$$\Rightarrow x_c=\frac{11}{5} \qquad \Rightarrow y_c=\frac{2}{5}$$

$$\therefore\ C\equiv\left(\frac{11}{5},\frac{2}{5}\right)$$

Then equation of BC is

$$\frac{y-6}{x+7}=\left(\frac{6-\frac{2}{5}}{-7-\frac{11}{5}}\right)=\left(\frac{\frac{28}{5}}{-\frac{46}{5}}\right)$$

$$= \frac{-28}{46} = \frac{-14}{23}$$

$$23y - 138 = -14x - 98$$

$$\underline{\underline{14x + 23y - 40 = 0}}$$

Coordinates of O,

$$x + 2y = x - y + 5$$

$$\Rightarrow \quad 3y = 5 \quad \Rightarrow \quad y = \frac{5}{3}$$

Then, $\quad x = -2 \times \frac{5}{3} = \frac{-10}{3}$

$$\therefore O \equiv \left(\frac{-10}{3}, \frac{5}{3}\right)$$

The centre I of the circle passing through A, M, O and N is the mid-point of AO.

$$\therefore I \equiv \left(\frac{1 - \frac{10}{3}}{2}, \frac{-2 + \frac{5}{3}}{2}\right) = \left(\frac{-7}{6}, \frac{-1}{6}\right)$$

Radius (r) of this circle is $\frac{1}{2}$ OA.

$$\therefore r = \frac{1}{2}\sqrt{\left(1 + \frac{10}{3}\right)^2 + \left(-2 - \frac{5}{3}\right)^2}$$

$$= \frac{1}{2}\sqrt{\left(\frac{13}{3}\right)^2 + \left(\frac{-11}{3}\right)^2}$$

$$= \frac{1}{6}\sqrt{169 + 121}$$

$$= \frac{1}{6}\sqrt{290}.$$

$\therefore$ The equation of that circle is

$$\left(x+\frac{7}{6}\right)^2+\left(y+\frac{1}{6}\right)^2 = \frac{290}{36} = \frac{145}{18}$$

$$x^2+\frac{7}{3}x+\frac{49}{36}+y^2+\frac{1}{3}y+\frac{1}{36} = \frac{145}{18}$$

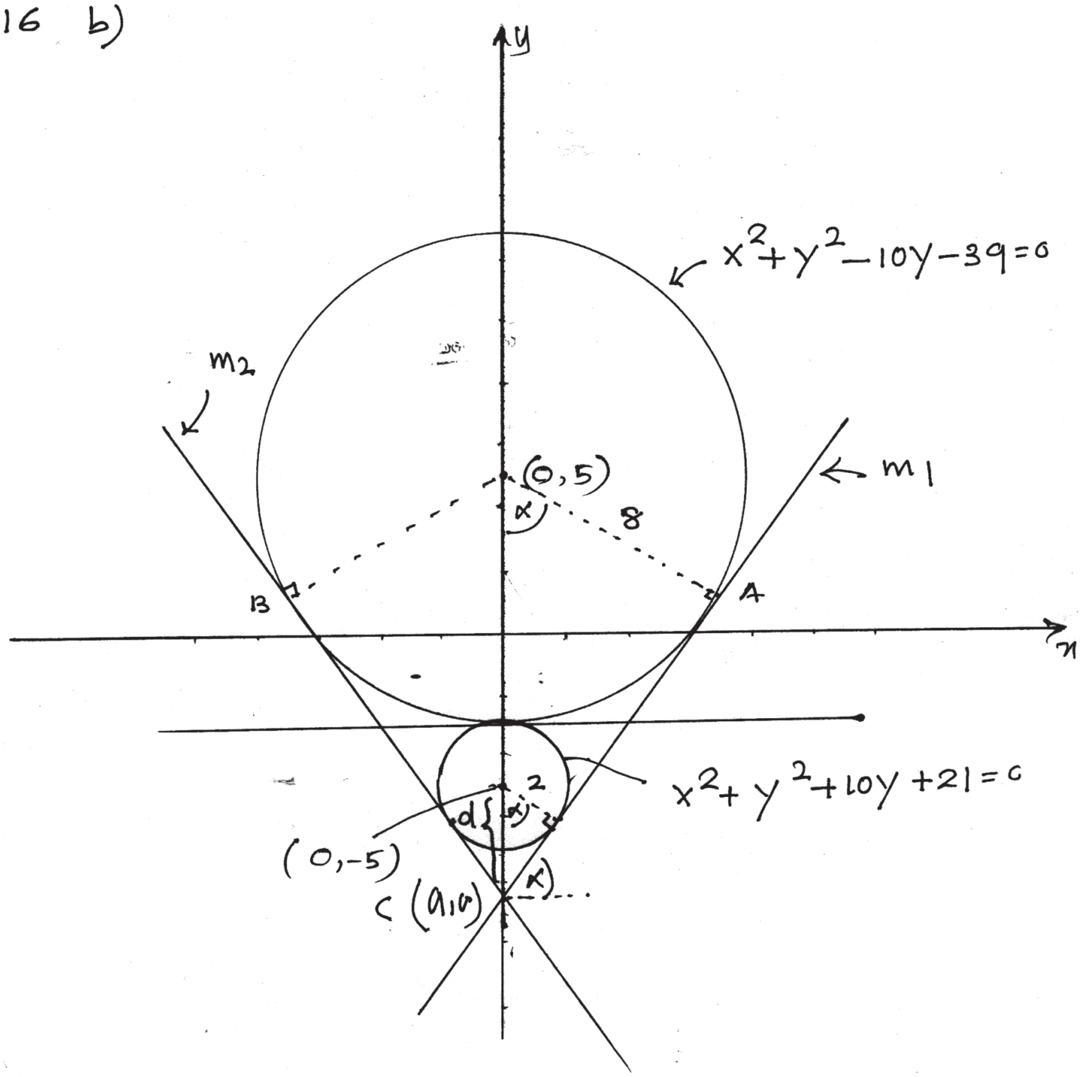
$$x^2+y^2+\frac{7}{3}x+\frac{1}{3}y+\frac{25}{18}-\frac{145}{18} = 0$$

$$x^2+y^2+\frac{7}{3}x+\frac{1}{3}y-\frac{120}{18} = 0$$

$$\underline{\underline{x^2+y^2+\frac{7}{3}x+\frac{1}{3}y-\frac{20}{3} = 0}}$$

70

16 b)



$x^2+y^2-10y-39=0$

$(0,5)$ - Center

$r=\sqrt{5^2+39}$

$=8$.

$x^2+y^2+10y+21=0$

$(0,-5)$ - Center

$r=\sqrt{5^2-21}$

$=2$.

$$\frac{8}{2}=\frac{10+d}{d}$$

$$4d=10+d$$

$$d=\underline{\underline{10/3}}$$

$$\cos\alpha=\frac{8}{10+\frac{10}{3}}$$

$$=\frac{8\times 3}{40}$$

$$=\frac{3}{5}$$

$m_1 = \tan\alpha$ $\quad c = \frac{-25}{3}$ $\quad a = -5 - \frac{10}{3}$

$= 4/3.$ $\quad\quad = \frac{-25}{3}$

$y = \frac{4}{3}x - \frac{25}{3}$ $\quad\quad c \equiv (0, -\frac{25}{3})$

$\underline{\underline{3y - 4x + 25 = 0}}$

$m_2 = \tan(180 - \alpha)$ $\quad c = -\frac{25}{3}$

$= -\tan\alpha$

$= \frac{-4}{3}$

$y = \frac{-4}{3}x - \frac{25}{3}$

$\underline{\underline{3y + 4x + 25 = 0}}$

$\therefore$ The Eq^n of the tangent

$3y - 4x + 25 = \underline{\underline{0}}$

$3y + 4x + 25 \underline{\underline{= 0}}$

$\underline{\underline{y = -3}}$.

Let $A \equiv (x_1\, y_1)$

$y_1 = 5 - 8\cos\alpha$ $\quad\quad x_1 = 8\sin\alpha$

$= 5 - 8 \times \frac{3}{5}$ $\quad\quad = 8 \times \frac{4}{5}$

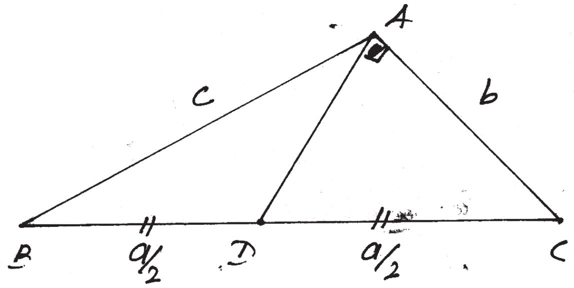
$= \underline{\underline{\frac{1}{5}}}$ $\quad\quad = \frac{32}{5}$

$\therefore A = \underline{\underline{(\frac{32}{5}, \frac{1}{5})}}$

Similarly

$\underline{\underline{B}} \equiv (-\frac{32}{5}, \frac{1}{5})$ 60

17) a. Sin rule $\boxed{20}$



In ΔABD, by sine formula

$$\frac{AD}{\sin B} = \frac{BD}{\sin(A-90°)}$$

$$AD = \frac{-a\sin B}{2\cos A} \quad \text{—①}$$

In ΔADC, $\frac{AD}{DC} = \sin C$

$$AD = \frac{a}{2}\sin C \quad \text{—②}$$

① = ② $\quad \frac{a\sin B}{2\cos A} = \frac{a}{2}\sin C$

$$\sin C\cos A = -\sin B$$

$$ck\left(\frac{b^2+c^2-a^2}{2bc}\right) = -kb$$

$$b^2+c^2-a^2 = -2b^2$$

$$\underline{\underline{3b^2 = a^2-c^2}}$$

$$\cos A\cos B = \left(\frac{b^2+c^2-a^2}{2bc}\right)\left(\frac{a^2+c^2-b^2}{2ac}\right)$$

$$= \frac{1}{4abc^2}(b^2-3b^2)(b^2+3b^2)$$

$$= \underline{\underline{\frac{-2b^2}{ac}}} = \frac{-2}{ac}\left(\frac{a^2-c^2}{3}\right) = \underline{\frac{2(c^2-a^2)}{3ac}} \quad \boxed{80}$$

b) $$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}.$$

$$\tan\theta + \tan\left(\theta + \frac{\pi}{3}\right) + \tan\left(\theta + \frac{2\pi}{3}\right) = 3$$

$$\tan\theta + \frac{\tan\theta + \sqrt{3}}{1 - \sqrt{3}\tan\theta} + \frac{\tan\theta - \sqrt{3}}{1 + \sqrt{3}\tan\theta} = 3$$

$$\frac{\tan\theta(1 - \sqrt{3}\tan\theta)(1 + \sqrt{3}\tan\theta) + (\tan\theta + \sqrt{3})(1 + \sqrt{3}\tan\theta) + (\tan\theta - \sqrt{3})(1 - \sqrt{3}\tan\theta)}{1 - 3\tan^2\theta} = 3$$

$$\frac{\tan\theta(1 - 3\tan^2\theta) + \sqrt{3}\tan^2\theta + 4\tan\theta + \sqrt{3} - \sqrt{3}\tan^2\theta + 4\tan\theta - \sqrt{3}}{1 - 3\tan^2\theta} = 3$$

$$\tan\theta - 3\tan^3\theta + 8\tan\theta = 3 - 9\tan^2\theta$$

$$\cancel{\tan} \quad 9\tan\theta - 3\tan^3\theta = 3 - 9\tan^2\theta$$

$$\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta} = 1$$

$$\tan 3\theta = 1$$

$$\tan 3\theta = \tan \pi/4$$

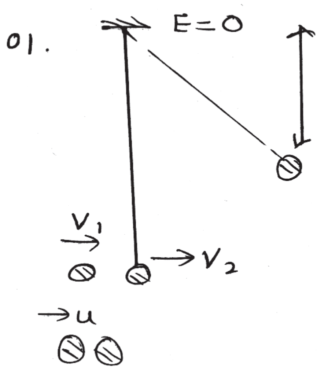
$$3\theta = n\pi + \frac{\pi}{4}.$$

$$\theta = \frac{n\pi}{3} + \frac{\pi}{12} \; ; n \in \mathbb{Z}$$

50

Part A

01.



By the principle of conservation of linear momentum.

$$mV_2 + mV_1 = mu$$

$$V_2 + V_1 = u \quad 5$$

Newton's Law of restitution

$$V_2 - V_1 = -e[0-u]$$

$$V_2 - V_1 = eu \quad 5$$

$$V_2 = \frac{u}{2}(1+e)$$

Using the principle of Conservation of energy

$$\frac{1}{2}mV_2^2 - mgl = -mg\frac{l}{2} \quad 5$$

$$\frac{1}{2}V_2^2 = gl/2$$

$$V_2 = \sqrt{gl} \quad 5$$

$$u = \frac{4}{3}\sqrt{gl}$$

$$\frac{4}{3}\sqrt{gl} \times \frac{1}{2}(1+e) = \sqrt{gl}$$

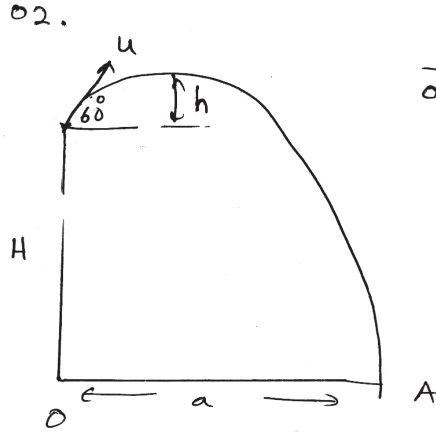
$$2 + 2e = 3$$

$$2e = 1$$

$$e = \frac{1}{2} \quad 5$$

25

02.



$\overrightarrow{OA}$ $S = ut$

$$a = \frac{u}{2}t \Rightarrow t = \frac{2a}{u} \quad 5$$

OA $\uparrow$ $s = ut + \frac{1}{2}at^2$

$$-H = \frac{u\sqrt{3}}{2} \times \frac{2a}{u} - \frac{1}{2}g\frac{4a^2}{u^2} \quad 5$$

$$-H = \sqrt{3}a - \frac{2ga^2}{u^2}$$

$$u^2 = \frac{2ga^2}{\sqrt{3}a + H} \quad 5$$

From O ↑ $\quad V^2 = u^2 + 2as$

$$0 = u^2 \times \frac{3}{4} - 2gh \quad (5)$$

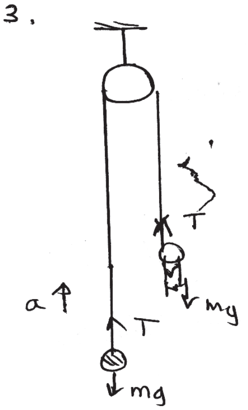
$$h = \frac{3u^2}{8g}$$

$$h = \frac{3}{8g}\left(\frac{2ga^2}{\sqrt{3}a + H}\right)$$

$$h = \frac{3a^2}{4(\sqrt{3}a + H)} \quad (5)$$

25

03.



Child - C

String - R

$a_{R,E} = \uparrow a$

$a_{C,R} = \uparrow f$

$a_{C,E} = a_{C,R} + a_{R,E}$

$= \uparrow f + \uparrow a \quad (5)$

$F = ma$

(M) $\uparrow \; T - Mg = Ma \quad — (1) \quad (5)$

$\downarrow \; mg - T = m(a - f) \quad — (2) \quad (5)$

(1)+(2) $\quad mg - Mg = (M+m)a - mf \quad (5)$

$$\frac{(m-M)g + mf}{M+m} = a \quad (5)$$

25

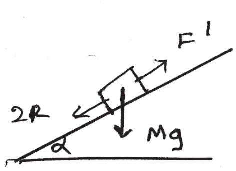
04.

$P = FV$

$$\frac{1000H}{V} = F \quad (5)$$

$\rightarrow \quad F - R = 0$

$$\therefore R = \frac{1000H}{V} N \quad (5)$$



$\nearrow \quad F' - 2R - Mg\sin\alpha = 0 \quad (5)$

$$F' = \frac{2000H}{V} + Mg\sin\alpha \quad (5)$$

$P = FV$

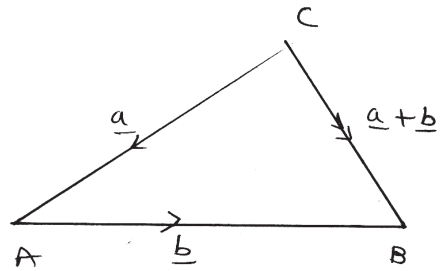
$$1000H = F'\frac{V}{3}$$

$$3000H = \left(\frac{2000H}{V} + Mg\sin\alpha\right)V \quad (5)$$

$1000H = MgV\sin\alpha$

25

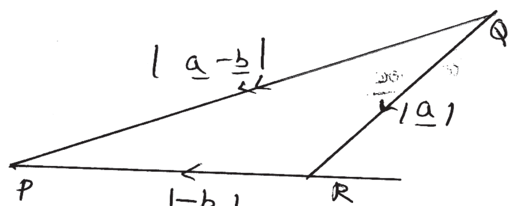
05.



$|\underline{a}| = |\underline{b}| = |\underline{a}+\underline{b}|$ (5)

ABC is an equilateral triangle. (5)

Angle between $\underline{a}$ and $\underline{b}$ is 120°. (5)



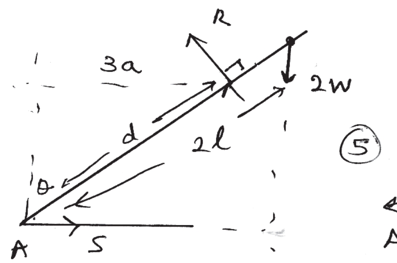
PQR is an equilateral triangle. (5)

$P\hat{R}Q = 60^\circ$

angle between $\underline{a}$ and $\underline{b}$ is 60° (5)

25

06.



(5)

$\uparrow \quad R \sin\theta = 2W$

$R = \dfrac{2W}{\sin\theta}$ (5)

$\sin\theta = \dfrac{3a}{d}$

$d = \dfrac{3a}{\sin\theta}$

A) $\quad R \times \dfrac{3a}{\sin\theta} = 2W \times 2l \sin\theta$ (10)

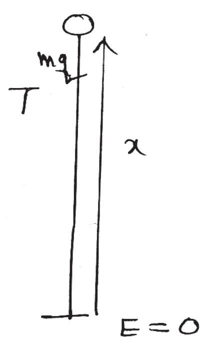
$\dfrac{2W}{\sin\theta} \times \dfrac{3a}{\sin\theta} = 4Wl\sin\theta$

$\sin^3\theta = \dfrac{3a}{2l}$ (5)

$\sin\theta = \left(\dfrac{3a}{2l}\right)^{1/3}$

25

07.



$\frac{1}{2} m \cdot 3gl$ (5) $= \frac{1}{2} m\dot{x}^2$ (5) $+ \frac{1}{2} \frac{mg(x-l)^2}{l}$ (5) $+ mgx$

$3gl = \dot{x}^2 + \dfrac{g(x-l)^2}{l} + 2gx$ (5)

$\dot{x}^2 = 3gl - \dfrac{g}{l}(x-l)^2 - 2gx$ (5)

25

08. $P(A \cap B') = P(A) - P(A \cap B)$ (5)

$$\frac{5}{12} = \frac{2}{3} - P(A \cap B)$$

$$P(A \cap B) = \frac{2}{3} - \frac{5}{12} = \frac{3}{12} = \frac{1}{4} \quad (5)$$

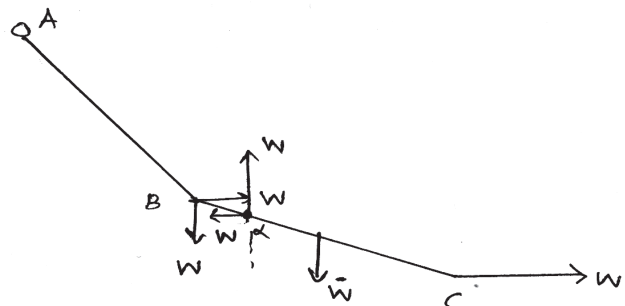
$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad (5)$$

$$= \frac{2}{3} + \frac{1}{2} - \frac{1}{4} \quad (5)$$

$$P(A \cup B) = \frac{11}{12} \quad (5)$$

25

09.



(5)

$B \circlearrowleft$ $W \times 2a \cos\alpha - Wa \sin\alpha = 0$ (10)

$$2 \cos\alpha = \sin\alpha$$

$$2 = \tan\alpha \quad (5)$$

The reaction on BC $\sqrt{2}W$ W W (5)

25

10.

Body	mass	distance to the centre of mass from O
Cone	$\frac{1}{3}\pi a^2 4a\rho$	$\frac{3}{4} \times 4a = 3a$ (5)
Cylinder	$\pi(2a)^2 4a\rho$	$6a$ (5)
Composite body	$\frac{52}{3}\pi a^3\rho$	$\bar{x}$

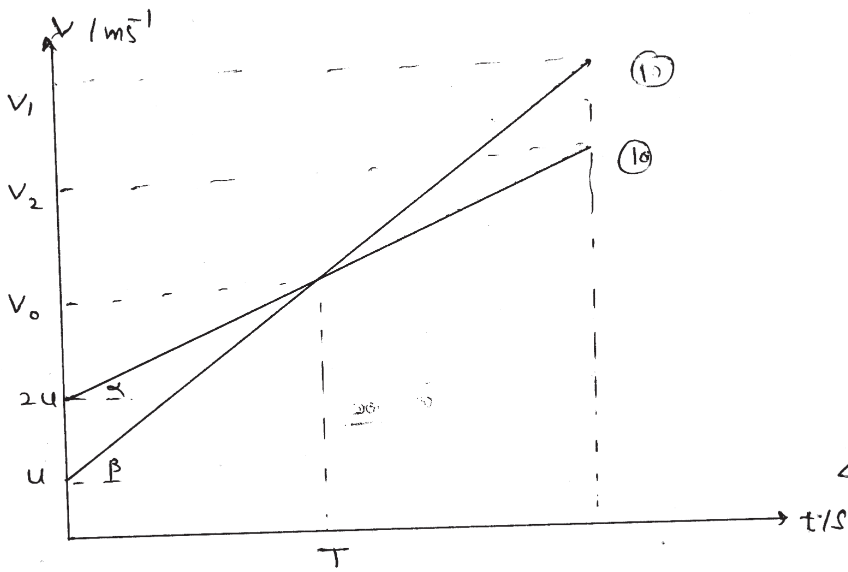
$$\frac{52\pi a^3\rho\,\bar{x}}{3} = \frac{4}{3}\pi a^3\rho \times 3a + 16\pi a^3\rho \times 6a \quad (10)$$

$$\bar{x} = 100a \times \frac{3}{52}$$

$$\bar{x} = \frac{75a}{13} \quad (5)$$

25

(11)



(20)

$\tan\alpha = a$ $\qquad\qquad$ $\tan\beta = 2a$

$a = \dfrac{V_2 - 2u}{t}$ (5) $\qquad\qquad$ $\dfrac{V_1 - u}{t} = 2a$ (5)

$V_2 = 2u + at$ $\qquad\qquad$ $V_1 = u + 2at$

$\tan\alpha = \dfrac{V_0 - 2u}{T}$ (5) $\qquad\qquad$ $\tan\beta = \dfrac{V_0 - u}{T} = 2a$ (5)

$a = \dfrac{V_0 - 2u}{T}$ $\qquad\qquad$ $V_0 = u + 2aT$

$2u + aT = u + 2aT$ (5)

$T = \dfrac{u}{a}$

(25)

Since the distances travelled by both cars with in the th- time are equal.

$\frac{1}{2}\, uT = \frac{1}{2}(V_1 - V_2)(t - T)$ (10)

$uT = (u + 2at - 2u - at)(t - T)$

$uT = (at - u)(t - T)$

$uT = at^2 - atT - ut + uT$

$t = 0,\quad t\,(at - u - aT) = 0$ (10) $\qquad aT = u$

$t = \dfrac{u + aT}{a}$

(20)

$t = \dfrac{2u}{a}$

$t = 2T$

The distance travelled by cars $= \frac{1}{2}(u+v_1)t$ (10)

$= \frac{1}{2} \times \frac{2u}{a}(u+u+2at)$ (5)

$= \frac{u}{a}\left(2u + 2a \times \frac{2u}{a}\right)$

$= \frac{6u^2}{a}$. (5)

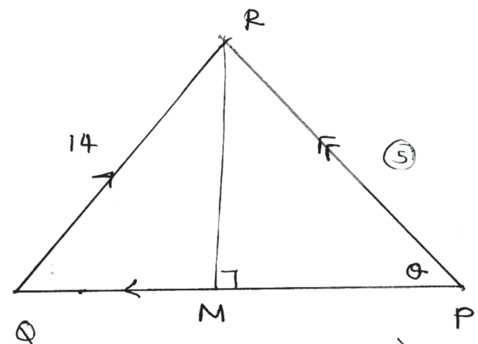
20

b) $V_{P,E} = \vec{V}$

$V_{Q,E} = 14$ (30°) (5)

$\underline{V}_{Q,P} = \underline{V}_{Q,E} + \underline{V}_{E,P}$

$PR = 14$ (30°) $+ V$ (5)



15

By using the cosine rule

$PR^2 = V^2 + 14^2 - 2V \times 14 \cos 60^\circ$ (10)

$PR = \sqrt{V^2 - 14V + 196}$ (5)

$\text{Time} = \frac{12\sqrt{79}}{PR}$ (5)

$PR = \frac{12\sqrt{79}}{6} = 2\sqrt{79}$

20

$\sqrt{V^2 - 14V + 196} = 2\sqrt{79}$ (5)

$V^2 - 14V - 120 = 0$ (5)

$(V-20)(V+6) = 0$

$V = 20,\ V \neq -6$ (5)

$V = 20\ ms^{-1}$ (5)

20

$\tan\theta = \frac{MR}{MP} = \frac{14 \sin \pi/3}{20 - 14\cos \pi/3} = \frac{7\sqrt{3}}{13}$ (5)

$\theta = \tan^{-1}\left(\frac{7\sqrt{3}}{13}\right)$ (5)

10

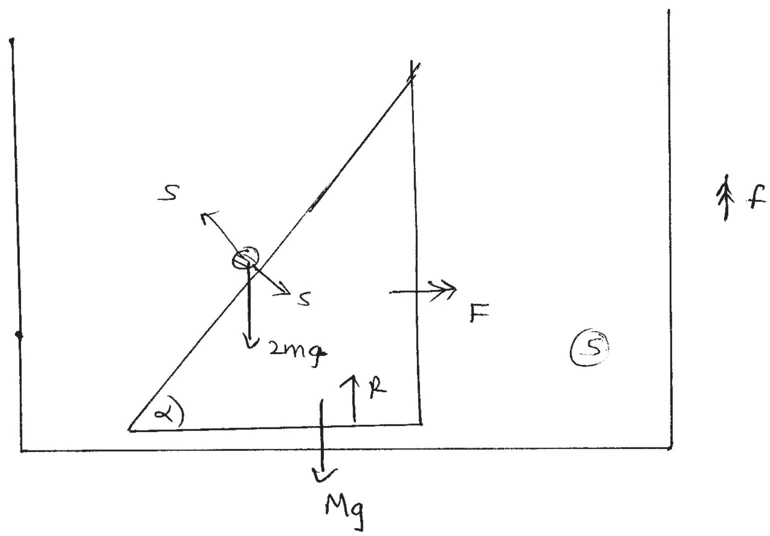
$$\tan\theta = \frac{MR}{MP}$$

$$= \frac{14 \sin \pi/3}{20 - 14 \cos \pi/3} \quad (5)$$

$$\tan\theta = \frac{7\sqrt{3}}{13} \qquad \theta = \tan^{-1}\left(\frac{7\sqrt{3}}{13}\right) \quad (5)$$

10

(12) a)



(5)

$\underline{a}_{L,E} = \uparrow f$ (5) $\qquad \underline{a}_{M,L} = \xrightarrow{} F \qquad \underline{a}_{2m,M} = a$ (along incline, α)

$\underline{a}_{M,E} = \underline{a}_{M,L} + \underline{a}_{L,E} \qquad \underline{a}_{2m,E} = \underline{a}_{2m,M} + \underline{a}_{M,E}$

$= \xrightarrow{} F + \uparrow f$ (5) $\qquad = a$ (along incline, α) $+ \uparrow f \xrightarrow{} F$

For the wedge and the particle (5)

20

$\rightarrow \underline{F} = m\underline{a}$

$0 = MF + 2m\,[F - a\cos\alpha]$ — ① (10)

for the particle (2m)

$$2mg \sin\alpha = 2m\,[a - f\sin\alpha - F\cos\alpha]$$

$$g \sin\alpha = a - f\sin\alpha - F\cos\alpha$$

$$(g + f)\sin\alpha = a - F\cos\alpha \quad — ② \ (10)$$

by ①, $\quad F\,(M + 2m) = 2m\,a\cos\alpha$

by ②,

$$(g+f)\sin\alpha = \frac{F(M+2m)}{2m\cos\alpha} - F\cos\alpha$$

$$2m(g+f)\sin\alpha\cos\alpha = F\left[(M+2m) - 2m\cos^2\alpha\right] \quad (10)$$

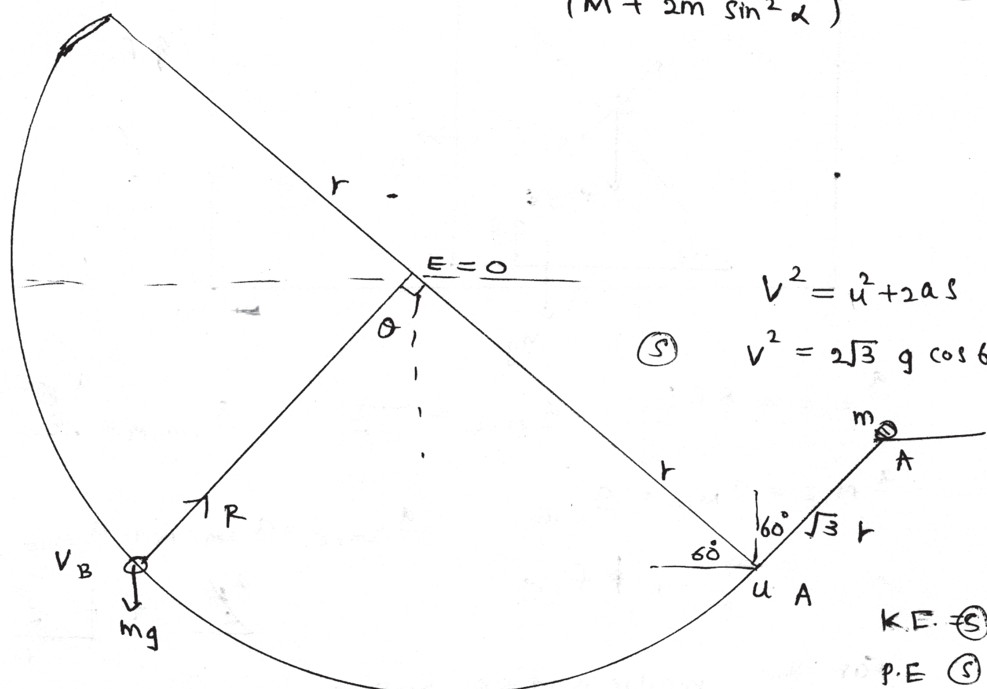
$$2m(g+f)\sin\alpha\cos\alpha = F(M+2m\sin^2\alpha)$$

$$F = \frac{2m(g+f)\sin\alpha\cos\alpha}{M+2m\sin^2\alpha} \quad (10)$$

$$a = \frac{2m(g+f)\sin\alpha\cos\alpha\,(M+2m)}{(M+2m\sin^2\alpha)\,2m\cos\alpha}$$

$$a = \frac{(g+f)\sin\alpha\,(M+2m)}{(M+2m\sin^2\alpha)} \quad (10)$$

50



(5)

$$V^2 = u^2 + 2as$$

$$V^2 = 2\sqrt{3}\, g\cos 60 \times r \quad (5)$$

K.E. (5)
P.E (5)
eq^n (5)

Energy at A = Energy at B

$$\frac{1}{2}mu^2 - mg\,r\sin 60 = \frac{1}{2}mv^2 - mg\,r\cos\theta$$

$$u^2 - \sqrt{3}\,gr = v^2 - 2gr\cos\theta$$

$$v^2 = u^2 - \sqrt{3}gr + 2gr\cos\theta \quad (5)$$

$$v^2 = \sqrt{3}gr - \sqrt{3}gr + 2gr\cos\theta$$

35

$F = ma$

$R - mg\cos\theta = \dfrac{mv^2}{r}$ (10)

$R = mg\cos\theta + \dfrac{m}{r}\left[u^2 - \sqrt{3}gr + 2gr\cos\theta\right]$ (5)

$R = 3mg\cos\theta + \dfrac{mu^2}{r} - \sqrt{3}mg$ (5)

when $v = 0$, $\theta = \theta_2$

$\cos\theta_2 = \dfrac{\sqrt{3}gr - u^2}{2gr}$ (5)

25

$u^2 = \sqrt{3}gr$, $\cos\theta_2 = 0$

$\theta_2 = \dfrac{\pi}{2}$ (5)

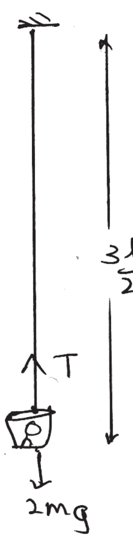
when $v = 0$, $R = mg\cos\pi/2$

$R = 0$ (5)

$\therefore$ when $v = 0$, $R = 0$. $\therefore$ The particle doesn't leave the surface. The maximum distance travelled by the particle in the circular path is $\dfrac{2\pi}{3} r$. (10)

20

(13)



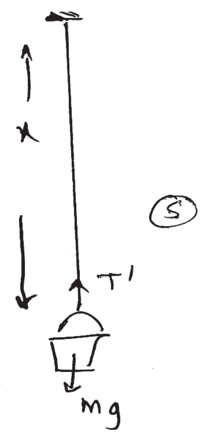
In equilibrium,

$T = 2mg$ (5)

$\lambda \cdot \dfrac{(l/2)}{l} = 2mg$ (5)

$\lambda = 4mg$ (5)

15



$$T' = mg\frac{(x-l)}{l} \quad (5)$$

$F = ma$

$$\downarrow \; mg - T' = m\ddot{x} \quad (10)$$

$$mg - \frac{4mg(x-l)}{l} = m\ddot{x}$$

$$\ddot{x} = -\frac{4g}{l}\left[x - 4l - \frac{l}{4}\right]$$

$$\ddot{x} = -\frac{4g}{l}\left[x - \frac{5l}{4}\right] \quad (5)$$

$$x - \frac{5l}{4} = X \qquad \omega = 2\sqrt{\frac{g}{l}}$$

differentiating w.r.t. t, $\ddot{x} = \ddot{X}$ (5)

$$\ddot{X} = -\frac{4g}{l}X$$

$$\ddot{X} = -\omega^2 X$$

$\therefore$ The motion is a simple harmonic motion. (5)

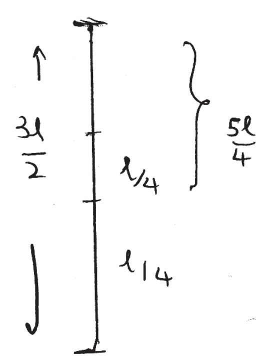
35

iii $\ddot{X} = 0$, $X = 0$ $\quad x - \frac{5l}{4} = 0$

centre $x = \frac{5l}{4}$ (5)

The amplitude of the motion $\Big\} = \frac{3l}{2} - \frac{5l}{4}$ (5)

$$= \frac{l}{4} \quad (5)$$



Since the amplitude of the motion is $l/4$ the velocity when the string comes to its natural length is zero. $\therefore$ The particle doesn't move under gravity.

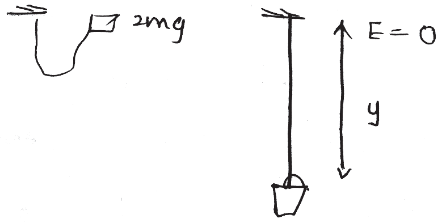
$\therefore$ The motion is simple harmonic completely. (5)

20

iv Period $= \frac{2\pi}{\omega}$ (5) $= \frac{2\pi}{2\sqrt{\frac{g}{l}}}$ (5) $= \pi\sqrt{\frac{l}{g}}$ (5)

15

v.



By the principle of Conservation of Energy.

$$\frac{1}{2}\times 2m\times 2gl = \frac{1}{2}\times 4mg\frac{(y-l)^2}{l} - 2mgy + \frac{1}{2}m\dot{y}^2$$

5 5 5 5

$$2gl^2 = 2g(y-l)^2 - 2gyl + l\dot{y}^2$$

$$l\dot{y}^2 + 2g(y-l)^2 - 2gyl - 2gl^2 = 0$$ 5

25

vi when $\dot{y} = 0$, 5

$$2g(y-l)^2 - 2gyl - 2gl^2 = 0$$

$$y^2 - 2yl + l^2 - yl - l^2 = 0$$

$$y(y-3l) = 0$$ 5

$$y = 0 \text{ or } y = 3l$$ 5

15

vii

$$l\dot{y}^2 + 2g(y-l)^2 - 2gyl - 2gl^2 = 0$$

differentiating w.r.t. t,

$$2\dot{y}\ddot{y}l + 4g(y-l)\dot{y} - 2g\dot{y}l = 0$$ 5

$2\dot{y} \neq 0$, $\quad \ddot{y}l + 2g(y-l) - \cancel{2}gl = 0$

$$\ddot{y}l + 2gy - 2gl - \cancel{2}gl = 0$$ 5

$$\ddot{y}l = -2gy + 3gl$$

$$= -2g\left[y - \frac{3l}{2}\right]$$ 5

$$\ddot{y} = -\frac{2g}{l}\left[y - \frac{3l}{2}\right]$$

When $\ddot{y} = 0$, $y = \frac{3l}{2}$

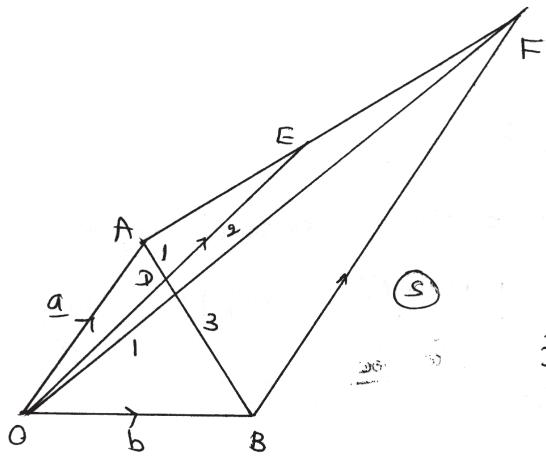
The Centre is at a distance $\frac{3l}{2}$ from O.

Amplitude $= 3l - \frac{3l}{2}$

$$= \frac{3l}{2}$$

25

14 a)



$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$ (5)

$= \underline{a} + \frac{1}{4}\overrightarrow{AB}$

$= \underline{a} + \frac{1}{4}(\underline{b} - \underline{a})$

$= \frac{3\underline{a} + \underline{b}}{4}$ (5)

$\overrightarrow{DE} = 2\overrightarrow{OD}$

$= \frac{3\underline{a} + \underline{b}}{2}$ (5)

(5)

$\overrightarrow{AE} = \overrightarrow{AD} + \overrightarrow{DE}$ (5) $= \frac{1}{4}(\underline{b} - \underline{a}) + \frac{3\underline{a} + \underline{b}}{2}$ (5)

$\overrightarrow{AE} = \frac{5\underline{a} + 3\underline{b}}{4}$

30

$\overrightarrow{BF} = \overrightarrow{BA} + \overrightarrow{AF}$ (5)

$\lambda\,\overrightarrow{OE} = \underline{a} - \underline{b} + \mu\,\overrightarrow{AE}$

$\lambda\,(3\overrightarrow{OD}) = \underline{a} - \underline{b} + \mu\left(\frac{5\underline{a} + 3\underline{b}}{4}\right)$

$3\lambda\left(\frac{3\underline{a} + \underline{b}}{4}\right) = \frac{(4 + 5\mu)\,\underline{a} + (3\mu - 4)\,\underline{b}}{4}$ (10)

$5\mu + 4 = 9\lambda$ (5) $\qquad 3\mu - 4 = 3\lambda$ (5)

$9\lambda - 5\mu = 4$ —(1) $\qquad 3\lambda - 3\mu = -4$ —(2)

by (1) & (2) $\quad \mu = 4$ (5), $\lambda = \frac{8}{3}$ (5)

35

$\overrightarrow{AF} = \mu\,\overrightarrow{AE}$

$\overrightarrow{AF} = 4\,\overrightarrow{AE}$

$AE : EF = 1 : 3$ (5)

$\overrightarrow{BF} = \lambda\,\overrightarrow{OE}$

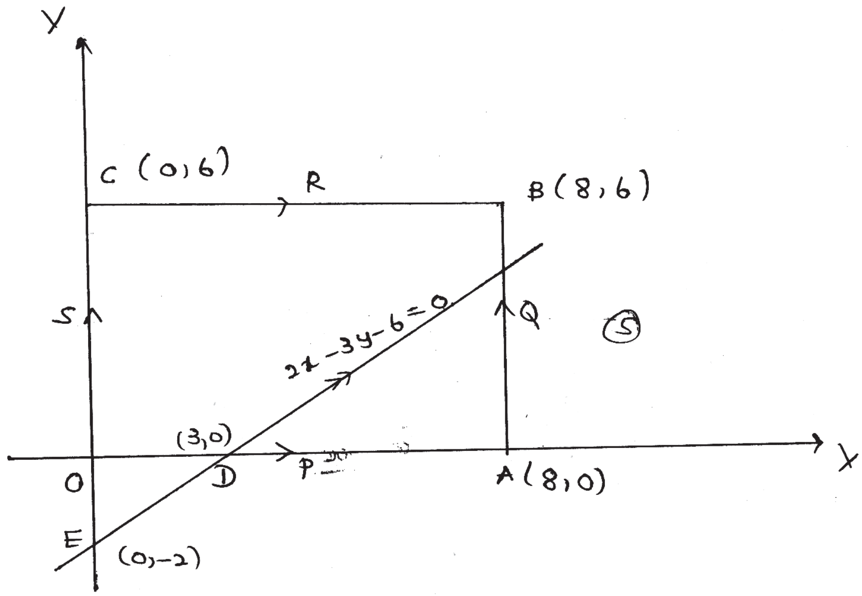
$= 3\lambda\left(\frac{3\underline{a} + \underline{b}}{4}\right)$

$= 3 \times \frac{8}{3}\left(\frac{3\underline{a} + \underline{b}}{4}\right)$ (5)

$\overrightarrow{BF} = 2\,(\underline{b} + 3\underline{a})$

10

b)



O↺ $8Q - 6R = 42$ — ① S

D↺ $5Q - 6R - 3S = 0$ — ② S

when $S = 2$, $5Q - 6R = 6$ — ③

by ① & ③ $Q = 12$ S

$R = 9$ S

E↺ $8Q - 8R - 2P = 0$ S

$8 \times 12 - 8 \times 9 - 2P = 0$

$P = 12$ S

$\rightarrow X = 12 + 9 = 21$ N S

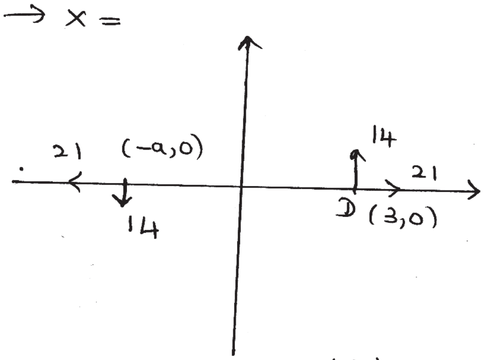
$\uparrow Y = 2 + 12 = 14$ N

$R = \sqrt{21^2 + 14^2}$

$= 7\sqrt{13}$ N S

35

$\rightarrow X =$



~~By~~

When a force $(-21, -14)$ is S added at the point $E(-a, 0)$ $(a > 0)$, it is equivalent to a couple.

$(a + 3)\,14 = 70$ S

$a = 2$ S

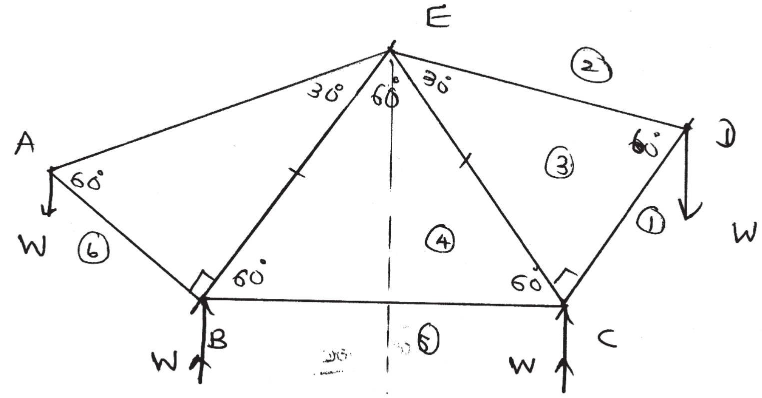
Additional force which makes an angle α with the negative direction of the X-axis, is $7\sqrt{13}$ N. S , $\alpha = \tan^{-1}\left(\frac{2}{3}\right)$

∴ The equation of the line of action is

$3y - 2x + 4 = 0$ 10

40

15) a)



$\uparrow \quad 2R = W$

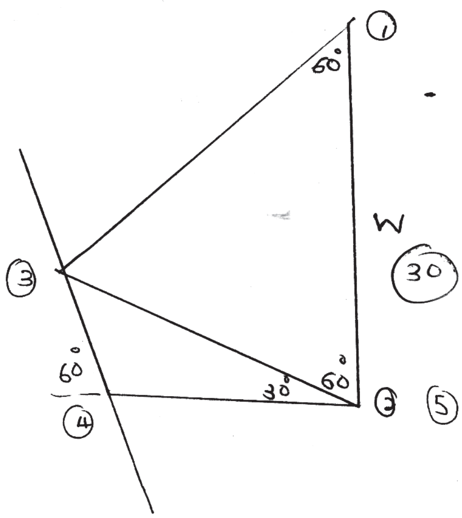
$R = \dfrac{W}{2}$ (5)

The System is symmetric about FE.

DE = AE

BA = CD (5)

EB = EC



(30)

$(2)(3) \sin 30^\circ = (3)(4) \sin 60$

$\dfrac{W}{2} = (3)(4) \dfrac{\sqrt{3}}{2}$

$(2)(3) \cos 30^\circ = (3)(4) \cos 60 + (4)(5)$

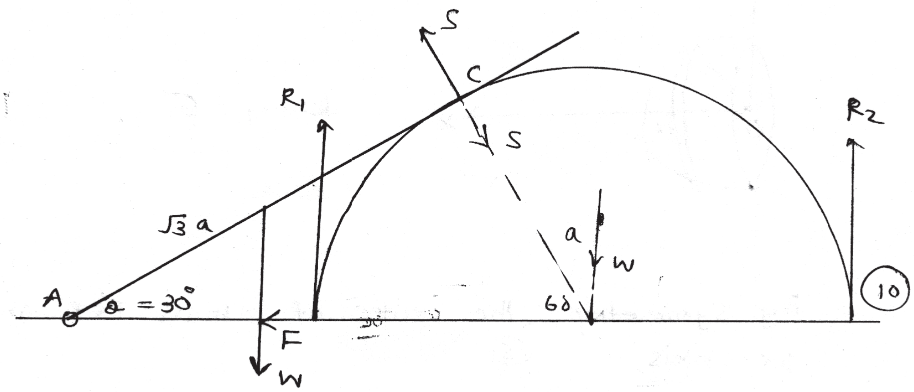
$\dfrac{W\sqrt{3}}{2} - \dfrac{W}{\sqrt{3}} \times \dfrac{1}{2} = (4)(5)$

$\dfrac{6W - 2W}{4\sqrt{3}} = \dfrac{4W}{4\sqrt{3}}$

Rod	Stresses		Magnitude	
	Thrust	Tension		
CD / BA	✓	—	W	(10)
DE / AE	—	✓	W	(10)
EC / EB	✓	—	$W/\sqrt{3}$	(10)
BC	✓	—	$W/\sqrt{3}$	(10)

80

b)



$\tan\theta = \dfrac{a}{\sqrt{3}a}$

$\theta = 30^\circ$

$\uparrow \; R_1 + R_2 - W - S\cos 30^\circ = 0$ (5)

$R_1 + R_2 = W + \frac{\sqrt{3}}{2} S$ (5)

$A \circlearrowleft \quad S\sqrt{3}\, a - W a \cos 30 = 0$ (10)

$S = W\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} = \frac{W}{2}$ (5)

$R_1 + R_2 = W + \frac{W}{2}\,\frac{\sqrt{3}}{2} = W\left[1 + \frac{\sqrt{3}}{4}\right]$ (5)

$\rightarrow \; S\cos 60^\circ - F = 0$ (5)

$F = \frac{W}{4}$ (5)

50

For the equilibrium

$\mu \geq \left|\frac{F}{R}\right|$

$\mu \geq \left|\frac{F}{R_1 + R_2}\right|$

$\mu \geq \left|\dfrac{W/4}{W\left[1 + \frac{\sqrt{3}}{4}\right]}\right|$

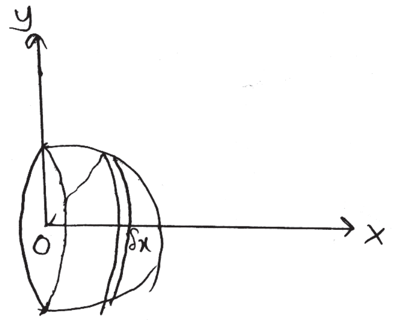
$\mu \geq \dfrac{1}{4 + \sqrt{3}}$ (10)

$\mu \geq \dfrac{4 - \sqrt{3}}{16 - 3}$

$\mu \geq \dfrac{4 - \sqrt{3}}{13}$ (10)

20

16)



density $= \sigma$

By symmetry the centre of mass lies on the ox axis. (5)

$\delta m = \pi (a^2 - x^2)\,\delta x\,\sigma$

$$\bar{x} = \frac{\int_0^a \pi (a^2 - x^2)\,\sigma x\,dx}{\int_0^a \pi (a^2 - x^2)\,\sigma\,dx} \quad (15) \quad = \frac{\int_0^a (a^2 x - x^3)\,dx}{(a^2 - x^2)\,dx} \quad (5)$$

$$= \frac{\left. \frac{a^2 x^2}{2} - \frac{x^4}{4} \right|_0^a \ (5)}{\left. a^2 x - \frac{x^3}{3} \right|_0^a \ (5)} = \underline{\underline{\frac{3}{8}a}} \quad (5)$$

40

By symmetry the centre of mass lies on the symmetric axis. (5)

Body	mass	distance to the centre of mass of the body
	$\frac{2}{3}\pi \times 27a^3\sigma = 18\pi a^3\sigma$ (5)	$\frac{9}{8}a$ (5)
	$2\pi a^3\sigma$ (5)	a (5)
	$3\pi a^3 k\sigma$ (5)	$\frac{9a}{2}$ (5)
Composite body	$(16+3k)\pi a^3\sigma$ (5)	$\bar{x}$

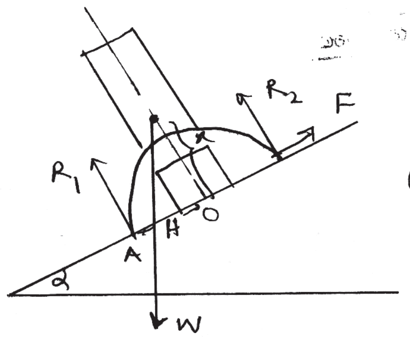
40

$$(16+3k)\,\pi a^3\sigma\bar{x} = 18\pi a^3\sigma \times \frac{9}{8}a - 2\pi a^3\sigma \times a + 3\pi a^3 k\sigma \times \frac{9a}{2}$$ (15)

$$(16+3k)\,\bar{x} = \frac{81a}{4} - 2a + \frac{27}{2}ka$$ (5)

$$\bar{x} = \frac{(73+54k)\,a}{4\,(16+3k)}$$

20



(10)

$\nearrow$ $R_1 + R_2 = W\cos\alpha$ (5)

$F = W\sin\alpha$ (5)

Since it is in equilibrium,

$$\mu \geq \frac{|F|}{R_1+R_2}$$ (10)

$$\mu \geq \frac{W\sin\alpha}{W\cos\alpha}$$ (5)

$$\mu \geq \tan\alpha$$

$OA > OH$ (10)

$3a > \bar{x}\tan\alpha$

$$3a > \frac{(73+54k)}{4(16+3k)}\,a\tan\alpha$$ (5)

$$\frac{12(16+3k)}{73+54k} > \tan\alpha$$

50

17) a) i If $A \cup B = S$, A and B are exhaustive events (10)

ii If $A \cap B = \phi$, A and B are mutually exclusive events. (10)

20

b) i) $A \cup A' = \Omega$ (5)

$P(A \cup A') = P(\Omega)$

$P(A) + P(A') = 1 \qquad (A \cap A') = \phi$

$\therefore\ P(A') = 1 - P(A)$ (5)

ii $A = (A\cap B) \cup (A \cap B')$ 5

$P(A) = P(A\cap B) + P(A\cap B')$ — ① 5 5 $[\because (A\cap B)\cap(A\cap B') = \phi]$

iii $A\cup B = B \cup (A\cap B')$ 5

$P(A\cup B) = P(B) + P(A\cap B')$ — ② 5 $[\because B\cap(A\cap B') = \phi]$

by ① & ②

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$ 5

40

$P(A'\cup B') = P(A') + P(B') - P(A'\cap B')$ 5

$\frac{17}{20} = \frac{3}{5} + \frac{3}{10} - P(A'\cap B')$

$P(A'\cap B') = \frac{1}{20}$ 5

$P(A\cup B) = 1 - P(A\cup B)'$ 5 $= 1 - \frac{1}{20} = \frac{19}{20}$ 5

$P(A\cap B) = 1 - P(A\cap B)' = 1 - \frac{17}{20} = \frac{3}{20}$ 5

$P(A'\cap B) = P(B) - P(A\cap B)$ 5 $= \frac{7}{10} - \frac{3}{20} = \frac{11}{20}$ 5

35

c) i) A = All the three balls are in the same color.

$P(A) = P(BBB) + P(RRR) + P(GGG)$

$= \frac{3}{12}\times\frac{2}{11}\times\frac{1}{10}$ 5 $+ \frac{4}{12}\times\frac{3}{11}\times\frac{2}{10}$ 5 $+ \frac{5}{12}\times\frac{4}{11}\times\frac{3}{10}$ 5

$= \frac{9}{132} = \frac{3}{44}$ 5

20

ii) $P(B,R,G) = \frac{3}{12}\times\frac{4}{11}\times\frac{5}{10}$ 5 $= \frac{1}{22}$ 5

10

iii) B = Getting balls of different colors

$P(B) = \frac{3}{12}\times\frac{4}{11}\times\frac{5}{10}\times 6$ 5 $= \frac{3}{11}$ 5

10

iv $P(R', R', R') = \frac{8}{12}\times\frac{7}{11}\times\frac{6}{10}$ 5 $= \frac{14}{55}$

05

v. $1 - P(R'R'R') = 1 - \frac{14}{55}$ 5 $= \frac{41}{55}$ 5

10