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Provincial Department of Education - NWP

10 E I

Second Term Test - Grade 12 - 2019

Index No : ....

Combined Mathematics I

Three hours only

Instructions:

- *This question paper consists of two parts.*
  Part A (Question 1 - 10) and **Part B** (Question 11 - 17)
- **Part A**
  Answer all questions. Write your answers to each question in the space provided. you may use additional sheets if more space is needed.
- **Part B**
  Answer five questions only. Write your answers on the sheets provided.
- *At the end of the time allocated, tie the answers of the two parts together so that* **Part A** *is on top of* **part B** *before handing them over to the supervisor.*
- *You are permitted to remove only* **Part B** *of the question paper from the Examination Hall.*

For Examiner's Use only

(10) Combined Mathematics I		
Part	Question No	Marks Awarded
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
	Total	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	
Paper 1 total		
Percentage		

Paper I	
Paper II	
Total	
Final Marks	

Final Marks

In Numbers	
In Words	

Marking Examiner	
Marks Checked by 1	
2	
Supervised by	

Combined mathematics 12 - I (Part A)

01) Roots of the equation $x^2 - \lambda x + \mu = 0$ are two consecutive integers. Prove that $\lambda^2 = 4\mu + 1$.

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02) Find the value of x satisfying the inequality $\frac{1}{(x+2)} \leq \frac{x}{3}$.

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03) Remainder when the polynomial $f(x) = x^m + nx$ is divided by $(x^2 - x - 2)$ is $2x + 6$. Where m and n are integers. Find the values of m and n.

04) Resolve $\frac{2x^3-x+3}{x(x-1)}$ in to partial fractions.

05) Evaluate $\lim_{x \to 0} \frac{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}{x}$.

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06) Solve the equation $log_2 x + \log_4 x + log_{16} x = \frac{21}{4}$.

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07) Co-ordinates of the vertices B and C of isosceles triangle ABC are $(-1,2)$ and $(4,-5)$ respectively. AB = AC. Find a pair of suitable coordinates for the vertex A.

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08) Function $f(x)$ is defined as $f(x) = \frac{x^2}{x^4+1}$ for $f: \mathcal{R} \longrightarrow \mathcal{R}$. Mention range of the function.

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09) If $tan^{-1} x + tan^{-1} y + tan^{-1} z = \pi$, prove that, $x + y + z = xyz$.

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10) Find the general solution of the equation $cosec\, x + \sec x = 2\sqrt{2}$.

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Combined mathematics 12 - I (Part B)

<u>Answer five questions only.</u>

11) a) Show that the sum and product of the roots of the equation $ax^2 + bx + c = 0$ are $-\frac{b}{a}$ and $\frac{c}{a}$ respectively.

If $\frac{k+1}{k}$ and $\frac{k+2}{k+1}$ are roots of the quadratic equation $ax^2 + bx + c = 0$, prove that $(a+b+c)^2 = b^2 - 4ac$ where $k \neq -1, 0$ and a, b, c are real numbers. Also prove that roots of the equation are always real.

(b) $f(x)$ is a quadratic function of x and it is divisible by $(2x+1)$. Remainders when $F(x)$ is divided by $(x-1)$ and $(2x-1)$ are -6 and -5 respectively. Find $f(x)$.

If α and β are roots of the equation $f(x) = 4\lambda x$, find the quadratic equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$. Where $\lambda \in \mathcal{R}$.

12) a) Mention domain of the following functions.

i) $y = 1 - \sqrt{1 - x^2}$ ii) $y = \sqrt{x^2 - 4x + 3}$

iii) $y = \sqrt{2x-1} + \sqrt{3-2x}$

b) Let, $p(x) = x^{\frac{1}{3}}$, $q(x) = (x^9 + x^6)^{\frac{1}{2}}$ and $r(x) = x\sqrt{1+x}$. Show that $q[p(x)] = r(x)$.

c) i) If $\frac{x\,(y+z-x)}{\log x} = \frac{y(z+x-y)}{\log y} = \frac{z(x+y-z)}{\log z}$, prove that $x^y . y^x = z^y y^z = x^z z^x$.

ii) Solve following simultaneous equations and find x and y.

$\log_9 xy = \frac{5}{2}$, $\log_3 x \,.\log_3 y = -6$

iii) If $\log_4 x = a$ and $\log_{12} x = b$ Prove that $\log_3 4 = \frac{b}{a-b}$ and $\log_3 48 = \frac{a+b}{a-b}$

13) a) (i) Sketch the graph of |2x+1|.

(ii) Sketch the graph of -|2x+1|.

(iii) Hence sketch the graphs of -|2x+1| + 4 and $|x+1|$ on a same coordinate plane Hence solve the inequality |2x+1| + |x+1| ≤ 4.

b) Evaluate following limits.

(i) $\lim\limits_{x \to 0} \frac{x\,tanx}{1-\cos x}$ (ii) $\lim\limits_{x \to \pi} \frac{1+\cos x}{tan^2 x}$ (iii) $\lim\limits_{x \to 0} \frac{1-cos^3 x}{x\sin x \cos x}$

c) Function $f(x)$ is defined as bellow.

$f(x) = \begin{Bmatrix} x+1 \ ; & x \leq 1 \\ 3 - ax^2 \ ; & x > 1 \end{Bmatrix}$ If the function is continuous find the value of a.

14) a) Show that the maximum value of $f(x) = (a - x)(x - b)$ as x varies is given by $\frac{1}{4}(a-b)^2$. Draw a rough sketch of the graph of $f(x)$ when $0 < b < a$.

b) It is given that c and d are roots of the equation $(x-a)(x-b) - k^2 = 0$. Prove that c and d are real as a, b, k are real numbers.
Also show that roots of the quadratic equation $(x-c)(x-d) + k^2 = 0$ are a and b.

c) (i) Write down quotient and the remainder when polynomial $2x^3 - x^2 + x + 1$ is divided by $(x-1)$ and write down the division algorithm. (

(ii) Write down the corresponding division algorithm when the above quotient is divided by $(x-3)$.

(iii) Deduce that the remainder when polynomial $2x^3 - x^2 + x + 1$ is divided by $(x-1)(x-3)(2x+7)$ is $(23x - 20)$.

15) a) Find the partial fractions of $\frac{1}{(1-x)(1+x)}$. Hence deduce the partial fractions of $\frac{1}{(1-x)^3(1+x)}$.

b) $A(-1,0)$ and $B(5,1)$ are two vertices of parallelogram $ABCD$. $(3,-5)$ is the coordinate at point where the side BC divides internally in the ratio $2:1$. Find the coordinates at the remaining vertices of the parallelogram. Hence find the length of the diagonals AC and BD.

c) If α and β are the roots of the equation $k^2x^2 + (kx+1)(x+k) + 1 = 0$, Show that $\alpha^2\beta^2 + (\alpha\beta + 1)(\alpha + \beta) + 1 = 0$.

16) a) Prove the identity $\frac{Sin^2 A - Sin^2 B}{\text{Sin } A \, Cos\, A - Sin\, B Cos\, B} = \tan(A+B)$.

b) Solve the equation $\tan 3\theta = 1$.
Represent $tan\, 3\theta$ in terms of $tan\, \theta$. Hence show that roots of the equation,
$t^3 - 3t^2 - 3t + 1 = 0$ are -1, $\tan\left(\frac{\pi}{12}\right)$ and $\tan\left(\frac{5\pi}{12}\right)$.
Hence find the value of $\tan\left(\frac{\pi}{12}\right)$. Where $t = \tan\theta$.

c) Solve the equation, $tan^{-1}x + tan^{-1}\left(\frac{x}{2}\right) + tan^{-1}\left(\frac{x}{3}\right) = \frac{\pi}{2}$.

17) a) Prove for any triangle using standard notation that, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.
Prove that $a = (b-c)\cos\left(\frac{A}{2}\right)\, cosec\left(\frac{B-C}{2}\right)$.
Length of the median, passes through vertex A of triangle ABC is m. It makes angles α and β with side AB and AC respectively. Prove that,

i.) $2m(\sin\alpha - \sin\beta) = a(\sin B - \sin C)$

ii.) $2m \sin\frac{1}{2}(\alpha - \beta) = (b-c)\sin\left(\frac{A}{2}\right)$

b) State the cosine rule for any triangle ABC in standard notation. hence prove that, if $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{a}{bc} + \frac{b}{ca}$ then $c = \frac{\pi}{2}$.

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Second Term Test - Grade 12 - 2019

Index No : ....

Combined Mathematics II

Three hours only

Instructions:

- *This question paper consists of two parts.*
  Part A (Question 1 - 10) and **Part B** (Question 11 - 17)
- **Part A**
  *Answer **all** questions. Write your answers to each question in the space provided. you may use additional sheets if more space is needed.*
- **Part B**
  Answer five questions only. Write your answers on the sheets provided.
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(10) Combined Mathematics II		
Part	Question No	Marks Awarded
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
	Total	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	
Paper 1 total		
Percentage		

Paper I	
Paper II	
Total	
Final Marks	

Final Marks

In Numbers	
In Words	

Marking Examiner	
Marks Checked by 1	
2	
Supervised by	

(Part A)

1) Position vectors of points A and B with respect to the origin O are $2\underline{i} + 3\underline{j}$ and $-\underline{i} + 5\underline{j}$ respectively. Find

$i)$ $\overrightarrow{AB}$ $\quad$ $ii)$ $|\overrightarrow{AB}|$

$iii)$ Vector with magnitude 3 times as $|\overrightarrow{AB}|$ in the direction of $\overrightarrow{OA}$.

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2) If $\underline{a} = 2\underline{i} - 5\underline{j}$ and $\underline{b} = k\underline{i} + 2\underline{j}$ are perpendicular to each other, find the value of k. Find,

$i)$ $\underline{b} - \underline{a}$ $\quad$ $ii)$ $|\underline{a}|$ and $|\underline{b} - \underline{a}|$

$iii)$ $\underline{a}.(\underline{b} - \underline{a})$ $\quad$ $iv)$ angle between $\underline{a}$ and $(\underline{b} - \underline{a})$

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3) Define vector product $\underline{a} \times \underline{b}$ for two non zero vectors $\underline{a}$ and $\underline{b}$.

$\underline{i}$ and $\underline{j}$ are unit vectors along the axes OX and OY respectively. Find.

If $\underline{a} = 4\underline{i} + 3\underline{j}$ and $\underline{b} = 2\underline{i} - 3\underline{j}$ Find $\underline{a} \times \underline{b}$ and $\underline{b} \times \underline{a}$

Prove that $\underline{a} \times \underline{b} \neq \underline{b} \times \underline{a}$

4) Resultant of two equal forces acting at a point inclined 2α to each other is twice than when they are inclined 2β to each other. Prove that $\cos\alpha = 2 \cos\beta$.

5) Two parallel forces of magnitudes $4N$ and $6N$ act on a rigid body at points A and B respectively. Find the magnitude of resultant force and the line of action when the forces are.

(i) like forces

(ii) unlike forces.

Where, $AB = 15\,m$

6) $ABCD$ is a square of side $2a$. Forces of magnitudes $1, 2, 3$ and $2\sqrt{2}$ Newtons act along the sides $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$ and $\overrightarrow{AC}$ respectively. Taking X and Y axes along AB and AD, show that the system is reduced to a couple and a resultant force and find magnitude of the couple and the resultant force.

7) Particle starts to move with a velocity $U\ ms^{-1}$ from point O travels with uniform velocity during time t and suddenly increase its velocity up to $ku\ (k > 1)$. Then it travels with velocity $ku\ ms^{-1}$ during time $\frac{t}{2}\ s$. Find its displacement after time $\frac{3t}{2}$ seconds.

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8) Particle which is projected up with velocity u from a point on the ground meets another particle which is released at the same instant from rest from a height of h. If velocities of the particles equals to each other when they are meeting, show that $u^2 = 2gh$.

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9) A particle of weight $6N$ hangs using two light inextensible strings which are inclined at an angle α and β to the vertical and the particle is in equilibrium. If tensions on the stings are $3N$ and $3\sqrt{3}\,N$, find the values of α and β.

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10) A particle of mass m is in equilibrium on a smooth inclined plane by applying a force P parallel to the inclined plane. Show that the inclination of the plane to the horizontal is $Sin^{-1}\left(\frac{P}{mg}\right)$.

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Combined Mathematics 12 - II (Part - B)

Answer five questions only.

11) Particle X is projected up with velocity $u\,ms^{-1}$ at $t = 0$ from point O. After time t if it reaches to a point, which is at height hm from point O , obtain an quadratic equation of t. Hence if t_1 and t_2 are the corresponding times taken by the particle reach to that point in two different ways, show that $t_1 t_2 = \frac{2h}{g}$ and $(t_2 - t_1) = \frac{2}{g}\sqrt{u^2 - 2gh}$. Show that the time taken by the particle to reach point A again after passing point A is $\frac{2}{g}\sqrt{u^2 - 2gh}$. Hence show that the maximum height reach by a particle which is projected up with velocity $u\,ms^{-1}$ is $H = \frac{u^2}{2g}$.

12) a) A motor vehicle starts from rest moves with uniform acceleration $a_1\,ms^{-2}$ to obtain its maximum velocity. As soon as reaches its maximum velocity it starts to retardate uniformly with $a_2 ms^{-2}$ to comes to rest at point B. If the total displacement of the particle is S, show that the total time taken is $\left[\frac{2S(a_1 + a_2)}{a_1 a_2}\right]^{\frac{1}{2}}$ seconds.

Also show that the maximum speed of the vehicle is, $V = \sqrt{\frac{2S(a_1\, a_2)}{a_1 + a_2}}\, ms^{-1}$

If both maximum acceleration and retardation is $a\, ms^{-2}$, show that the minimum time taken to the journey is $2\sqrt{\frac{S}{a}}$.

b) There is a notice indicating that the maximum velocity of the road due to construction of the road is $V_o ms^{-1}$ $(V_0 < V)$. Motor vehicle is moving with velocity V_0 , through a certain distance between AB. Show that the additional time taken due to construction of the road for the motion between A and B is given by $\frac{S(V - V_0)^2}{V_0 V^2}$.

13) Position vectors of the points P and Q with respect to O is $\underline{p}$ and $\underline{q}$ respectively. R is a point on PQ such that $PR{:}PQ = \lambda{:}1$. If $\underline{r}$ is the position vector of R, show that $\underline{r} = p + \lambda\,(q - p)$.

$OACB$ is parallogoram. Sides $\overrightarrow{OA}$ and $\overrightarrow{OB}$ represents vector $\underline{a}$ and $\underline{b}$ respectively. L and M are mid points of sides AC and CB respectively. Find $\overrightarrow{OL}$ and $\overrightarrow{OM}$.
X is a point on AM such that $AX : XM, 2{:}3$. Show that points $O.X.L$ are collinear.

14) $\underline{a}$ and $\underline{b}$ are non-parallel and non-zero vectors. If $\alpha a + \beta b = 0$ prove that $\alpha = 0$ and $\beta = 0$. $PQRS$ is a trapezium such that $\overrightarrow{SR} = \frac{1}{3}\overrightarrow{PQ}$. Let $\overrightarrow{PQ} = \underline{P}$ and $\overrightarrow{PS} = \underline{q}$. X is on QR such that $\overrightarrow{QX} = \frac{2}{3}\overrightarrow{QR}$. Y is the intersecting point of PX and QS and it satisfy $\overrightarrow{QY} = \lambda\,\overrightarrow{QS}$. Show that $\overrightarrow{PX} = \frac{5}{9}\underline{p} + \frac{3}{2}\underline{q}$ and $\overrightarrow{PY} = (1-\lambda)\underline{p} + \lambda\,\underline{q}$. Hence find the value of λ.

15) $\underline{i}$ and $\underline{j}$ are the unit vectors along OX and OY axes of the rectangular cartesian coordinate system. Forces of magnitudes $\left(3\underline{i}+\underline{j}\right)$, $\left(2\underline{i}+4\underline{j}\right)$, $\left(\underline{i}+5\underline{j}\right)$, $\left(P\underline{i}+Q\underline{j}\right)$ act at points with position vectors $(-6\underline{i})$, $\left(-\underline{i}-4\underline{j}\right)$, $\left(\underline{i}+2\underline{j}\right)$, $\left(2\underline{i}+3\underline{j}\right)$ respectively.

i. Find the value of P, Q if the system reduces to a couple of magnitude G.

ii. If the system of forces reduces to a single force of magnitude Yj at point $\left(\underline{i}.\underline{j}\right)$ find the values of G and Y.

16) a) $ABCDEF$ is a regular hexagon of side a. Forces of magnitudes $4P, 2P, P, 3P, 2P$ and $5P$ acts along the sides AB, BC, DC, ED, EF and AF respectively. Find magnitude and direction of the resultant force and the point where the line of action of the resultant force meats side AB.
Now a couple of magnitude $10\sqrt{3}\,Pa$ in the direction of $FEDCBA$ and a force of magnitude Q are introduced to the system. If the new system of forces is in equilibrium, find the magnitude and direction of the force Q and its line of action.

b) $ABCDEF$ is a light inextensible string. Its two ends A and F are attached to two points A and F on horizontal zeling and particles of mass M are attach at points B,C,D, E on the string such that five identical strings are formed. String AB and Ef are inclided at an angle $\theta \left(< \frac{\pi}{2}\right)$ to the vertical and strings BC and QE are inclined at an angle $\beta \left(< \frac{\pi}{2}\right)$ to the vertical. CD part of the string is horizontal and system is in equilibrium, Prove that,

i. Tension in the string AB is $2Mg\sec\theta$

ii. $2\tan\theta = \tan\beta$

iii. Tension in the string BC is $Mg\sqrt{4\,tan^2\theta + 1}$

iv. Tension of the CD part of the string is, $2Mgtan\theta$.

17) a) Two particles A and B starts from rest at $t = 0$ and travels with constant acceleration $f\ ms^{-2}$ through the time t. After time t force on the particle B is removed and hence it travels with uniform velocity and particle A moves as before. Use velocity time graph for the motion of A on the frame of reference of B to show that the distance between two particles A and B after time $t_1 + t_2$ is $\frac{ft_1^2}{2}$.

b) Particle P is projected vertically up with velocity $2u\ ms^{-1}$ at $t = 0$ from point O. When particles P is at its maximum height, particle Q is projected vertically up with velocity $3u\ mS^{-1}$ from he same point O. Draw velocity time graph for the motion of Q with respect to P and hence show that the time taken to collide to two particles is $\frac{7u}{3g}$ seconds.

(1). $x^2 - \lambda x + \mu = 0 \begin{cases} \alpha \\ \alpha + 1 \end{cases}$

$2\alpha + 1 = \lambda$ (5)

$\alpha^2 + \alpha = \mu$

$\left(\frac{\lambda - 1}{2}\right)^2 + \frac{\lambda - 1}{2} = \mu$ (5)

$(\lambda - 1)^2 + 2(\lambda - 1) = 4\mu$ (5)

$\lambda^2 - 1 = 4\mu$ (5)

$\underline{\underline{\lambda^2 = 4\mu + 1}}$ (5)

25

(2). $\frac{1}{x+2} \leq \frac{x}{3}$

$\frac{1}{x+2} - \frac{x}{3} \leq 0$ (5)

$\frac{3 - x^2 - 2x}{3(x+2)} \leq 0$ (5)

$\frac{x^2 + 2x - 3}{3(x+2)} \geq 0$

$\frac{(x+3)(x-1)}{3(x+2)} \geq 0$ (5)

$\begin{array}{ccccccc} (-) & & (+) & & (-) & & (+) \\ & -3 & & -2 & & 1 & \end{array}$ (5)

Solution is,

$\underline{\underline{x \in [-3, -2) \cup [1, \infty)}}$ (5)

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3. $x^m + nx \equiv (x^2 - x - 2)\,\phi(x) + 2x + 6$

$x^m + nx \equiv (x-2)(x+1)\,\phi(x) + 2x + 6$ 5

$x = 2$

$2^m + 2n = 10$ ——— (1) 5

$x = -1$

$(-1)^m - n = 4$ ——— (2)

$(-1)^m + 2^{m-1} = 9$ 5

5 $\underline{\underline{m = 4}}$, $\underline{\underline{n = -3}}$ 5

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4. $\dfrac{2x^3 - x + 3}{x(x-1)} = (Ax + B) + \dfrac{C}{x} + \dfrac{D}{(x-1)}$ 10

$2x^3 - x + 3 = (Ax + B)(x^2 - x) + C(x-1) + Dx$

$x^3 \rightsquigarrow \quad 2 = A$

$x^2 \rightsquigarrow \quad 0 = -A + B$

$x \longrightarrow \quad -1 = -B + C + D$ 10

$x^0 \longrightarrow \quad 3 = -C$

$B = 2 \qquad D = 4$

$\therefore$

$$\dfrac{2x^3 - x + 3}{x(x-1)} \equiv (2x + 2) + \dfrac{(-3)}{x} + \dfrac{4}{(x-1)}$$ 5

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5. $$\lim_{x\to 0} \frac{\sqrt{1+\sin x} - \sqrt{1-\sin x}}{x}$$

$$= \lim_{x\to 0} \frac{(1+\sin x - 1 + \sin x)}{x\left(\sqrt{1+\sin x} + \sqrt{1-\sin x}\right)} \quad 5$$

$$= \lim_{x\to 0} \frac{2\sin x}{x} \cdot \lim_{x\to 0} \frac{1}{\left(\sqrt{1+\sin x} + \sqrt{1-\sin x}\right)}$$

5 5

$$= 2 \times 1 \times \frac{1}{2} \quad 5$$

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$$= 1 \quad 5$$

6. $$\log_2 x + \log_4 x + \log_{16} x = \frac{21}{4}$$

$$\frac{1}{\log_x 2} + \frac{1}{2\log_x 2} + \frac{1}{4\log_x 2} = \frac{21}{4} \quad 5$$

If, $\log_x 2 = t$ 5

$$\frac{1}{t} + \frac{1}{2t} + \frac{1}{4t} = \frac{21}{4}$$

5

$$\frac{4+2+1}{4t} = \frac{21}{4}$$

$$\frac{7}{4t} = \frac{21}{4}$$

5 $t = \frac{1}{3}$

But, $\log_x 2 = \frac{1}{3} \Rightarrow x^{1/3} = 2$

$$x = 2^3$$

$$x = 8 \quad 5$$

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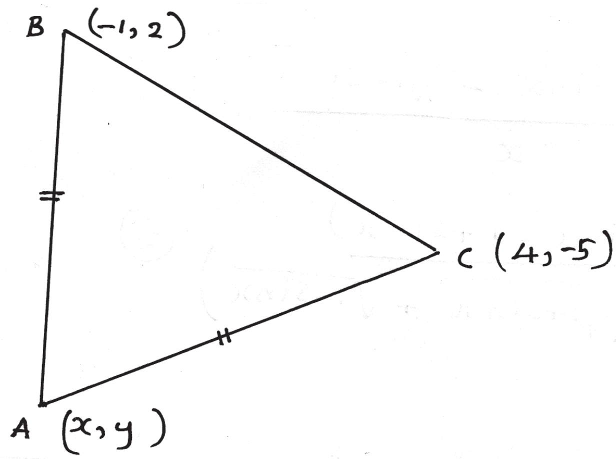
(7)



$$(x+1)^2 + (y+2)^2 = (x-4)^2 + (y+5)^2 \quad (5) \quad (5)$$

$$10x - 14y - 36 = 0$$

$$5x - 7y - 18 = 0 \quad (5)$$

(give marks to correct 2 points) (10) 25

(8). $f(x) = \dfrac{x^2}{x^4+1}$; $f : \mathbb{R} \longrightarrow \mathbb{R}$.

clearly that $f(x) \geq 0$, (5)

then,
$$(x^2-1)^2 \geq 0 \quad (5)$$

$$x^4 - 2x^2 + 1 \geq 0$$

$$x^4+1 \geq 2x^2$$

$$1 \geq \frac{2x^2}{x^4+1} \quad (5)$$

$$(5) \quad \frac{1}{2} \geq \frac{x^2}{x^4+1}$$

∴ range of function

$$R_f = \left[0, \tfrac{1}{2}\right] \quad (5)$$

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9. $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$

Let, $\tan^{-1}x = \alpha$, $\tan^{-1}y = \beta$, $\tan^{-1}z = \gamma$ 5

then,

$$\alpha + \beta + \gamma = \pi$$

$$\alpha + \beta = \pi - \gamma$$

$$\tan(\alpha+\beta) = \tan(\pi - \gamma) \quad 5$$

$$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = -\tan\gamma$$

5

$$\frac{x+y}{1-xy} = -z \quad 5 .$$

$$x + y = xyz - z$$

$$\underline{\underline{x + y + z = xyz}} \quad 5$$

25

10. $\operatorname{cosec} x + \sec x = 2\sqrt{2}$

$$\frac{\sin x + \cos x}{\sin x \cos x} = 2\sqrt{2} \quad 5$$

$$\frac{1}{\sqrt{2}}(\sin x + \cos x) = 2\sin x\cos x$$

$$\sin(x + \pi/4) = \sin 2x$$

$$\sin(x + \pi/4) - \sin 2x = 0 \quad 5$$

$$2\cos\left(\frac{3x}{2} + \frac{\pi}{8}\right)\sin\left(\pi/8 - x/2\right) = 0 \quad 5$$

25

$$\cos\left(\frac{3x}{2} + \frac{\pi}{8}\right) = 0 \quad ; \quad \sin\left(\frac{\pi}{8} - \frac{x}{2}\right) = 0$$

$$\frac{\pi}{8} - \frac{x}{2} = m\pi + (-1)^n \cdot 0$$

5 $\underline{\underline{\frac{3x}{2} + \frac{\pi}{8} = 2n\pi \pm \frac{\pi}{2} , n \in \mathbb{Z}}}$

5 $\underline{\underline{\frac{x}{2} = \frac{\pi}{8} - m\pi , m \in \mathbb{Z}}}$

(11) give marks to proof $\boxed{30}$

$$ax^2+bx+c=0 \begin{cases} \dfrac{k+1}{k} \\ \dfrac{k+2}{k+1} \end{cases}$$

When, $\dfrac{k+1}{k}$ and $\dfrac{k+2}{k+1}$ roots of the equation,

$$(5) \quad \frac{k+1}{k} + \frac{k+2}{k+1} = \frac{-b}{a} \quad \text{—(1)}$$

$$\frac{k+2}{k} = \frac{c}{a} \quad (5) \quad \text{—(2)}$$

$$k = \frac{2a}{c-a} \quad (5)$$

From (1).

$$(5) \quad 1+\frac{1}{k} + \frac{kc}{a(k+1)} = \frac{-b}{a}$$

$$1+\frac{c-a}{2a} + \frac{2c}{(c-a)\left(\frac{2a}{c-a}+1\right)} = \frac{-b}{a} \quad (5)$$

$$\frac{c-a}{2a} + \frac{2c}{a+c} = -\frac{(a+b)}{a}$$

$$(5) \quad \frac{c^2-a^2+4ac}{2(a+c)} = -(a+b)$$

$$c^2-a^2+4ac+2(a^2+ab+ac+bc) = 0 \quad (5)$$

$$(5) \quad a^2+b^2+c^2+4ac+2ab+2ac+2bc = b^2$$

$$\underline{\underline{(a+b+c)^2 = b^2-4ac}} \quad (5) \quad (5) \qquad \boxed{50}$$

For real roots,

$\Delta_x > 0$

(5) $b^2 - 4ac = (a+b+c)^2 > 0$ (5) $\boxed{10}$

$\therefore$ equation has real roots always.

b). $f(x) = (2x+1)(ax+b)$ (5)

$f(1) = -6$

$f(\frac{1}{2}) = -5$

$3(a+b) = -6$

$a+b = -2$ ——— (1) (5)

$2(\frac{a}{2} + b) = -5$

$a + 2b = -5$ ——— (2) (5)

$\underline{\underline{b = -3}}$ (5) $\underline{\underline{a = 1}}$

$\therefore f(x) = (2x+1)(x-3)$

$\underline{\underline{= 2x^2 - 5x - 3}}$ (5) $\boxed{25}$

$2x^2 - 5x - 3 = 4\lambda x$; $\lambda \in \mathbb{R}$

$2x^2 - (5+4\lambda)x - 3 = 0$ (5) α, β

$\alpha + \beta = \frac{5+4\lambda}{2}$ (5)

$\alpha\beta = \frac{-3}{2}$ (5)

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$$

$$= -\frac{(5+4\lambda) \times 2}{2 \times 3}$$

$$= -\frac{(5+4\lambda)}{3} \quad 10$$

$$\frac{1}{\alpha \cdot \beta} = -\frac{2}{3}$$

$\therefore$ The equation is,

$$5 \quad x^2 + x\left(\frac{5+4\lambda}{3}\right) - \frac{2}{3} = 0$$

$$\underline{\underline{3x^2 + (5+4\lambda)x - 2 = 0}} \quad 5$$

$\boxed{35}$

12 (a).

(i) $y = 1 - \sqrt{1-x^2}$

$$\underline{\underline{D_y = [-1, 1]}} \quad 5$$

(ii). $y = \sqrt{x^2 - 4x + 3}$

$$= \sqrt{(x-2)^2 - 1}$$

$$\underline{\underline{D_y = \mathbb{R} - (1, 3)}} \quad 5$$

(iii). $y = \sqrt{2x-1} + \sqrt{3-2x}$

$$\underline{\underline{D_y = \left[\frac{1}{2}, \frac{3}{2}\right]}} \quad 5$$

$\boxed{15}$

b). $p(x) = x^{1/3}$, $q(x) = (x^9 + x^6)^{1/2}$, $r(x) = x\sqrt{1+x}$

Hence,

$q[p(x)] = q(x^{1/3})$

(5) $= \left[(x^{1/3})^9 + (x^{1/3})^6\right]^{1/2}$

$= \left[x^3 + x^2\right]^{1/2}$ (5)

$\underline{\underline{q[p(x)] = x\sqrt{x+1}}}$ (5) $\boxed{15}$

c). (i) $\dfrac{x(y+z-x)}{\log x} = \dfrac{y(z+x-y)}{\log y} = \dfrac{z(x+y-z)}{\log z} = \dfrac{1}{k}$ (5)

$\log x = kx(y+z-x)$

$\log y = ky(z+x-y)$ (5)

$\log z = kz(x+y-z)$

$y\log x + x\log y = kxy(2z) = 2kxyz$ —— (1)

$y\log z + z\log y = 2kxyz$ —— (2) (5)

Similarly,

$z\log x + x\log z = 2kxyz$ —— (3)

From (1), (2) and (3), (10)

$y\log x + x\log y = y\log z + z\log y = z\log x + x\log z$

(5) $\log x^y + \log y^x = \log z^y + \log y^z = \log x^z + \log z^x$

$\log(x^y . y^x) = \log(z^y . y^z) = \log(x^z . z^x)$ (5)

(5) $\therefore$ $\underline{\underline{x^y y^z = z^y . y^z = x^z . z^x}}$ $\boxed{40}$

c). ii. $\log_9 xy = \frac{5}{2}$; $\log_3 x \cdot \log_3 y = -6$ —— (2)

$$\frac{1}{2\log_{xy} 3} = \frac{5}{2} \quad 5$$

$$\log_{xy} 3 = \frac{1}{5}$$

$$\frac{1}{\log_3 xy} = \frac{1}{5} \quad 5$$

$$\Rightarrow \log_3 x + \log_3 y = 5 \quad \text{—— (1)}$$

From (2);

$$\log_3 x - \frac{6}{\log_3 x} = 5 \quad 5$$

When $\log_3 x = t$,

$$t^2 - 5t - 6 = 0 \quad 5$$

$$(t - 6)(t + 1) = 0$$

$t = 6$ or $t = -1$

$\log_3 x = 6$ $\qquad$ $\log_3 x = -1$ 5

$\underline{\underline{x = 3^6}}$ $\qquad$ $x = 3^{-1}$

5 $\qquad$ $\underline{\underline{x = \frac{1}{3}}}$ 5

From (2);

$$6\log_3 3 \cdot \log_3 y = -6$$

$$\log_3 y = -1$$

$$\underline{\underline{y = \frac{1}{3}}}$$

$$\log_3\left(\frac{1}{3}\right) \cdot \log_3 y = -6$$

$$\left[\log_3(1) - \log_3 3\right] \log_3 y = -6$$

$$(0 - 1)\log_3 y = -6$$

$$\underline{\underline{y = 3^6}} \quad 5 \qquad \boxed{40}$$

(iii). $\log_4 x = a$, $\log_{12} x = b$

$\log_{12} x = b$

$\dfrac{1}{\log_x 12} = b$ 5

$\log_x(3\times 4) = \dfrac{1}{b}$ 5

$\log_x 3 + \log_x 4 = \dfrac{1}{b}$

$\log_x 3 = \dfrac{1}{b} - \dfrac{1}{a}$ 5 $\left(\log_x 4 = \dfrac{1}{a} \right)$

Then,

$\log_3 4 = \dfrac{\log_x 4}{\log_x 3}$ 5

$= \dfrac{1}{a\left(\frac{1}{b} - \frac{1}{a}\right)}$

$\underline{\underline{\log_3 4 = \dfrac{b}{a-b}}}$ 5 $\boxed{25}$

$\log_3 48 = \log_3 (4^2 \times 3)$ 5

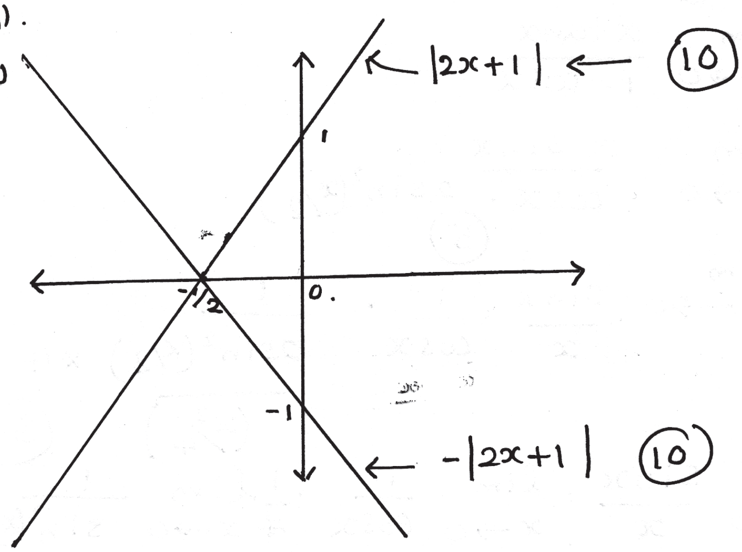
$= 2\log_3 4 + \log_3 3$ 5

$= \dfrac{2b}{a-b} + 1$

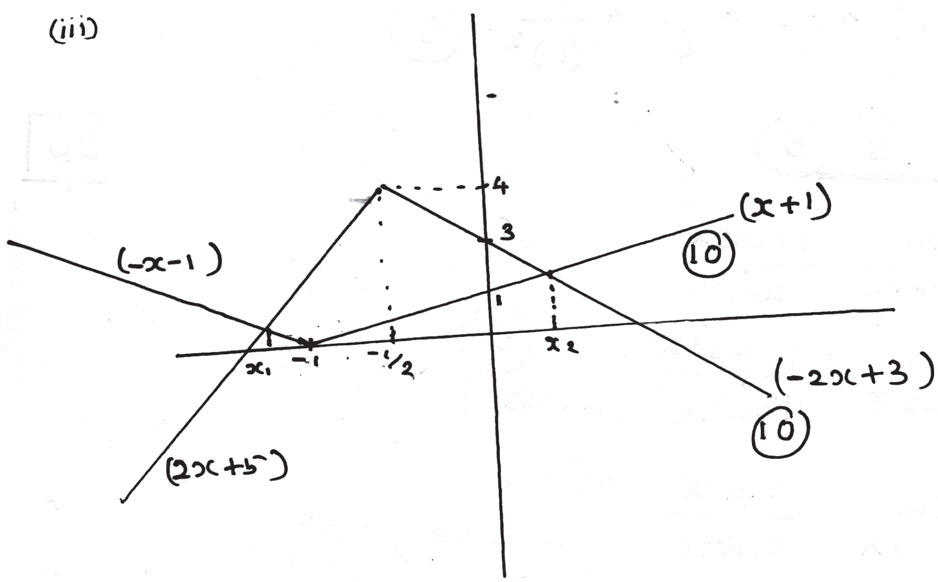
$\underline{\underline{\log_3 48 = \dfrac{a+b}{a-b}}}$ 5 $\boxed{15}$

(13) (a).

(i)



(iii)



$$|2x+1| + |x+1| \leq 4$$

$$|x+1| \leq 4 - |2x+1| \quad (5)$$

$\underset{\sim}{x_1}$

$$2x+5 = -x-1$$

(5) $$3x = -6$$

$$x = -2$$

$\underset{\sim}{x_2}$

$$-2x+3 = x+1 \quad (5)$$

$$3x = 2$$

$$x = 2/3$$

$\therefore$ solution space is $x \in \left[-2, \frac{2}{3}\right]$

(5) $\boxed{60}$

b). (i) $\lim\limits_{x \to 0} \dfrac{x \tan x}{1 - \cos x}$

$= \lim\limits_{x \to 0} \dfrac{x \sin x}{\cos x \cdot 2\sin^2(x/2)}$ 5

$= \lim\limits_{x \to 0} \dfrac{\sin x}{x} \cdot \dfrac{1}{\cos x} \cdot \dfrac{1}{\dfrac{2\sin^2(x/2)}{(x^2/4)} \times 4}$ 5

$= \lim\limits_{x \to 0} \dfrac{\sin x}{x} \cdot \lim\limits_{x \to 0} \dfrac{1}{\cos x} \cdot \dfrac{1}{8} \lim\limits_{x \to 0} \dfrac{1}{\dfrac{\sin^2(x/2)}{(x^2/4)}}$ 5

$= 1 \times \dfrac{1}{1} \times \dfrac{1}{8} \times \dfrac{1}{(1)^2}$ 5

$= \dfrac{1}{8}$ 5

$\boxed{25}$

ii). $\lim\limits_{x \to \pi} \dfrac{1 + \cos x}{\tan^2 x}$

$= \lim\limits_{x \to \pi} \dfrac{1 - \cos^2 x}{\tan^2 x \cdot (1 - \cos x)}$ 5

$= \lim\limits_{x \to \pi} \dfrac{\sin^2 x \cdot \cos^2 x \cdot}{\sin^2 x \quad (1 - \cos x)}$ 5

$= \lim\limits_{x \to \pi} \dfrac{\cos^2 x}{(1 - \cos x)}$ 5

$= \dfrac{(-1)^2}{1 - (-1)}$ 5

$= \dfrac{1}{2}$ 5

$\boxed{25}$

(iii). $$\lim_{x\to 0} \frac{1-\cos^3 x}{x\sin x\cos x}$$

$$= \lim_{x\to 0} \frac{(1-\cos x)(1+\cos x+\cos^2 x)}{x\sin x\cos x} \quad (5)$$

$$= \lim_{x\to 0} \frac{2\sin^2 x/2\,(1+\cos x+\cos^2 x)}{2x\sin x/2\cos x/2\cos x} \quad (5)$$

$$= \lim_{x\to 0} \frac{\sin(x/2)}{2(x/2)} \cdot \lim_{x\to 0} \frac{(1+\cos x+\cos^2 x)}{\cos x/2\,\cos x}. \quad (5)$$

$$= 1\times\frac{1}{2}\times\frac{(1+1+1^2)}{1\times 1} \quad (5)$$

$$= \frac{3}{2} \quad (5)$$

$\boxed{25}$

e). $$f(x) = \begin{cases} x+1; & x\leq 1 \\ 3-ax^2; & x>1 \end{cases}$$

For "f" to be continuous at $x=1$, We must have.

$$2 = f(1) = \lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = 3-a$$
(5) (5)

Thus, $a=1$ (5)

$\boxed{15}$

(14) (a). $f(x) = (a-x)(x-b)$

$$f(x) = -x^2 + x(a+b) - ab$$

$$= -\{x^2 - x(a+b) + ab\}$$

5 $$= -\left\{\left(x - \frac{a+b}{2}\right)^2 + ab - \left(\frac{a+b}{2}\right)^2\right\}$$

Then;

$$f(x)_{max} = \left(\frac{a+b}{2}\right)^2 - ab \quad 5$$

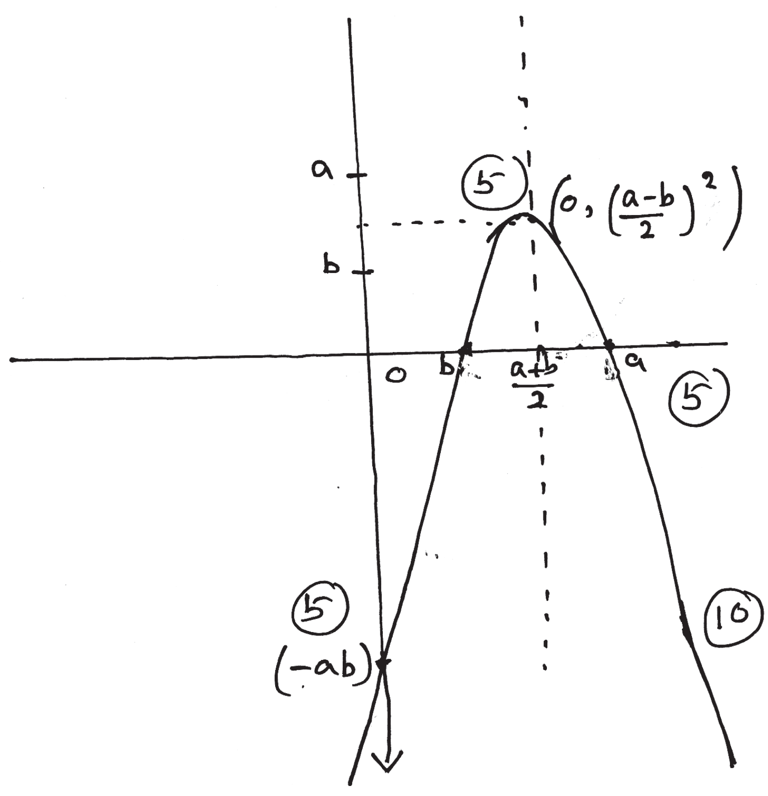
$$= \frac{1}{4}(a^2 - 2ab + b^2)$$

$$f(x)_{max} = \frac{1}{4}.(a-b)^2 \quad 5 \quad \text{when} \quad x = \frac{a+b}{2}$$

$\boxed{15}$

graph of the function f ;.

when $0<b<a$,



$\boxed{25}$

(b). $(x-a)(x-b) - k^2 = 0 < \begin{matrix} c \\ d \end{matrix}$

For real roots, $x^2 - x(a+b) + ab - k^2 = 0$ 5

$\Delta_x \geq 0$,

$\Delta_x = (a+b)^2 - 4 \cdot 1 \cdot (ab - k^2)$ 10

$= a^2 + 2ab + b^2 - 4ab + 4k^2$

$= a^2 - 2ab + b^2 + 4k^2$ 5

$= (a-b)^2 + 4k^2 \geq 0$ 5

$\therefore$ eq$^{\underline{n}}$ have real roots. 5

$(x-c)(x-d) + k^2 = 0 < \begin{matrix} \lambda \\ \mu \end{matrix}$ $x^2 - x(c+d) + k^2 + cd = 0$ 5

5 $\lambda + \mu = c + d$ — (1)

$\lambda\mu = k^2 + cd$ — (2)

5

But, $c + d = a + b$

$cd = ab - k^2$ 5

From (2)

$\lambda\mu = k^2 + ab - k^2$

$\lambda\mu = ab$ — (A) 5

From (1).

$\lambda + \mu = a + b$ — (B) 5

$\therefore \underline{\underline{\lambda = a}}$ $\underline{\underline{\mu = b}}$. 5

roots of the given equation are a and b.

$\boxed{65}$

c) (i) $2x^3 - x^2 + x + 1 \equiv (x-1)(2x^2 + x + 2) + 3$ 15

ii). $(2x^2 + x + 2) \equiv (x-3)(2x+7) + 23$ 15

iii). $2x^3 - x^2 + x + 1 \equiv (x-1)\left\{(x-3)(2x+7) + 23\right\} + 3$ 5

$\equiv (x-1)(x-3)(2x+7) + 23(x-1) + 3$ 5

$\therefore$ Remainder is $23(x-1) + 3$ 5

$= \underline{\underline{23x - 20}}$

$\boxed{45}$

15. $\dfrac{1}{(1-x)(1+x)} = \dfrac{A}{(1-x)} + \dfrac{B}{(1+x)}$ 5

$1 = A(1+x) + B(1-x)$

$x^0 \to A + B = 1$

$x^1 \to A - B = 0$ 5

$A = B = \frac{1}{2}$ 5

$\therefore \dfrac{1}{(1-x)(1+x)} = \dfrac{1}{2(1-x)} + \dfrac{1}{2(1+x)}$ 5

$\dfrac{1}{(1-x)^3(1+x)} = \dfrac{1}{(1-x)^2}\left\{\dfrac{1}{2(1-x)} + \dfrac{1}{2(1+x)}\right\}$ 10

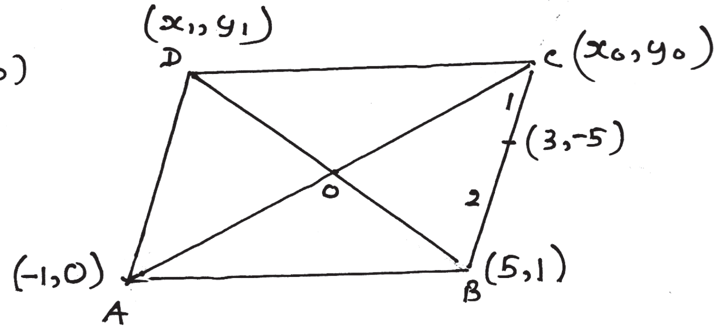
$= \dfrac{1}{2}\dfrac{1}{(1-x)^3} + \dfrac{1}{2}\cdot\dfrac{1}{(1-x)}\left\{\dfrac{1}{2(1-x)} + \dfrac{1}{2(1+x)}\right\}$ 10

$= \dfrac{1}{2(1-x)^3} + \dfrac{1}{4(1-x)^2} + \dfrac{1}{4}\left\{\dfrac{1}{2(1-x)} + \dfrac{1}{2(1+x)}\right\}$ 10

$\boxed{50}$

$= \underline{\underline{\dfrac{1}{2(1-x)^3} + \dfrac{1}{4(1-x)^2} + \dfrac{1}{8(1-x)} + \dfrac{1}{8(1+x)}}}$

(b)



$$\frac{2x_0+5}{3} = 3 \qquad \frac{2y_0+1}{3} = -5$$

$$2x_0 = 4 \qquad 2y_0 = -16$$

$$x_0 = 2 \qquad y_0 = -8$$

$\therefore$ $C \equiv (2, -8)$ (10)

Therefore,

$$O \equiv \left(\frac{2-1}{2}, \frac{-8}{2}\right)$$

$O \equiv \left(\frac{1}{2}, -4\right)$ (10)

$$\therefore \frac{x_1+5}{2} = \frac{1}{2} \qquad \frac{y_1+1}{2} = -4$$

$$x_1 = -5 \qquad y_1 = -9$$

$D \equiv (-5, -9)$ (10)

Length of AC $= \sqrt{(2+1)^2 + (-8)^2} = \sqrt{73}$ units. (10)

Length of BD $= \sqrt{(5+5)^2 + (1+9)^2} = 10\sqrt{2}$ units (10)

$\boxed{50}$

c). $k^2x^2 + (kx+1)(x+k) + 1 = 0$

$$(k^2+k)x^2 + (k^2+1)x + k + 1 = 0 \quad \begin{matrix} \alpha \\ \beta \end{matrix}$$

10

Then;

$$\alpha + \beta = \frac{-(k^2+1)}{k(k+1)} \quad 5$$

$$\alpha\beta = \frac{k+1}{k(k+1)} = \frac{1}{k} \quad 5$$

Consider;

$$\alpha^2\beta^2 + (\alpha\beta+1)(\alpha+\beta) + 1$$

$$= \left(\frac{k+1}{(k+1)k}\right)^2 + \left(\frac{k+1}{k(k+1)} + 1\right) - \left(\frac{k^2+1}{(k+1)k}\right) + 1 \quad 10$$

$$= \frac{1}{k^2} - \frac{\cancel{(k+1)}}{k} \frac{(k^2+1)}{k\,\cancel{(k+1)}} + 1 \quad 10$$

$$= \frac{1 - k^2 - 1}{k^2} + 1 \quad 5$$

$$= -1 + 1$$

$$= 0 \quad 5$$

$\boxed{50}$

(16)

(a). $\dfrac{\sin^2 A - \sin^2 B}{\sin A\cos A - \sin B\cos B}$

$$= \frac{(\sin A - \sin B)(\sin A + \sin B)}{\frac{1}{2}\sin 2A - \frac{1}{2}\sin 2B} \quad (10) \quad (10)$$

(5) (5)

$$= \frac{2 \times 2 . \cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) \times 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{2\cos(A+B)\sin(A-B)}$$

(5)

$$= \frac{\sin(A+B)\cancel{\sin(A-B)}}{\cos(A+B)\cancel{\sin(A-B)}}$$

$\boxed{40}$

$$= \underline{\underline{\tan(A+B)}} \quad (5)$$

b). $\tan 3\alpha = 1$

$\tan 3\alpha = \tan(\pi/4)$

$3\alpha = n\pi + \pi/4$ (5)

$\alpha = \dfrac{n\pi}{3} + \dfrac{\pi}{12}$; $n \in \mathbb{Z}$ (5)

$\left\{\alpha = \pi/12 , \dfrac{5\pi}{12} , - - - \right\}$

(5)

But, $\tan 3\alpha = \dfrac{3\tan\alpha - \tan^3\alpha}{1 - 3\tan^2\alpha} = 1$ (10)

$\Rightarrow 3\tan\alpha - \tan^3\alpha - 1 + 3\tan^2\alpha = 0$ (5)

If, $t = \tan\alpha$

$t^3 - 3t^2 - 3t + 1 = 0$ —— (1)

(5)

From (1).

$(t+1)(t^2-4t+1)=0$

$t=-1$ 5

$\alpha = \pi/12 \; , \; 5\pi/12$. 5

$\therefore$ above eq$^{\underline{n}}$ has 3 roots,

$\underline{\underline{\tan\alpha = -1 \; , \; \tan(\pi/12) \; , \; \tan\left(\frac{5\pi}{12}\right)}}$. 5

Consider, $(t^2-4t+1)=0$

5 $t = \dfrac{4 \pm \sqrt{16-4\cdot1\cdot1}}{2}$

$t = \dfrac{4 \pm \sqrt{12}}{2}$ 5

$t = 2 \pm \sqrt{3}$

$\tan(\pi/12) < \tan\left(\frac{5\pi}{12}\right)$ 5

$\therefore \underline{\underline{\tan\left(\pi/12\right) = 2-\sqrt{3}}}$ 5

$\boxed{70}$

c). $\tan^{-1}x + \tan^{-1}\frac{x}{2} + \tan^{-1}\frac{x}{3} = \frac{\pi}{2}$

Let $\alpha = \tan^{-1}x$, $\beta = \tan^{-1}\frac{x}{2}$, $\gamma = \tan^{-1}\frac{x}{3}$

$\tan\alpha = x$, 5 $\tan\beta = \frac{x}{2}$, $\tan\gamma = \frac{x}{3}$

$$\alpha + \beta + \gamma = \pi/2$$

$$\alpha + \beta = \pi/2 - \gamma \quad 5$$

$$\tan(\alpha + \beta) = \tan(\pi/2 - \gamma)$$

5

$$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \cot\gamma \quad 5$$

$$\frac{x + x/2}{1 - x \cdot \frac{x}{2}} = \frac{3}{x} \quad 5$$

$$\frac{2x + x}{2 - x^2} = \frac{3}{x}.$$

$x \neq (0, \pm\sqrt{2})$ 5

5 $3x^2 = 6 - 3x^2$

$$6x^2 = 6$$

$$\underline{\underline{x = \pm 1}} \quad 5$$

$\boxed{40}$

(17) (a) give marks to proof sine-Rule $\boxed{30}$

$$a = (b-c)\cos(A/2)\operatorname{cosec}\left(\frac{B-C}{2}\right)$$

$$\frac{a}{b-c} = \cos(A/2)\operatorname{cosec}\left(\frac{B-C}{2}\right)$$

(5)

L.H.S. $\dfrac{a}{b-c}$

From sine rule,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{\lambda} \quad (5)$$

$$= \frac{\lambda \sin A}{\lambda \sin B - \lambda \sin C}$$

$$= \frac{\sin A}{\sin B - \sin C}$$

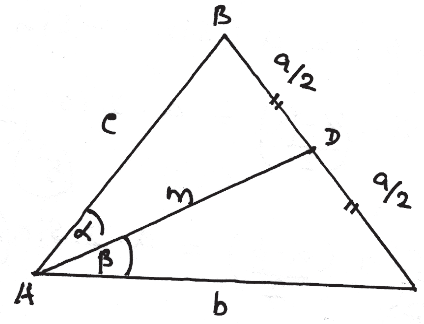
$$= \frac{2\cancel{\sin(A/2)}\cos(A/2)}{2\cancel{\cos\left(\frac{B+C}{2}\right)}\sin\left(\frac{B-C}{2}\right)} \quad (5) \quad (\because A+B+C=\pi)$$

$$= \cos(A/2)\operatorname{cosec}\left(\frac{B-C}{2}\right)$$

$$\therefore \frac{a}{b-c} = \cos(A/2)\operatorname{cosec}\left(\frac{B-C}{2}\right) \quad (5)$$

$$a = (b-c)\cos(A/2)\operatorname{cosec}\left(\frac{B-C}{2}\right)$$

$\boxed{20}$



5

(i) $2m(\sin\alpha - \sin\beta) = a(\sin B - \sin C)$

From sine - Rule.
for, ABD Δ,

$$\frac{\sin B}{m} = \frac{\sin\alpha}{a/2} \quad \text{—— (1)}$$

5

for ACD Δ,

$$\frac{\sin C}{m} = \frac{\sin\beta}{a/2} \quad \text{—— (2)}$$

5

(1) and (2),

$$a\sin B = 2m\sin\alpha$$

$$a\sin C = 2m\sin\beta$$

5

$\therefore$ $2m(\sin\alpha - \sin\beta) = a(\sin B - \sin C)$

5

25

ii). Considering, above result,

$$2m(\sin\alpha - \sin\beta) = a(\sin B - \sin C)$$

$$2m \cdot 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right) = a \cdot 2\cos\left(\frac{B+C}{2}\right)\sin\left(\frac{B-C}{2}\right)$$

5 5

$$2m \cdot 2\cos(A/2)\sin\left(\frac{\alpha-\beta}{2}\right) = a \cdot 2 \cdot \sin(A/2)\sin\left(\frac{B-C}{2}\right)$$

5 5

But,

$$a = (b-c)\cos(A/2)\operatorname{cosec}\left(\frac{B-C}{2}\right)$$

$$\underbrace{2m\cos(A/2)\operatorname{cosec}\left(\frac{B-C}{2}\right)}\sin\left(\frac{\alpha-\beta}{2}\right) = a\sin(A/2)$$

10

$$2m \cdot \frac{a}{(b-c)}\sin\left(\frac{\alpha-\beta}{2}\right) = a\sin(A/2) \quad 5$$

$$2m\sin\frac{1}{2}(\alpha-\beta) = (b-c)\sin(A/2) \quad 5$$

$\boxed{40}$

b). Cosine Rule. — 5

If,

$$\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{a}{bc} + \frac{b}{ca}$$

$$\frac{b^2+c^2-a^2}{2bac} + \frac{a^2+c^2-b^2}{2abc} + \frac{a^2+b^2-c^2}{2abc} = \frac{ca^2+b^2c}{abc^2}$$

10

$$\frac{a^2+b^2+c^2}{2abc} = \frac{a^2+b^2}{abc}$$

$$a^2+b^2+c^2 = 2(a^2+b^2)$$

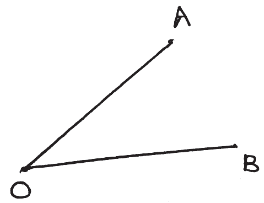
$$a^2+b^2-c^2 = 0 \quad 10$$

$$5 \quad \frac{a^2+b^2-c^2}{2ab} = \cos C = 0$$

$$C = \pi/2 \quad 5$$

$\boxed{35}$

01)



$\overrightarrow{OA} = 2\underline{i} + 3\underline{j}$

$\overrightarrow{OB} = -\underline{i} + 5\underline{j}$

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -\underline{i} + 5\underline{j} - 2\underline{i} - 3\underline{j}$

$= -3\underline{i} + 2\underline{j}$ 5

$|\overrightarrow{AB}| = \sqrt{3^2+2^2} = \sqrt{13}$ 5

The unit vector in the direction of $\overrightarrow{OA} = \dfrac{\overrightarrow{OA}}{|\overrightarrow{OA}|} = \dfrac{2\underline{i} + 3\underline{j}}{\sqrt{4+9}}$ 5

$= \dfrac{2\underline{i} + 3\underline{j}}{\sqrt{13}}$ 5

The vector in the direction of $\overrightarrow{OA} = \dfrac{2\underline{i} + 3\underline{j}}{\sqrt{13}} \times 3\sqrt{13}$

$= \underline{\underline{6\underline{i} + 9\underline{j}}}$ 5

25

02) $\underline{a} = 2\underline{i} - 5\underline{j}$ $\quad \underline{b} = k\underline{i} + 2\underline{j}$

Since $\underline{a} \perp \underline{b}$

$\underline{a} \cdot \underline{b} = |\underline{a}|\,|\underline{b}| \cos 90 = 0$

$(2\underline{i} - 5\underline{j}) \cdot (k\underline{i} + 2\underline{j}) = 0$ 5

$2k - 10 = 0 \Rightarrow k = 5$

i) $\underline{b} - \underline{a} = k\underline{i} + 2\underline{j} - 2\underline{i} + 5\underline{j} = 3\underline{i} + 7\underline{j}$ 5

ii) $|\underline{a}| = \sqrt{2^2 + 5^2} = \sqrt{29}$

$|\underline{b} - \underline{a}| = \sqrt{3^2 + 7^2} = \sqrt{58}$ 5

If the angle between $\underline{a}$ and $\underline{b} - \underline{a}$ is θ,

$\underline{a} \cdot (\underline{b} - \underline{a}) = |\underline{a}|\,|\underline{b} - \underline{a}| \cos\theta$

$$\cos\theta = \frac{\underline{a}\cdot(\underline{b}-\underline{a})}{|\underline{a}|\cdot|\underline{b}-\underline{a}|} \quad \text{— (1)}$$

$$\underline{a}\cdot(\underline{b}-\underline{a}) = (2\underline{i}-5\underline{j})\cdot(3\underline{i}+7\underline{j}) = -29 \qquad (5)$$

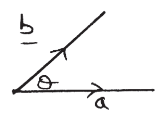
$$(1) \Rightarrow \quad \cos\theta = \frac{-29}{\sqrt{29}\times\sqrt{58}} = \frac{-1}{\sqrt{2}}$$

$$\therefore \theta = \underline{\underline{135^\circ}} \qquad (5)$$

25

03) $\underline{a} = 4\underline{i} + 3\underline{j}$

$\underline{b} = 2\underline{i} - \underline{j}$



$\underline{a}\times\underline{b} = |\underline{a}||\underline{b}| \sin\theta \, \underline{n}$. (5) where $\underline{n}$ is a unit vector perpendicular to both $\underline{a}$ and $\underline{b}$ such that $\underline{a}$, $\underline{b}$ and $\underline{n}$ are in the right handed orientation.

$$\underline{a}\times\underline{b} = (4\underline{i}+3\underline{j})\times(2\underline{i}-\underline{j}) \qquad (5)$$

$$= 8\,\underline{i}\cdot\underline{i} - 4\,\underline{i}\cdot\underline{j} + 6\,\underline{j}\cdot\underline{i} - 3\,\underline{j}\cdot\underline{j}$$

$$= 6\,\underline{j}\cdot\underline{i} - 4\,\underline{i}\cdot\underline{j} \qquad (5)$$

$$\underline{b}\times\underline{a} = (2\underline{i}-\underline{j})\times(4\underline{i}+3\underline{j}) \qquad (5)$$

$$= 8\,\underline{i}\times\underline{i} + 6\,\underline{i}\times\underline{j} - 4\,\underline{j}\times\underline{i} \mp 12\,\underline{j}\times\underline{j}$$

$$= 6\,\underline{i}\times\underline{j} - 4\,\underline{j}\times\underline{i} \qquad (5)$$

$$\underline{a}\times\underline{b} \neq \underline{b}\times\underline{a}$$

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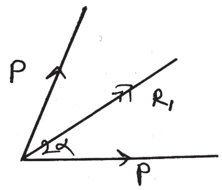
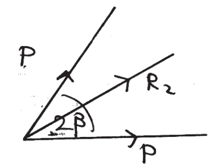
04)





$R_1 = 2R_2$

$\therefore R_1^2 = 4R_2^2$ (5)

$R_1^2 = P^2 + P^2 + 2P^2 \cos 2\alpha$ — (1) (5)

$R_2^2 = P^2 + P^2 + 2P^2 \cos 2\beta$ — (2) (5)

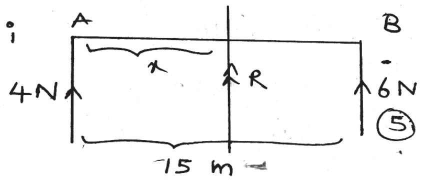
$P^2+P^2+2P^2\cos 2\alpha = 4[P^2+P^2+2P^2\cos 2\beta]$

$2P^2(1+\cos 2\alpha) = 8P^2[1+\cos 2\beta]$ (5)

$\cos\alpha = 2\cos\beta$ (5)

25

05) i

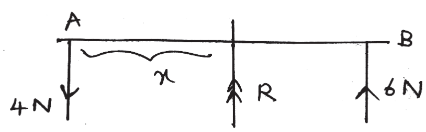


(5)

$\uparrow R = 10\,N$ (5)

A↶ $10x = 15 \times 6 \Rightarrow x = 9\,m$ (5)

ii



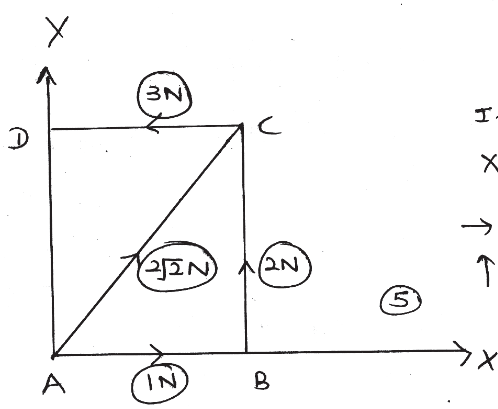
$\uparrow R = 6-4\,N = 2N$ (5)

A↶ $2 \times x = 6 \times 15$

$x = 45\,m$ (5)

25

06)



(5)

If reduces to a single force

$X \neq 0$ and $Y \neq 0$ (5)

$\rightarrow X = 1-3+2\sqrt{2}\cos 45 = 0$ (5)

$\uparrow Y = 2+2\sqrt{2}\cos 45 = 4N$ (5)

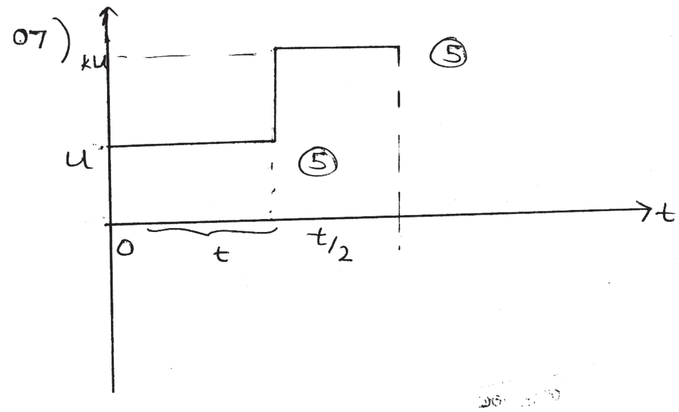
$\therefore$ The system reduces to a single force.

A↶ $G = 2\times 2a + 3\times 2a$

$= 10\,a\ Nm$ (5)

25

07)



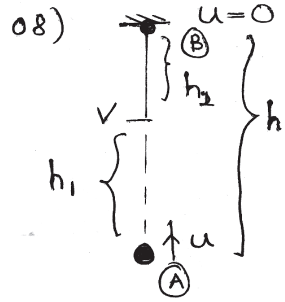
$S = \text{Area}$

$= ut + ku \times \frac{t}{2}$ (5) (5)

$= \frac{ut}{2}(2+k)$ m (5)

25

08)



for (A) $\uparrow$ $v^2 = u^2 + 2as$

$= u^2 - 2gh_1$ —(1) (5)

for (B) $\downarrow$ $v^2 = u^2 + 2as$

$v^2 = 2gh_2$ —(2) (5)

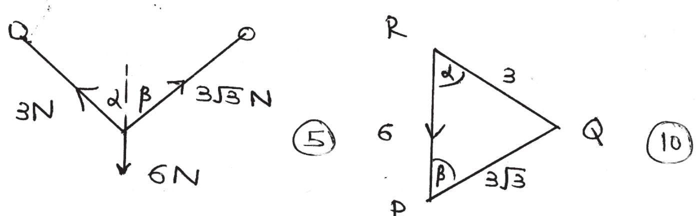
by (1) & (2) $\Rightarrow$ $u^2 - 2gh_1 = 2gh_2$ (10)

$u^2 = 2g(h_1 + h_2)$

$u^2 = 2gh$ (5)

25

08)

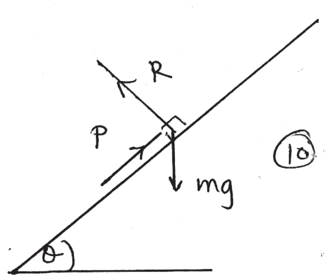


Since $6^2 = (3\sqrt{3})^2 + 3^2$, $P\hat{Q}R$ is a right angle.

$\tan\beta = \frac{3}{3\sqrt{3}} \Rightarrow \beta = 30^\circ$ (5) | $\tan\alpha = \frac{3\sqrt{3}}{3} \Rightarrow \alpha = 60^\circ$ (5)

25

10)



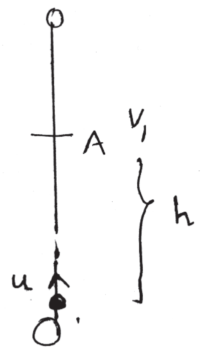
$\nearrow$ $P - mg\sin\theta = 0$ (10)

$\sin\theta = \frac{P}{mg}$

$\theta = \sin^{-1}\left(\frac{P}{mg}\right)$ (5)

25

11)



For the motion O - A ,

$S = ut + \frac{1}{2}at^2$

$h = ut - \frac{1}{2}gt^2$

$gt^2 - 2ut + 2h = 0$

This is a quadratic equation in t. 40

$t_1 + t_2 = \frac{2u}{g}$ (10)

$t_1 t_2 = \frac{2h}{g}$ (10)

$(t_2 - t_1)^2 = (t_2 + t_1)^2 - 4t_1t_2$ (10)

$= \frac{4u^2}{g^2} - \frac{8h}{g}$

$t_2 - t_1 = \frac{2}{g}\sqrt{u^2 - 2gh}$ (10)

40

$\uparrow\ V^2 = u^2 + 2as$

$V^2 = u^2 - 2gh \Rightarrow V = \sqrt{u^2 - 2gh}$ (10)

For the motion AOA ,

$\uparrow\ S = ut + \frac{1}{2}at^2$

$0 = \sqrt{(u^2 - 2gh)}\, t_2 - \frac{1}{2}gt_2^2$ (10)

$0 = t_2\left[\sqrt{u^2 - 2gh} - \frac{gt_2}{2}\right]$ (5)

$t_2 = 0$ (5) or $t_2 = \frac{2}{g}\sqrt{u^2 - 2gh}$ (5)

$\therefore$ The time taken $= \frac{2}{g}\sqrt{u^2 - 2gh}$ (5)

40

Velocity at the height h,

since $V = \sqrt{u^2 - 2gh}$ (10)

When h is maximum, $V = 0$. (10)

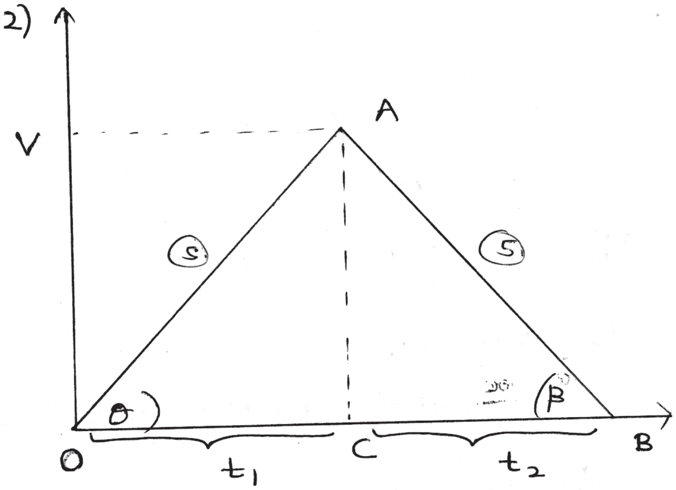
If the maximum height is H,

$0 = \sqrt{u^2 - 2gH}$

$\therefore H = \frac{u^2}{2g}$ (10)

30

12)



10

$\tan\theta = a_1 \ ms^{-2}$ $\qquad$ $\tan\beta = a_2 \ ms^{-2}$

From OAC Δ, $\tan\theta = \dfrac{V}{t_1} = a_1$ (5)

$$t_1 = \frac{V}{a_1} \quad (5)$$

$$\tan\beta = \frac{V}{t_2} = a_2 \quad (5)$$

$$t_2 = \frac{V}{a_2} \quad (5)$$

$S =$ Area of OAB Δ $= \dfrac{1}{2} V . (t_1 + t_2)$ (5)

$$V = \frac{2S}{t_1 + t_2} \quad (5)$$

① + ② $\quad t_1 + t_2 = \dfrac{V (a_1 + a_2)}{a_1 a_2}$ (5)

$$V = \frac{2S a_1 a_2}{V (a_1 + a_2)}$$

$$V = \sqrt{\frac{2S a_1 a_2}{a_1 + a_2}} \quad (5)$$

$$t_1 + t_2 = \left[\frac{2S (a_1 + a_2)}{a_1 a_2}\right]^{1/2} \quad (10)$$

50

$a_1 = a \; ms^{-2}$ $\qquad a_2 = a \; ms^{-2}$

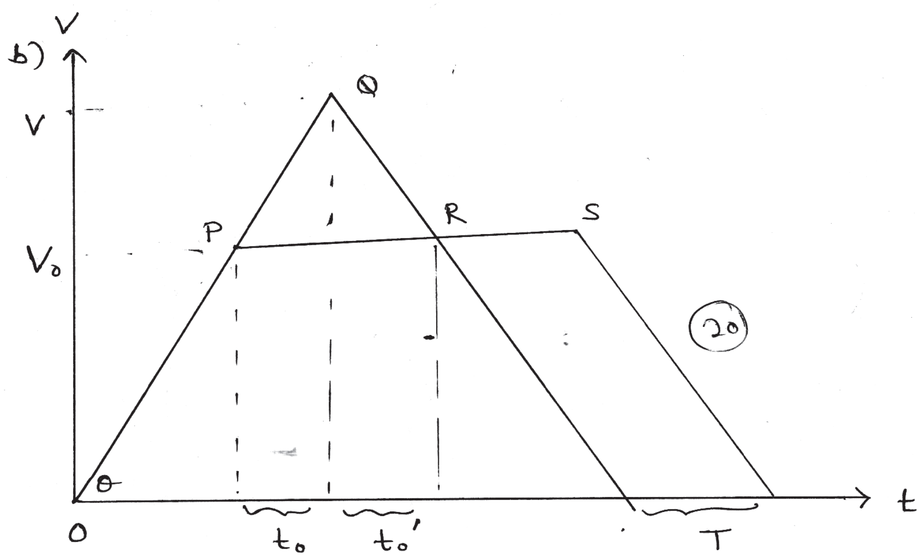
$$t_1 + t_2 = V\left[\frac{1}{a} + \frac{1}{a}\right] = \frac{2V}{a}$$

$$t_1 + t_2 = \left[2S\frac{(a+a)}{a^2}\right]^{\frac{1}{2}} = 2\sqrt{\frac{S}{a}} \quad (10)$$

$$\text{minimum time} = 2\sqrt{\frac{S}{a}} \quad (10)$$

20

b)



(20)

20

since the distances travelled are equal,

Area of PQR = Area of RSXY (5)

$$\frac{1}{2}(V - V_0)(t_0 + t_0') = V_0 T \quad \text{—(A)} \quad (5)$$

$$a_1 = \frac{V - V_0}{t_0} \Rightarrow t_0 = \frac{V - V_0}{a_1} \quad (10)$$

$$a_2 = \frac{V - V_0}{t_0'} \Rightarrow t_0' = \frac{V - V_0}{a_2} \quad (10)$$

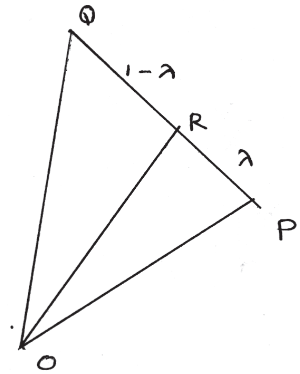
$$(A) \Rightarrow \frac{1}{2}(V - V_0)^2\left(\frac{1}{a_1} + \frac{1}{a_2}\right) = V_0 T \quad (5)$$

$$V = \sqrt{\frac{2Sa_1a_2}{a_1 + a_2}} \Rightarrow \frac{2S}{V^2} = \frac{1}{a_1} + \frac{1}{a_2} \quad (10)$$

$$T = \frac{S(V - V_0)^2}{V_0 V^2} \quad (5)$$

50

(13)

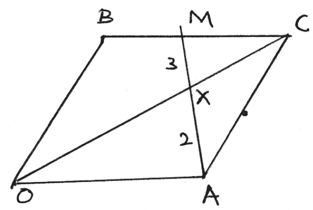


$$\frac{PR}{PQ} = \frac{\lambda}{1}$$

$$\overrightarrow{PR} = \lambda \overrightarrow{PQ} = \lambda \left[\overrightarrow{PO} + \overrightarrow{OQ} \right]$$
$$= \lambda (\underline{q} - \underline{p})$$

$$\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PR}$$
$$= \underline{p} + \lambda (\underline{q} - \underline{p})$$

50



$$\overrightarrow{OA} = \underline{a}, \quad \overrightarrow{OB} = \underline{b}$$

$$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = \underline{a} + \frac{1}{2}\underline{b}$$ (20)

$$\overrightarrow{OM} = \underline{b} + \frac{1}{2}\underline{a}$$ (20)

$$\overrightarrow{OX} = \overrightarrow{OA} + \frac{2}{5}\overrightarrow{AM}$$ (10)

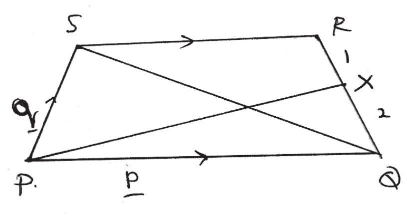
$$= \underline{a} + \frac{2}{5}\left(-\underline{a} + \underline{b} + \frac{1}{2}\underline{a}\right)$$

$$= \underline{a} + \frac{2}{5}\left(\underline{b} - \frac{1}{2}\underline{a}\right)$$ (20)

$$= \frac{4}{5}\underline{a} + \frac{2\underline{b}}{5}$$ (10)

$$= \frac{4}{5}\left(\underline{a} + \frac{1}{2}\underline{b}\right) = \frac{4}{5}\overrightarrow{OC}$$ (20)

(14) $\alpha \underline{a} + \beta \underline{b} = 0 \Rightarrow \alpha = 0, \beta = 0$ for the ~~Proof~~ (40)



$$\overrightarrow{PX} = \overrightarrow{PQ} + \frac{2}{3}\overrightarrow{QR}$$ (10)

$$= \underline{p} + \frac{2}{3}(QP + PR)$$

$$= \underline{p} + \frac{2}{3}\left(-p + q + \frac{p}{3}\right)$$ (10)

$$= \underline{p} - \frac{4p}{9} + \frac{2}{3}q = \frac{5q}{9} + \frac{2q}{3}$$ (20)

$$\overrightarrow{PY} = \overrightarrow{PQ} + \overrightarrow{QY}$$
$$= (1-\lambda)\underline{p} + \lambda \underline{q}$$ (20)

$$\overrightarrow{PY} = \mu \overrightarrow{PQ}$$

$$(1-\lambda)\underline{p} + \lambda q = \mu\left(\frac{5}{9}p + \frac{2}{3}q\right)$$

$$\underline{p}\left(1 - \lambda - \frac{5\mu}{9}\right) + \underline{q}\left(\lambda - \frac{2\mu}{3}\right) = 0$$ (20)

Since $p \neq 0, q \neq 0$ (10)

$$\lambda = \frac{2}{3}\mu, \quad \lambda + \frac{5\mu}{9} = 1$$

$$\mu = \frac{9}{10}$$

$$\lambda = \frac{6}{11}$$ (20)

(15) If the resultant is R,

$R = 3\underline{i}+\underline{j} + 2\underline{i}+4\underline{j} + \underline{i}+5\underline{j} + P\underline{i} + Q\underline{j}$

$= (6+P)\underline{i} + (10+Q)\underline{j}$ (30)

To be equivalent to a couple G, $R = 0$ (10)

$\Sigma X_i = 0$ $\Sigma Y_i = 0$

$6+P=0$ (10) $10+Q=0$ (10)

$P=-6$ $Q=-10$

∴ The system reduces to a single force acting at $\underline{i}+\underline{j}$

$R = Y\underline{j}$

$Y\underline{j} = (6+P)\underline{i} + (10+Q)\underline{j}$ (20)

$\Sigma X_i = 0.$

$6+P=0$, $P=-6N$

$Y = 10+Q$ (20)

$Y\underline{j}$ is equivalent to a single force, the moments about the point $\underline{i}+\underline{j}$ should be equal to zero. (10)

The moment about O, = moments about Y_i (10)

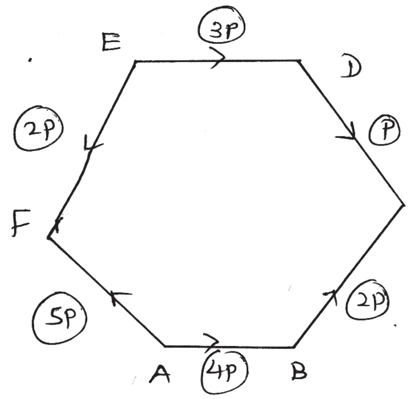
$2Q+5-4-6-(-8+2+3P) = Y\times 1$

$2Q -5+8-2-3P = Y$

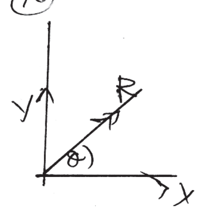
$Q = -9$ (30)

$Y = 1$

(16)



(10)



$\rightarrow X = 4P + 3P - 4P\cos 60 = 5P$ (5)

$\uparrow Y = 7P\cos 30 - 3P\cos 30 = 2\sqrt{3}P$ (5)

$$R = \sqrt{X^2+Y^2} = \sqrt{(5P)^2+(2\sqrt{3}P)^2} = \sqrt{37}P \quad 5$$

$$\tan\theta = \frac{Y}{X} = \frac{2\sqrt{3}}{5} \Rightarrow \theta = \tan^{-1}\left(\frac{2\sqrt{3}}{5}\right) \quad 5$$

30

$\curvearrowleft A \quad -Y \times x = -2P\cos 60 \times 2a\cos 30 + 3P\times 2a\cos 30 +$
$P\cos 30 \times a + P\cos 60 \times 2a\cos 30 - 2P\cos 30 \times a$

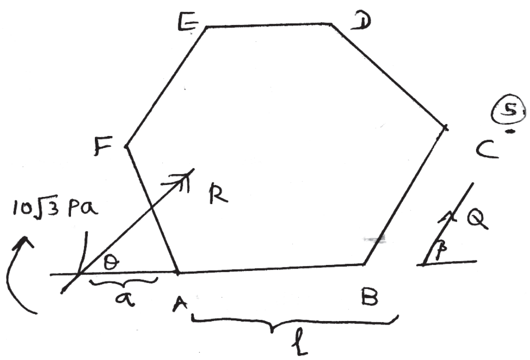
$= \sqrt{3}Pa + 3\sqrt{3}Pa$

$-2\sqrt{3}Pa = 2\sqrt{3}Pa$

$x = -a \quad 10$

10

It lies at a distance a from A on the produced line BA.



Since the new system is in equilibrium

$\rightarrow X = 0$

$\uparrow Y = 0$

$\curvearrowleft A = 0 \quad 5$

$\rightarrow \quad R\cos\theta + Q\cos\beta = 0$

$Q\cos\beta = -R\cos\theta = -8P \quad 5$

$Q\sin\beta = -R\sin\theta = -2\sqrt{3}P \quad 5$

$\tan\beta = \tan\theta$

$\beta = \theta = \tan^{-1}\left(\frac{2\sqrt{3}}{5}\right) \quad 5$

$R^2 = Q^2 \Rightarrow R = Q \quad 5$

$\curvearrowleft A \quad R\sin\theta \times a = 10\sqrt{3}Pa - Q\sin\beta \times \ell$

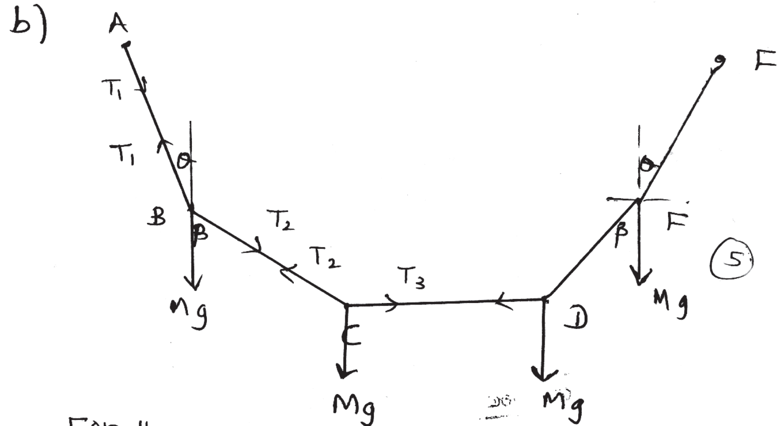
$2\sqrt{3}P \times a = 10\sqrt{3}Pa - 2\sqrt{3}P \times \ell$

$2\sqrt{3}P\ell = 8\sqrt{3}Pa \quad 10$

$\ell = 4a$

40

b)



(5)

05

For the equilibrium of B,

$\uparrow\ T_1 \cos\theta = Mg + T_2 \cos\beta$ (10)

$T_1 \sin\theta = T_2 \sin\beta$ (10)

$T_1 = 2Mg \sec\theta$ (10)

$2Mg \sec\theta \sin\theta = Mg \dfrac{\sin\beta}{\cos\beta}$ (10)

$2\tan\theta = \tan\beta$

For the eq^n of C,

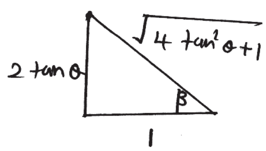
$\uparrow\ T_2 \cos\beta = Mg$

30

For the equilibrium at C,

$T_3 = T_2 \sin\beta = \dfrac{Mg}{\cos\beta} \sin\beta$ (10)

$T_3 = Mg \tan\beta = 2Mg \tan\theta$



10

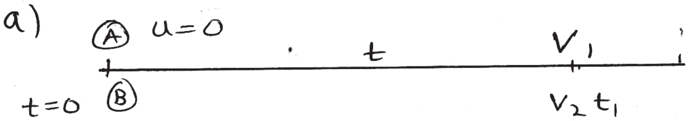
$\cos\beta = \dfrac{1}{\sqrt{4\tan^2\theta+1}}$ (5) 10

The tension in the part BC, $T_2 = \dfrac{Mg}{\cos\beta}$ (10)

$= Mg\sqrt{4\tan^2\theta+1}$

15

(17) a)



$\underline{V}_{A,E} = 0$ $\quad V_{B,A} = 0$

$\underline{a}_{B,A} = \underline{a}_{B,E} + \underline{a}_{E,A}$ (5)

$= \rightarrow f + \leftarrow f)$

$\underline{a}_{A,B} = 0$ (5), $\underline{V}_{A,B} = 0$ (5)

when $t=t$, $V_{A,E} = V_1$

$V_{B,E} = V_2 = V_1$

15

$t=t$ $\quad V_{AB} = V_{AE} + V_{EB}$

$= V_1 - V_2 = 0$ (10)

$\underline{a}_{A,B} = \underline{a}_{A,E} + \underline{a}_{E,B} = 0$ (5)

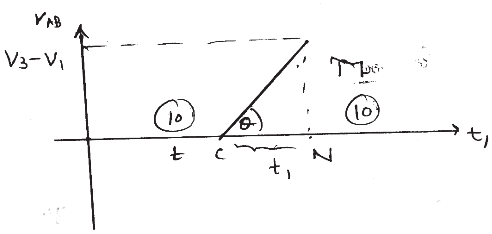
when $t = t + t_1$, $V_{AE} = V_3$

$V_{BE} = V_1$

$V_{A,B} = V_{A,E} + V_{E,B} = V_3 - V_1$ 5

$\underline{a}_{A,B} = \underline{a}_{A,E} + \underline{a}_{E,B} = f + 0$ 5

25



$\tan\theta = f$ 5

$= \dfrac{V_3 - V_1}{t_1} = f$

20

The distance between A and B = Area of LMN.

$= \frac{1}{2} t_1 (V_3 - V_1)$

$= \frac{1}{2} f t_1^2$ 10

15

b)

P, 2u

Q = 3u

at $t=0$, $V_{P,E} = \uparrow 2u$

$\underline{V}_{Q,P} = V_{Q,E} + V_{E,P}$ 10

$= 0 + (-2u) = -2u$

$\underline{a}_{Q,P} = \underline{a}_{Q,E} + \underline{a}_{E,P}$

$= 0 + g$ 5

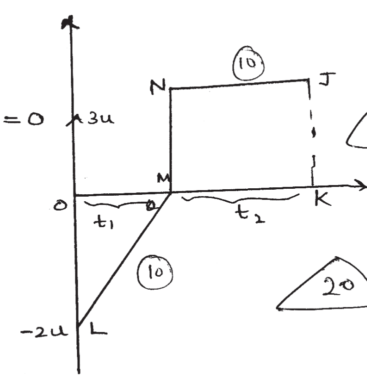
15

when $t = t$,

$V_{P,E} = 0$, $V_{Q,E} = 3u$

$\underline{V}_{Q,P} = \underline{V}_{Q,E} + \underline{V}_{E,P} = 3u + 0$ 10

$\underline{a}_{Q,P} = \underline{a}_{QE} + \underline{a}_{E,P} = (-g) + (g) = 0$ 5



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$\tan\theta = g = \dfrac{2u}{t_1}$ 5 $\Rightarrow t_1 = \dfrac{2u}{g}$

$0 = \frac{1}{2} \times 2u \times t_1 - 3u\, t_2$ 5

$3u\, t_2 = u t_1$

$t_2 = \dfrac{t_1}{3}$ 5

Total time taken to collide $= t_1 + t_2$ 5

$= \dfrac{2u}{g} + \dfrac{2u}{3g}$

$= \dfrac{8u}{3g}$ 5

25