Provincial Department of Education - NWP

10 E I

First Term Test - Grade 13 - 2019

Index No : ....

Combined Mathematics I

Three hours only

Instructions:

* *This question paper consists of two parts.*
  Part A (Question 1 - 10) and **Part B** (Question 11 - 17)
* **Part A**
  *Answer **all** questions. Write your answers to each question in the space provided. you may use additional sheets if more space is needed.*
* **Part B**
  Answer five questions only. Write your answers on the sheets provided.
* *At the end of the time allocated, tie the answers of the two parts together so that* **Part A** *is on top of* **part B** *before handing them over to the supervisor.*
* *You are permitted to remove only* **Part B** *of the question paper from the Examination Hall.*

For Examiner's Use only

(10) Combined Mathematics I		
Part	Question No	Marks Awarded
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
	Total	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	
Paper 1 total		
Percentage		

Paper I	
Paper II	
Total	
Final Marks	

Final Marks

In Numbers	
In Words	

Marking Examiner	
Marks Checked by 1	
2	
Supervised by	

Combined Maths 13 - I (Part A)

01. If there is no remainder when the polynomial $ax^3 + bx^2 + cx + d$ is divided by $\left(x^2 + k^2\right)$ show that, $ab = cd$.

02. Using the graphs solve the inequality $2 + |x - 4| > (x - 4)^2$.

03. Find the range of values of a such that the equation $(a-2)x^2 - 3(a+2)x + 6a = 0$ has two real and distinct roots. Here $a \in \Re$.

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04. Evaluate, $\lim_{\theta \to \frac{\pi}{2}} \dfrac{1-\sin\theta}{\left(\frac{\pi}{2}-\theta\right)\cos\theta}$.

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05. Find the area bounded by the curve $y = x(x-2)$ and the line $y = -\dfrac{x}{2}$ using integration.

06. Show that the equation of the normal line to the parabola $y^2 = 4x$ at the point $(t^2, 2t)$, is $y + tx = 2t + t^3$.

If the normal line meets this parabola again at the point $(T^2, 2T)$, show that $t^2 + tT + 2 = 0$.

07. Each of two parallel lines makes an angle α with the positive direction of x axis . One line passes through the point (h, k) and the other line passes through the point (m, n). Show that the perpendicular distance between the lines is $|(h-m)\sin\alpha-(k-n)\cos\alpha|$.

08. Let $A=(2,-1)$ and $B=(4,-3)$. $C=(3t,-t)$ is a point on the perpendicular bisector of AB. Here $t\in R$ Find the value of t and find the coordinates of D such that $ACBD$ form a rhombus.

09. Solve $2\tan^{-1}(x-1)+\tan^{-1}x=\dfrac{\pi}{2}$.

10. Find the values of x in the range $[-\pi,\pi]$, which satisfies the equation $\cos 2x+3\sin x=2$.

Part B

❖ **Answer only 05 questions**.

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11. *a.* If and α, β are the roots of the equation, $ax^2+bx+c=0$ find the value of $(\alpha-\beta)^2$ in terms of a, b and c. Obtain the roots of the equation $(c-b+a)x^2+(b-2a)x+a=0$ in terms of α, β.

b. (i) If there is a common root for the equations $ax^2+a^2x+1=0$ and $bx^2+b^2x+1=0$, show that the quadratic equation $abx^2+x+a^2b^2=0$ is satisfied by the other roots of them.

ii. Show that for real x, there is no real value of the expression $\dfrac{x^2+2x-1}{2x-1}$ between 1 and 2 .

c. Let, $f(x)=ax^3+bx^2+x+2=0$. It is given that $(x-1)$ is a factor of $f(x)$ and the remainder when $f(x)$ is divided by $(x+1)$ is -6. Find the values of a and b. Express $f(x)$ as a product of linear factors for these values of a and b.

12. *a.* Sketch the graph of $y=|x|-2$

Hence obtain the graph of $y=\big||x|-2\big|$. Solve the inequality $\dfrac{\big||x|-2\big|}{2}>1$ using above graph.

b. $f(x)$ is a polynomial of degree 4 and it is divisible by x^2+2. When it is divided by $(x+1)^2(x-2)$ the reminder is $6x^2-3x$. Find the polynomial. Show that there are no real roots for the equation $f(x) = 0$.

c. Express $\dfrac{4x^2-x+2}{x(x+1)^2}$ in partial fractions.

d. When $a, b, c \in \Re$, show that $\log_a c=\dfrac{\log_b c}{\log_b a}$. Hence deduce that $\log_a c=\dfrac{1}{\log_c a}$.

Show that $\log_a x\cdot\log_b x+\log_b x\cdot\log_c x+\log_c x\cdot\log_a x=\dfrac{\log_a x\cdot\log_b x\cdot\log_c x}{\log_{abc} x}$.

13. The equation of a curve is given by $y=\dfrac{3x+4}{(2x+1)(x-2)}$.

i. Express $\dfrac{3x+4}{(2x+1)(x-2)}$ in partial fractions.

Hence find the partial fractions of $\dfrac{(3x+4)^2}{(2x+1)^2(x-2)^2}$.

ii. Show that $\dfrac{dy}{dx}=\dfrac{2}{(2x+1)^2}-\dfrac{2}{(x-2)^2}$. Hence or otherwise show that there is a turning point at $x=-3$. Find x coordinates of other stationary points on the curve.

iii. Find, $\dfrac{d^2y}{dx^2}$ and determine the nature of the turning point at $x=-3$.

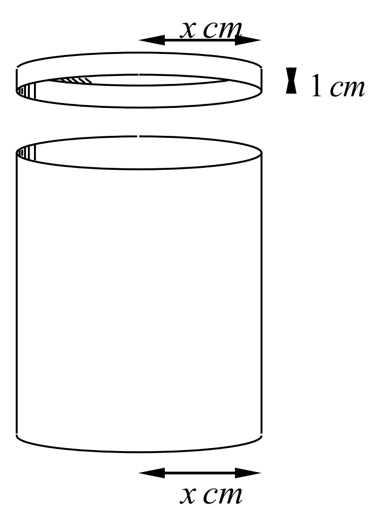
iv. Sketch the graph of $y=\dfrac{3x+4}{(2x+1)(x-2)}$ by indicating the turning points and asymptotes.

v. Find $\displaystyle\int\frac{3x+4}{(2x+1)(x-2)}dx$ and hence show that the area bounded by the curve, x axis and the lines $x=4$ and $x=12$ is $\ln 15$.

Find the volume of the solid generated by rotating this region about x axis.

14. *a.* Let $f(x)=\dfrac{(x-1)(x-5)}{(x-4)(x-2)}$ for $x\neq 0, x\neq 2$. Show that $f'(x)=\dfrac{6(x-3)}{(x-2)^2(x-4)^2}$ and find values of x such that $f'(x)=0$.

Sketch the graph of $y=f(x)$ by indicating the turning points and asymptotes.



b. It is needed to make a biscuit tin using a thin metal sheet of area $80\pi\ cm^2$ without any wastage. The radius of the lid same as the radius of the tin $x\,cm$. The lid overlaps the tin by $1\,cm$ as shown in the figure. The volume of tin is $V\ cm^3$ Here x is a variable.

i. Show that $V=\pi(40x-x^2-x^3)$

ii. Using the derivative of *v*, find the radius when the volume is maximum.

15. *a.* Let $f(x)=\dfrac{1}{x(x-1)^2}$. Express $f(x)$ in partial fractions.

Evaluate $\int f(x)dx$. Hence or otherwise evaluate $\displaystyle\int\frac{1}{(e^t-1)^2}\,dt$.

b. Using the integration by parts, find $\int xe^{-2x}dx$.

c. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$.

Using the above result, evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin^3 x}{\sin x + \cos x} dx$.

16. In the parallelogram $PQRS$, the equations of the sides PQ, QR, RS and SP are $ax+by+c_1=0, lx+my+n_1=0, ax+by+c_2=0$ and $lx+my+n_1=0$ respectively. Show that the area of parallelogram is $\frac{|c_1-c_2||n_1-n_2|}{|am-bl|}$.

In the rhombus $ABCD$ the equations of the sides AB and AC are $x-y+1=0$ and $2x-y-1=0$ respectively. If the side BC passes through the point $(5,-6)$, find the equations of BC, CD and DA .

Using the results of the first part, find the area of the rhombus $ABCD$.

17. a. If $\theta = 36^\circ$, show that $\sin 3\theta = \sin 2\theta$ Hence deduce that $\cos 36^\circ = \frac{\sqrt{5}+1}{4}$.

Further show that, $\sin 12^\circ \sin 48^\circ \sin 54^\circ = \frac{1}{8}$.

b. State the Sine rule for a triangle with usual notation.

The angular bisector of the interior angle $\hat{A}$ meets BC at D. Show that $AD = \frac{2bc}{(b+c)}\cos\frac{A}{2}$.

Find the length of the angular bisector of the interior angle $\hat{B}$.

If the angular bisectors of the interior angles $\hat{A}$ and $\hat{B}$ are equal in length, show that $\sin B\cos\left(\frac{A-C}{2}\right) = \sin A\cos\left(\frac{B-C}{2}\right)$.

c. Express $f(x) = 2\cos^2 x + 4\sin x\cos x - 2\sin^2 x$ in the form $f(x) = A\sin(2x+\alpha)$. Here $A>0$ and α is an acute angle. Hence draw a rough graph of $y=f(x)$ in the range $-\pi \le x \le \pi$.

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Provincial Department of Education - NWP

10 | E | II

First Term Test - Grade 13 - 2019

Index No : ....

Combined Mathematics II

Three hours only

Instructions:

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	15	
	16	
	17	
	Total	
Paper 1 total		
Percentage		

Paper I	
Paper II	
Total	
Final Marks	

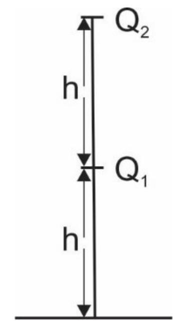
Final Marks

In Numbers	
In Words	

Marking Examiner	
Marks Checked by 1	
2	
Supervised by	

(Part A)

01) A particle Q_1 is released from the highest point of the ground floor of a building with two stores with equal height from the ground level. At the same instant another particle Q_2 is projected vertically downwards with initial velocity u from the highest point of the first floor, such that both particles meet each other at the ground level. (see the diagram). If the time taken for both particles to meet each other is T, draw the $v-t$ graphs for the motion of both particles in the same diagram. Hence show that $u = \frac{gT}{2}$. Here g is the acceleration due to gravity.

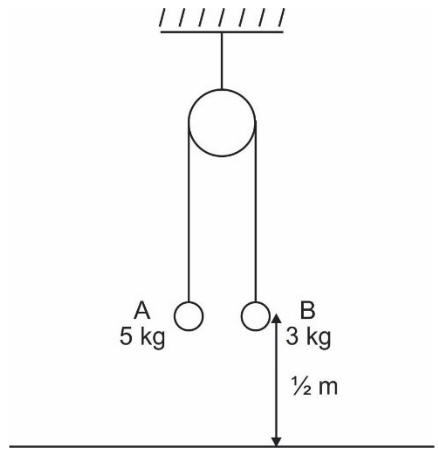


02) A sphere A of mass $2m$ moving with an initial velocity u on a smooth horizontal table, collides directly with a smooth sphere B of mass m with the same size. If the coefficient of restitution for the collision is e $(0 < e < 1)$, obtain the velocity of the sphere B after the collision. If $e = \frac{1}{4}$, show that the impulsive force on the particle B is $\frac{5mu}{6}$.

03) A particle projected under gravity at an angle θ with the horizontal from a point O, passes through the maximum point P which is inclined at an angle α to the horizontal from O, show that $\tan\theta = 2\tan\alpha$.

04) A train engine of mass M metric tonnes, power $H\ Kw$ travels along a horizontal road with a constant velocity $v\ ms^{-1}$ under a resistive force of R N. Show that $R = \frac{\mathrm{H \times 10^3}}{\mathrm{V}}$. For this engine to travel upwards along an inclined plane of 30^0 to the horizontal with the same constant velocity v and under the same resistive force, show that the new power of the engine should be $\left(\frac{Mgv}{2} + H\right) kw$.

05) Two particles A and B with masses $5kg$ and $3\,kg$ respectively are connected by a light inextensible string passing over a smooth light pulley fixed to a ceiling. Initially the two particles are held in a same horizontal level at a height of $\frac{1}{2}m$ above the floor with the strings taught and vertical. (see the diagram). When the system is released gently show that the acceleration of the system is $\frac{g}{8}\ m$. If the strings are long enough for the particle A to hit the ground and the particle B doesn't hit the pulley, show that the particle B reaches a height of $\frac{9}{8}\ m$ above the ground level for the first time. Here g is the acceleration due to gravity.



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06) The position vectors of two non collinear points A and B with respect to the origin O are $\underline{a}$ and $\underline{b}$ respectively. C is a point on OA such that $\overrightarrow{OC} = \frac{2}{3}\overrightarrow{OA}$. D is a point on OB such that $\overrightarrow{OD} = 2\overrightarrow{OB}$ and E is a point on AB such that $\overrightarrow{AE} = \frac{1}{2}\overrightarrow{AB}$.

i) Find $\overrightarrow{CE}$ and $\overrightarrow{ED}$ in terms of $\underline{a}$ and $\underline{b}$

ii) Show that C, E and D are collinear and find the ratio $CE : ED$.

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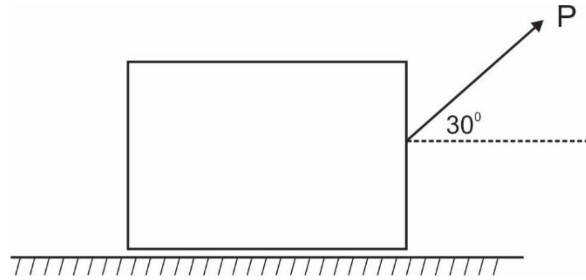
07) A force $P\ (< 2w)$ inclined at an angle 30^0 to the horizontal is applied on a wooden block of weight w, which is placed on a rough horizontal table (See the figure). If the coeffient of friction between the table and the wooden block is μ , show that the maximum value of P that should be applied such that the block is in equilibrium without moving is $\frac{2\mu w}{\sqrt{3}+\mu}$.



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08) A smooth peg is at a horizontal distance $2\sqrt{2}\,a$ from a smooth wall. An uniform rod AB of weight W and length $16a$ is in equilibrium in a vertical plane with the end A against the wall and a point on the rod is in contact with the peg. Find the angle which the rod makes with the horizontal and show that the reaction on the rod by the peg is $\sqrt{2}\,w$.

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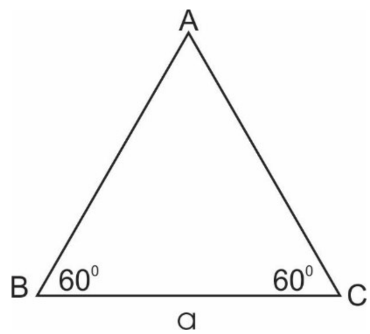
09) The forces $3P, 4P$ and $5P$ act along the sides AB, BC and CA respectively of an equilateral tringle of side a in the direction indicated by the order of the letters.

i) Find the magnitude and the direction of the resultant.

ii) Assuming that the line of action of the resultant acts at a point, which is on the line BC produced at a distance of $\frac{3a}{2}$ from C, a single force is applied at B for the system to be reduced to a couple. Find the moment of the couple.



10) A particle P of mass m is placed on the top of a fixed sphere of center O and radius a and it is gently displaced from its position of equilibrium. Find the angle which OP makes with upward vertical and the velocity of the particle when it leaves the sphere.

Combined Maths 13 - II (Part B)

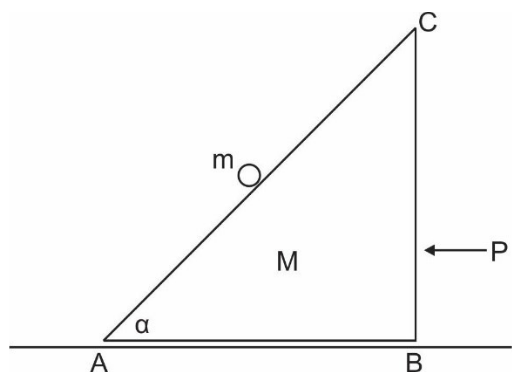
11) (a) A motor car A which moves with uniform velocity u along a straight level road, passes a police check point X. After time T from that instant a police car B starts from rest at X and moves with a uniform acceleration f along a level road parallel to it, to catch the car.

After obtaining the velocity of $2u$, the police car moves with the constant velocity. Draw velocity time graphs for both motions in the same diagram. Hence when B reaches its maximum velocity, show that the distance between two cars A and B is UT. Also prove that the time taken for the motor car B to pass the motor car A, from the moment which the motor car A passes the check point X is $2\left(T+\frac{u}{f}\right)$.

(b) Two airports B and C are situated at equal distance d due North and South of the airport A respectively. On a day when there is a wind blowing with constant velocity v, in a direction making an angle θ towards East from North, two planes moving with equal uniform velocity u relative to wind, begin their journeys to B and C from A at the same.

Using the principle of relative velocity draw the velocity triangles for both planes in the same diagram. Hence show that $t_1 - t_2 = \frac{2dv\cos\theta}{v^2-u^2}$, here t_1 and t_2 are the time taken by two planes to complete above journeys respectively. If both planes return to the airport A again, deduce that they reach A at the same instant.

12) A smooth wedge of mass M with vertical cross section ABC is kept on a smooth horizontal table. A particle of mass m is placed on a face of the wedge inclined at an angle α to the horizontal. As shown in the figure, a horizontal force P is applied on the other face such that the face containing the particle moves forward. By applying the equations of motion for the system in the direction BA and for the particle in the direction CA,



(i) show that the acceleration of the wedge relative to the earth is $\frac{P - mg\sin\alpha\cos\alpha}{M+m\sin^2\alpha}$.

(ii) show that the acceleration of the particle relative to the wedge is $\frac{(M+m)g\sin\alpha - P\cos\alpha}{M+m\sin^2\alpha}$

(iii) show that if $P \le \frac{(M+m)g\sin\alpha}{g\cos\alpha}$, the particle m moves upwards along the inclined plane or it comes to rest on the plane.

(iv) If the system is released gently, find acceleration of the wedge relative to earth and the acceleration of the particle relative to the wedge. Further show that the acceleration of the particle relative to the earth is $\frac{g\sin\alpha}{M+m\sin^2\alpha}\{M^2+ m\sin^2\alpha\,(m+2M)\}^{\frac{1}{2}}$.

13) (a) A smooth circular tube of radius a and center O is fixed in a vertical plane. A smooth particle of mass m is kept on the lowest point of the tube. A velocity of u is applied on the particle such that it moves in a vertical circular path. When the particle makes an angle θ with the downward vertical through the center, show that the velocity v of the particle is given by $v^2 = u^2 + 2ag\cos\theta - 2ag$ and the reaction R on the particle m by the tube is given by $R = \frac{m}{a}(u^2 + 3ga\cos\theta - 2ga)$.

If the particle leaves its circular motion, show that $2ag < u^2 < 5ag$.

Also show that $u^2 > 5ag$, if the particle completes its circular motion.

(b) A ship sails with uniform velocity u in a straight path. A gun is fixed to the ship making an angle θ with the horizontal such that it aims the direction of motion of the ship. At a certain instant a bullet is released from the gun with a velocity $\sqrt{3}\,u$ so that it hit a castle of height h which is at a horizontal distance d from the gun. Assuming that the gun is in the level of sea, if the bullet hits castle, show that

$$gd^2 + 2u^2\left(1 + \sqrt{3}\cos\theta\right)^2 h - 2u^2\sqrt{3d}\,\sin\theta\left(1 + \sqrt{3}\cos\theta\right) = 0$$

14) (a) $\underline{a}$ and $\underline{b}$ are two non-zero, non-parallel vectors. When α and β are two scalars, prove that $\alpha\underline{a} + \beta\underline{b} = 0$ if and only if $\alpha = 0$ and $\beta = 0$.

$OABC$ is a parallelogram. $\overrightarrow{OA} = \underline{a}$ and $\overrightarrow{OB} = \underline{b}$. M and N are the midpoints of BC and AC respectively. Let $OP = \lambda\, ON$ and $AP = \mu\, AM$ (λ and μ are scalars). Find the values of λ and μ. Also find the ratios $OP:PN$ and $AP:PM$.

(b) A system of three coplanar forces (measured in Newtons) and their points of action are shown in the table below.

Point of action	Position vector	Force
A	$3\underline{i} + 2\underline{j}$	$4\underline{i} + 2\underline{j}$
B	$(-3\underline{i} + 0\,\underline{j})$	$-\underline{i} - \underline{j}$
C	$2\,\underline{i} - 3\,\underline{j}$	$-3\underline{i} - \underline{j}$

Here $\underline{i}$ and $\underline{j}$ are the unit vectors along the axes OX and OY in the rectangular Cartesian coordinate plane respectively. Lengths are measured in meters.

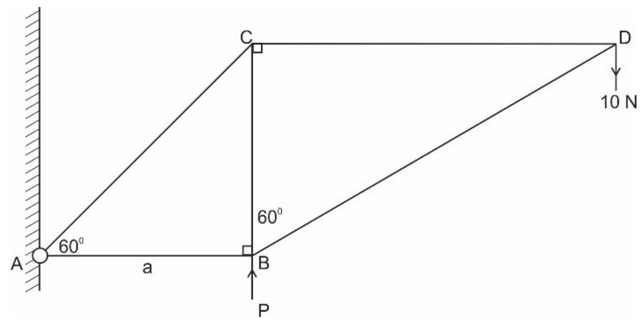
Mark the components of these three forces on the Cartesian coordinate plane indicating the coordinates of the points of action.

Show that the system is equivalent to a couple only. Find the additional force that should be applied for the system to be in equilibrium, other than the force $\left(\frac{5}{\sqrt{2}}\,\underline{i} + \frac{5}{\sqrt{2}}\,\underline{j}\right)$ applied at the origin. Also find the point of action of that additional force.

15) (a) An uniform rod AB of length $2a$ and weight w inclined at an angle $\tan^{-1}\frac{4}{3}$ to the floor rests in a vertical plane perpendicular to the wall with the end A on a rough horizontal floor and the end B against a rough vertical wall. The coefficient of friction between the rod and the floor and the rod and the wall is $\frac{1}{2}$.

A horizontal force P is applied at the point C on the rod where $AC = \frac{a}{4}$, so that the rod is in limiting equilibrium of slipping towards the wall. Show that $P = \frac{5W}{3}$.

(b) The framework shown in the following figure is made by joining five light rods AB, BC, AC, CD and BD freely at their ends.



It is given that $AB = a$ and $C\hat{A}B = C\hat{B}D = 60^0$. A weight of $10N$ is suspended from D and the frame work is hinged to a vertical wall at A. The framework is kept in equilibrium in a vertical plane by a vertical force applied at B, so that rods AB and CD are horizontal and the rod BC is vertical. Find the value of P.

Draw a stress diagram using Bow's notation. Hence find the stresses in the five rods stating whether they are tensions or thrusts.

16) (a) Four uniforms smooth rods AB, BC, CD and DA each of length $2a$ and weight w are smoothly jointed at A, B, C and D so as to form a rhombus. The rods AB and CD are placed on two smooth pegs on the same horizontal level at a distance $2l$ apart such that the joint A is above C . A weight $2w$ is hung at C. The rhombus is kept in equilibrium symmetrically in a vertical plane with $B\hat{A}D = 120^0$

(i) Find the reaction on a rod by a peg.

(ii) Show that the reaction at the joint B is $\frac{\sqrt{43}\, w}{2}$ and find the inclination of the reaction to the horizontal.

(iii) Also show that $l = \sqrt{3}\, a$.

(b) The end A of an uniform rod AB of weight w and length $2a$ is hinged freely to a vertical wall and the end B is a attached to a point C on the wall by a means of a light inextensible string of length $2a$. C is a point on the wall at a height $d(< 4a)$ vertically above A. The rod is in equilibrium in a vertical plane perpendicular to the wall.

(i) Show that the tension in the string is $\frac{Wa}{d}$.

(ii) Show that the reaction of the hinge A is $\left(\frac{2a^2 + d^2}{2}\right)^{\frac{1}{2}} \frac{w}{d}$.

(iii) Find the angle made by the reaction of the hinge at A with the horizontal.

17) (a) An engine of a train with mass 200 metric tonnes moves along a straight level road with a constant velocity of $72\ kmh^{-1}$ against a constant resistance of $30\ 000N$. Find the power of the train engine in kilowatts.

Then another cabin of mass 100 metric tonnes is connected to the engine by a connecting rod and it is pulled along the same road. The resistance against the engine remains unchanged and the resistance of the cabin is $10000N$. If the engine works with the same power, find the acceleration of the train and the tension of the connecting rod when the velocity of the train is $36\ kmh^{-1}$.

Subsequently the train with the cabin climb up a hill inclined at $sin^{-1}\left(\frac{1}{30}\right)$ with the same power under the same resistive forces with an acceleration of $\frac{1}{10}\ ms^{-2}$. Then show that the velocity of the rain is $\frac{15}{4}\ ms^{-1}$. (The acceleration due to gravity is $10\ ms^{-2}$)

(b) Two smooth spheres A and B with the same radii moves in the same direction on a smooth horizontal table and collide directly. The masses of A and B are $3m$ and $2m$ respetively. Also the velocities of A and B are $2u$ and u respectively. The coefficient of restitution between the spheres is e. After the collision find the velocities of the spheres.

Show that the impulse $J = \frac{6mu\,(1+e)}{5}$.

Also show that the loss of kinetic energy due to collision is $\frac{J}{2}\,(1-e)u$.

(01) $ax^3 + bx^2 + cx + d = (x^2 + k^2)(Ax + B)$ (5)

$x^3 \rightsquigarrow \; a = A$ — ①

$x^2 \rightsquigarrow \; b = B$ — ②

$x \rightsquigarrow \; c = k^2 A$ — ③ (10)

constant $\rightsquigarrow \; d = k^2 B$ — ④

$①/②$ $\quad \frac{a}{b} = \frac{A}{B}$ — ⑤ $\qquad \frac{③}{④}$ $\quad \frac{c}{d} = \frac{A}{B}$ — ⑥ (5)

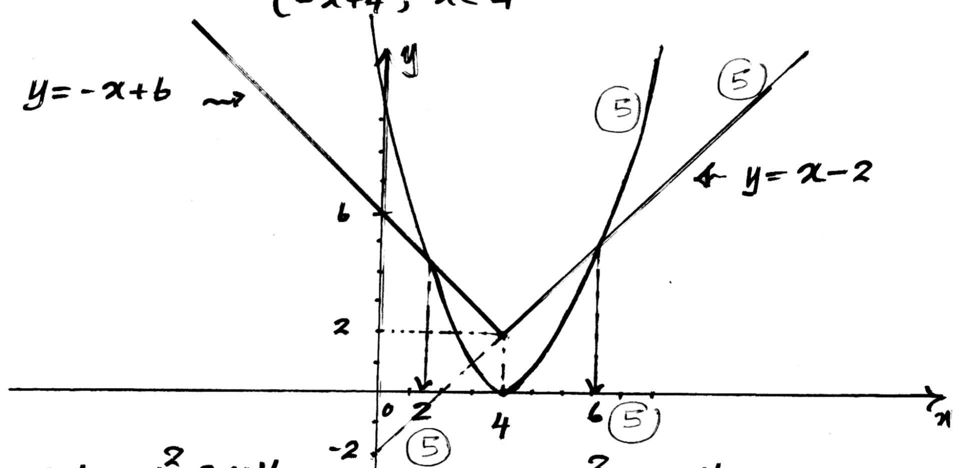
$⑤ = ⑥$ $\quad \frac{a}{b} = \frac{c}{d}$ (5)

$\underline{\underline{a : b = c : d}}$

$\boxed{25}$

(02)

$$|x-4| = \begin{cases} x-4 \; ; & x \geq 4 \\ -x+4 \; ; & x < 4 \end{cases}$$



$-x+6 = x^2 - 8x + 16$

$x^2 - 7x + 10 = 0$

$(x-5)(x-2) = 0$

$\underline{\underline{x=5}} \quad \underline{\underline{x=2}}$

$x - 2 = x^2 - 8x + 16$

$x^2 - 9x + 18 = 0$

$(x-6)(x-3) = 0$

$\underline{\underline{x=6}} \quad \underline{\underline{x=3}}$

$\boxed{25}$

$\underline{\underline{x \in (-\infty, 2) \cup (6, \infty)}}$ (5)

(03) $(a-2)x^2 - 3(a+2)x + 6a = 0.$

For real and distinct roots.

$a \neq 2$ and $\Delta > 0$ (5)

$\{-3(a+2)\}^2 - 4(a-2)6a > 0$ (5)

$9(a^2+4a+4) - 24a(a-2) > 0$

$3(a^2+4a+4) - 8a(a-2) > 0$

$3a^2+12a+12-8a^2+16a > 0$

$-5a^2+28a+12 > 0$

$5a^2-28a-12 < 0$

$(a-6)(5a+2) < 0$ (5)

	$(-\infty, -2/5)$	$(-2/5, 6)$	$(6, \infty)$
$(a-6)(5a+2)$	$+$	$-$	$+$ (5)

$\Rightarrow a \in \left(-\frac{2}{5}, 2\right) \cup (2, 6)$ (5)

$\boxed{25}$

(04) $\lim_{\theta \to \pi/2} \frac{1-\sin\theta}{\left(\frac{\pi}{2}-\theta\right)\cos\theta}.$

let $\theta - \pi/2 = x$

$\Rightarrow \theta = \pi/2 + x.$

$$\lim_{\theta \to \pi/2} \frac{1-\sin\theta}{(\pi/2-\theta)\cos\theta} = \lim_{x \to 0} \frac{1-\sin(\pi/2+x)}{-x\cos(\pi/2+x)}$$

$$= \lim_{x \to 0} \frac{1-\cos x}{x\sin x} \quad (5)$$

$$= \lim_{x \to 0} \frac{\sin^2 x}{x\sin x(1+\cos x)} \quad (5)$$

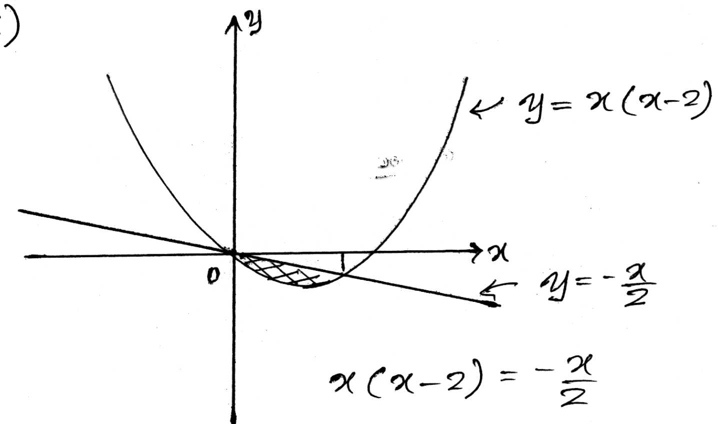
$$= \underbrace{\lim_{x \to 0}\left(\frac{\sin x}{x}\right)}_{1} \times \lim_{x \to 0} \frac{1}{1+\cos x} \quad (5)$$

$= 1 \times \frac{1}{2}$ (5)

$= \underline{\underline{\frac{1}{2}}}$ (5)

$\boxed{25}$

(05)



$x(x-2) = -\frac{x}{2}$

$2x^2 - 4x + x = 0$

$x(2x-3) = 0$

$x = 0$ or $x = 3/2$

$$\text{Area} = \int_0^{3/2} -x(x-2)\,dx - \int_0^{3/2} -\frac{x}{2}\,dx \quad (10)$$

$$= \left\{-\frac{x^3}{3} + \frac{2x^2}{2}\right\}_0^{3/2} + \left\{\frac{x^2}{4}\right\}_0^{3/2} \quad (5)$$

$$= \left\{-\frac{1}{3} \times \frac{27}{8} + \frac{9}{4}\right\} + \frac{1}{4}\left\{\frac{9}{4}\right\} \quad (5)$$

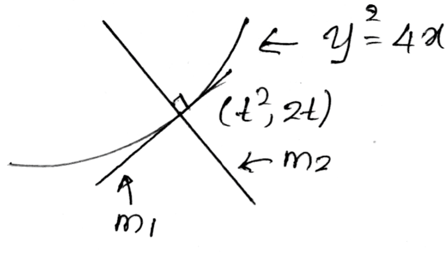
$$= \left\{-\frac{9}{8} + \frac{9}{4}\right\} + \frac{9}{16}$$

$$= 9\left\{\frac{1}{8}\right\} + \frac{9}{16}$$

$$= \underline{\underline{\frac{27}{16}}} \quad (5)$$

$\boxed{25}$

(06)



$$y^2=4x$$

$$2y\frac{dy}{dx}=4$$

$$\frac{dy}{dx}=\frac{2}{y}$$

$$m_1=\left(\frac{dy}{dx}\right)_{(t^2,2t)}=\frac{2}{2t}=\frac{1}{t} \quad (5)$$

$$\Rightarrow m_1m_2=-1.$$

$$m_2=-t \quad (5)$$

$\therefore$ Eq^n of the normal

$$\frac{y-2t}{x-t^2}=-t \quad (5)$$

$$y-2t=-tx+t^3$$

$$y+tx=2t+t^3$$

Since $(T^2, 2T)$ is on the normal

$$2T+tT^2=2t+t^3 \quad (5)$$

$$t^3+2t-tT^2-2T=0$$

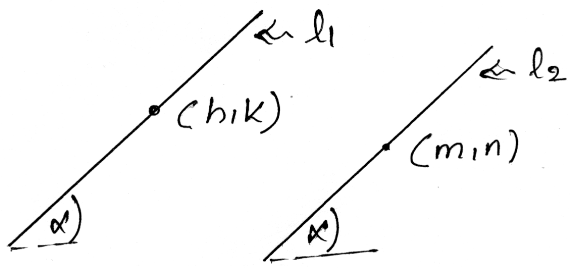
$$(t-T)(t^2+tT+2)=0$$

$$t-T\neq 0$$

$$\Rightarrow \underline{\underline{t^2+tT+2=0}} \quad (5)$$

$\boxed{25}$

(07)



Eq^n of the stright line l_1

$$\frac{y-k}{x-h} = \tan\alpha$$

$$y\cos\alpha - k\cos\alpha = x\sin\alpha - h\sin\alpha$$

$$y\cos\alpha - x\sin\alpha + h\sin\alpha - k\cos\alpha = 0. \quad (10)$$

Simillarly

Eq^n of the stright line l_2

$$y\cos\alpha - x\sin\alpha + m\sin\alpha - n\cos\alpha = 0. \quad (5)$$

$\therefore$ Pependicular distance =

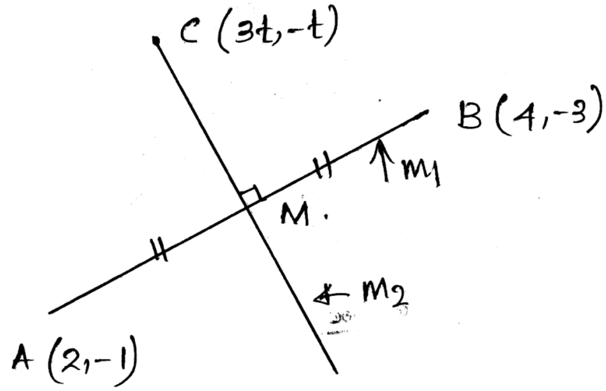
$$= \left| \frac{h\sin\alpha - k\cos\alpha - m\sin\alpha + n\cos\alpha}{\sqrt{\cos^2\alpha + \sin^2\alpha}} \right| \quad (5)$$

$$= \left| (h-m)\sin\alpha + (n-k)\cos\alpha \right|$$

$$= \underline{\underline{\left| (h-m)\sin\alpha - (k-n)\cos\alpha \right|}} \quad (5)$$

$\boxed{25}$

(08)



$$m_1 = \frac{-1+3}{2-4} = \frac{2}{-2} = -1$$

$$M \equiv \left(\frac{4+2}{2}, \frac{-3-1}{2}\right) \equiv (3,-2) \quad (5)$$

$$m_1 m_2 = -1$$
$$(-1)\, m_2 = -1$$
$$m_2 = 1$$

$\therefore$ Eq^n of the perpendicular bisector.

$$\frac{y+2}{x-3} = 1$$

$$y+2 = x-3$$

$$y - x + 5 = 0 \quad (5)$$

Since $C(3t,-t)$ is on the line $y-x+5=0$

$$-t - 3t + 5 = 0$$
$$4t = 5$$
$$t = 5/4 \ (5) \Rightarrow C \equiv \left(\frac{15}{4}, \frac{-5}{4}\right) \ (5)$$

Let $D \equiv (\bar{x}\ \bar{y})$

$$3 = \frac{\bar{x} + \frac{15}{4}}{2} \qquad -2 = \frac{\bar{y} - \frac{5}{4}}{2}$$

$$6 = \bar{x} + \frac{15}{4} \qquad \bar{y} = \frac{5}{4} - 4$$

$$\bar{x} = \frac{24-15}{4} = \frac{9}{4} \qquad = \frac{-11}{4}$$

$$D \equiv \left(\frac{9}{4}, \frac{-11}{4}\right) \quad (5)$$

(09) $\underbrace{2\tan^{-1}(x-1)}_{\alpha} + \underbrace{\tan^{-1}(x)}_{\beta} = \pi/2$

let $\alpha = \tan^{-1}(x-1)$ and $\beta = \tan^{-1}(x)$

$\Rightarrow \tan\alpha = x-1$ and $\Rightarrow \tan\beta = x$

$2\alpha + \beta = \pi/2$

$2\alpha = \pi/2 - \beta$ (5)

$\tan(2\alpha) = \tan(\pi/2 - \beta)$ (5)

$$\frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{1}{\tan\beta}$$ (5)

$$\frac{2(x-1)}{1-(x-1)^2} = \frac{1}{x}.$$ (5)

$2x(x-1) = 1 - x^2 + 2x - 1$

$2x^2 - 2x = -x^2 + 2x$

$3x^2 - 4x = 0$

$x(3x-4) = 0$

$x = 0$ or $x = 4/3$
#

$\therefore x = 4/3$ (5)

25

(10) $\cos 2x + 3\sin x = 2$

$1 - 2\sin^2 x + 3\sin x = 2$

5

$2\sin^2 x - 3\sin x + 1 = 0$

$2\sin^2 x - 2\sin x - \sin x + 1 = 0$

$2\sin x\,(\sin x - 1) - 1\,(\sin x - 1) = 0$

$(\sin x - 1)(2\sin x - 1) = 0$

$\sin x = 1$ or $\sin x = \frac{1}{2}$ 5

$\sin x = \sin \frac{\pi}{2}$ $\qquad$ $\sin x = \sin \frac{\pi}{6}$

$x = n\pi + (-1)^n \frac{\pi}{2}\ ;\ n \in \mathbb{Z}$ 5 $\qquad$ $x = n\pi + (-1)^n \frac{\pi}{6}\ ;\ n \in \mathbb{Z}$ 5

When $n = 0$

$\underline{x = \frac{\pi}{2}}$

When $n = 1$

$\underline{\underline{x = \frac{\pi}{2}}}$

When $n = 0$

$\underline{\underline{x = \frac{\pi}{6}}}$

When $n = 1$ 5

$\underline{\underline{x = \frac{5\pi}{6}}}$

$\boxed{25}$

(11) (a) $ax^2+bx+c=0 \rightarrow \alpha, \rightarrow \beta$

$\alpha+\beta = \frac{-b}{a}$ (5) $\alpha\beta = \frac{c}{a}$ (5)

$(\alpha-\beta)^2 = \alpha^2+\beta^2-2\alpha\beta$ (5)

$= (\alpha+\beta)^2 - 2\alpha\beta - 2\alpha\beta$ (5)

$= (\alpha+\beta)^2 - 4\alpha\beta$

$= \left(\frac{-b}{a}\right)^2 - 4\frac{c}{a}$ (5)

$= \frac{b^2}{a^2} - \frac{4ac}{a^2}$

$= \frac{b^2-4ac}{a^2}$ (5)

$(c-b+a)x^2+(b-2a)x+a=0$

roots are $\frac{(2a-b)\pm\sqrt{b^2-4ab+4a^2-(4ac-4ab+4a^2)}}{2(c-b-a)}$

$= \frac{(2a-b)\pm\sqrt{b^2-4ac}}{2(c-b-a)}$ (10)

$= \frac{(2-b/a)\pm\sqrt{(b/a)^2-4\,c/a}}{2(c/a-b/a-1)}$ (5)

$= \frac{2+\alpha+\beta\pm\sqrt{(\alpha-\beta)^2}}{2(\alpha\beta+\alpha+\beta+1)}$ (5)

55

(5)

$\therefore$ roots are $\frac{1+\alpha}{(\alpha+1)(\beta+1)}$ and $\frac{1+\beta}{(\alpha+1)(\beta+1)}$ //

(b)(i) Let the roots of $ax^2+a^2x+1=0$ are α, β. Also let $bx^2+b^2x+1=0$.'s roots are α, θ. (5)

$\therefore \quad a\alpha^2+a^2\alpha+1=0$ — ① and (5)

$b\alpha^2+b^2\alpha+1=0$ — ②

$① - ② \Rightarrow (a-b)\alpha^2+(a^2-b^2)\alpha=0$

$\therefore (a-b)\alpha(\alpha+a+b)=0$

$\therefore \quad \alpha=-a-b$

Also $\alpha+\beta=-a$
$\alpha+\theta=-b$ $\Rightarrow \beta=b$ (5) and $\theta=a$. (5)

$\therefore$ the quadratic equation whose roots are β, θ, is given by

$x^2-(\beta+\theta)x+\beta\theta=0$ (5)

i.e. $x^2-(a+b)x+ab=0$

But, since $\beta=b$ is a root of ①

$ab^2+a^2b+1=0$

i.e. $(a+b)=\frac{-1}{ab}$ (10)

$\therefore \quad x^2+\frac{1}{ab}x+ab=0$

i.e. $abx^2+x+a^2b^2=0$ (5) 40

(ii) Let $y=\dfrac{x^2+2x-1}{2x+1}$

i.e. $x^2+(2-2y)x+y-1=0$ (5)

for real values of

$(2-2y)^2-4.1.(y-1) \geq 0$ (5)

i.e. $4(y-1)^2 - 4(y-1) \geq 0$

i.e. $4(y-1)(y-1-1) \geq 0$

$\therefore \quad 4(y-1)(y-2) \geq 0.$ (5)

$\therefore$ This inequality and equation are satisfied when $y \geq 2$ or $y < 1$. (10)

25

(c). $f(x) = ax^3 + bx^2 + x + 2$

(5)

$f(1) = a + b + 3 = 0 \Rightarrow a + b = -3$ — ①

$f(-1) = -a + b + 1 = -6 \Rightarrow -a + b = -7$ — ② (5)

① + ② $\Rightarrow \quad 2b = -10 \Rightarrow b = -5$

$\therefore$ ① $\Rightarrow \quad a = -3 - (-5) = 2$

$\therefore f(x) = 2x^3 - 5x^2 + x + 2$ (5)

$f(x) = (x-1)(2x^2 + Ax - 2)$ (5)

$\Rightarrow \quad = 2x^3 + Ax^2 - 2x - 2x^2 - Ax + 2$

$= 2x^3 + (A-2)x^2 - (A+2)x + 2$

$= 2x^3 - 5x^2 + x + 2$

Compairing coefficients.

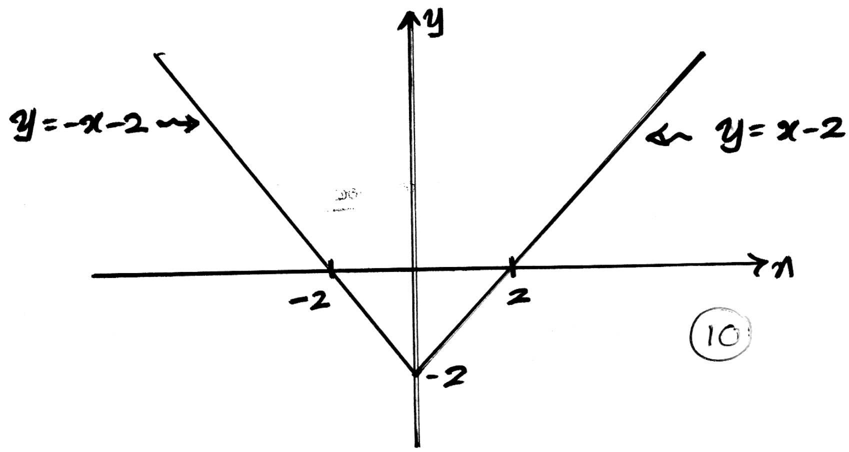
$\underline{x^2} \quad A - 2 = -5 \Rightarrow A = -3$ (5)

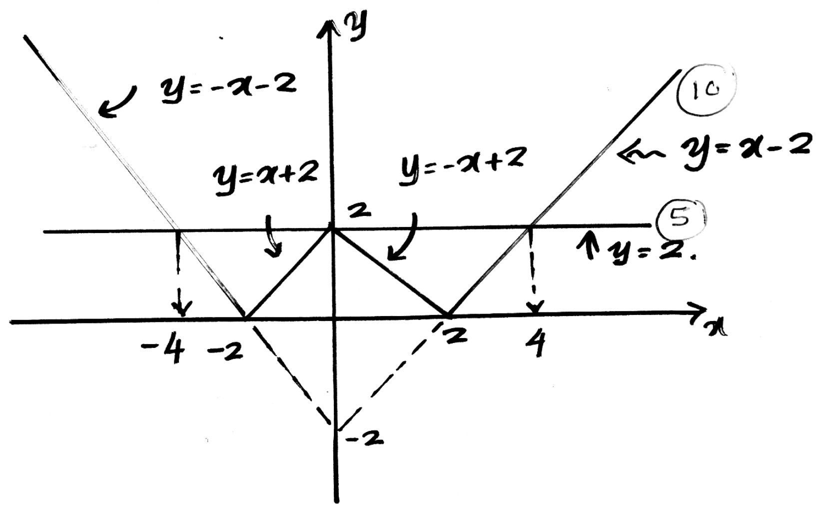
$\therefore f(x) = (x-1)(2x^2 - 3x - 2)$ (5)

$= \underline{(x-1)(x-2)(2x+1)}$ (10)

40

(12) a. $|x| = \begin{cases} x \; ; & x \geq 0 \\ -x \; ; & x < 0 \end{cases}$





$$\frac{||x|-2|}{2} > 1$$

$$||x|-2| > 2$$

$$\underline{\underline{x \in (-\infty, -4) \cup (4, \infty)}} \quad 5$$

30

$$f(x) = \underline{\underline{(x^2+2)(x^2-x+1)}} \quad 5$$

$$f(x) = 0$$

$$(x^2+2)(x^2-x+1) = 0$$

$$x^2+2 \neq 0 \qquad x^2-x+1=0$$

$$\Delta = (-1)^2 - 4\times1\times1$$

$$= -3 \quad 5$$

$$\Delta < 0$$

$\therefore$ The eq^n has not real roots.

55

(c) $$\frac{4x^2-x+2}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2} \quad 10$$

$$4x^2-x+2 \equiv A(x+1)^2 + Bx(x+1) + Cx$$

$x^2 \rightsquigarrow 4 = A+B$ — ①

$x \rightsquigarrow -1 = 2A+B+C$ — ②

Constant $\rightsquigarrow$ $\underline{\underline{2 = A}}$ 5

① $\Rightarrow$ $\underline{\underline{B=2}}$ 5

② $\Rightarrow$ $-1 = 4+2+C$

$\underline{\underline{C=-7}}$ 5

$$\frac{4x^2-x+2}{x(x+1)^2} = \underline{\underline{\frac{2}{x} + \frac{2}{x+1} - \frac{7}{(x+1)^2}}}$$

25

(b) $f(x) = (x^2+2)(Ax^2+Bx+C)$

$$= (x+1)^2(x-2)(Mx+N) + bx^2-3x \quad 15$$

$$(x^2+2)(Ax^2+Bx+C) \equiv (x+1)^2(x-2)(Mx+N) + bx^2-3x$$

$$\equiv (x^2+2x+1)(x-2)(Mx+N) + bx^2-3x$$

$$\equiv (x^3+2x^2+x-2x^2-4x-2)(Mx+N) + bx^2-3x$$

$$\equiv (x^3-3x-2)(Mx+N) + bx^2-3x$$

$x^4 \rightsquigarrow A = M$ — (1)

$x^3 \rightsquigarrow B = N$ — (2)

$x^2 \rightsquigarrow C+2A = -3M+6$

$C+2A = -3A+6$

$C+5A = 6$ — (3)

$x \rightsquigarrow 2B = -3N-2M-3$

$= -3B-2A-3$

$5B+2A = -3$ — (4)

constant $\rightsquigarrow 2C = -2N$

$C = -B$ 15

$C+B = 0$ — (5)

(3) and (5) $\Rightarrow$ $-B+5A = 6$ — (A)

$5B+2A = -3$ — (4)

(A) $\times 5 +$ (4) $\quad 27A = 30-3$

$\underline{\underline{A=1}}$ 5

(4) $5B+2 = -3$

$\underline{\underline{B=-1}}$ 5

(5) $\Rightarrow$ $\underline{\underline{C=1}}$ 5

d) $\log_a c = \dfrac{\log_b c}{\log_b a}$

Let $\log_a c = x$, $\log_b c = y$ and $\log_b a = z$

$\Rightarrow c = a^x$ — ① $\Rightarrow b^y = c$ — ② $\Rightarrow b^z = a$ — ③

① = ② $a^x = b^y$

from ③

$(b^z)^x = b^y$

$b^{xz} = b^y$

$xz = y$

$x = \dfrac{y}{z}$

$\log_a c = \dfrac{\log_b c}{\log_b a}$ 10

$\log_a c = \dfrac{\log_c c}{\log_c a}$ 5

$= \dfrac{1}{\log_c a}$ 5

$\log_a x \cdot \log_b x + \log_b x \cdot \log_c x + \log_c x \cdot \log_a x$

$= \dfrac{1}{\log_x a \cdot \log_x b} + \dfrac{1}{\log_x b \cdot \log_x c} + \dfrac{1}{\log_x c \log_x a}$

$= \dfrac{\log_x c + \log_x a + \log_x b}{\log_x a \cdot \log_x b \cdot \log_x c}$ 40

20

$= \dfrac{\log_x abc}{\log_x a \cdot \log_x b \cdot \log_x c} = \dfrac{\dfrac{1}{\log_{abc} x}}{\dfrac{1}{\log_a x} \cdot \dfrac{1}{\log_b x} \cdot \dfrac{1}{\log_c x}} = \dfrac{\log_a x \cdot \log_b x \cdot \log_c x}{\log_{abc} x}$

(13). $\dfrac{3x+4}{(2x+1)(x-2)} = \dfrac{A}{2x+1} + \dfrac{B}{x-2}$

$3x+4 = A(x-2) + B(2x+1)$

$x \rightsquigarrow \; 3 = A + 2B \rightsquigarrow (1)$

Constant $\rightsquigarrow \; 4 = -2A + B \rightsquigarrow (2)$

$(1) \times 2 + (B)$; $\; 10 = 5B$

$\underline{\underline{B = 2}}$

$(1) \Rightarrow \; 3 = A + 4$

$\underline{\underline{A = -1}}$

$$\frac{3x+4}{(2x+1)(x-2)} = \underline{\underline{\frac{2}{x-2} - \frac{1}{2x+1}}} \quad (5)$$

$$\frac{(3x+4)^2}{(2x+1)^2(x-2)^2} = \left\{\frac{2}{x-2} - \frac{1}{2x+1}\right\}^2$$

$$= \frac{4}{(x-2)^2} - \frac{4}{(x-2)(2x+1)} + \frac{1}{(2x+1)^2}$$

$$= \frac{4}{(x-2)^2} - \frac{4}{5}\left\{\frac{1}{x-2} - \frac{2}{2x+1}\right\} + \frac{1}{(2x+1)^2}$$

$$= \underline{\underline{\frac{8}{5(2x+1)} - \frac{4}{5(x-2)} + \frac{4}{(x-2)^2} + \frac{1}{(2x+1)^2}}} \quad (10)$$

15

ii) $y = \dfrac{2}{x-2} - \dfrac{1}{2x+1}$

$$\frac{dy}{dx} = 2 \times (-1)(x-2)^{-2} - (-1)(2x+1)^{-2} \times 2$$

$$= \frac{-2}{(x-2)^2} + \frac{2}{(2x+1)^2}$$

$$= \underline{\underline{\frac{2}{(2x+1)^2} - \frac{2}{(x-2)^2}}} \quad 5$$

For turning point; $\dfrac{dy}{dx} = 0.$

$$\frac{dy}{dx} = \frac{2(x-2)^2 - 2(2x+1)^2}{(2x+1)^2(x-2)^2}$$

$$= \frac{2\{x^2 - 4x + 4 - 4x^2 - 4x - 1\}}{(2x+1)^2(x-2)^2}$$

$$= \frac{-2\{3x^2 + 8x + 3\}}{(2x+1)^2(x-2)^2}$$

$$= \frac{-2\{(3x+1)(x+3)\}}{(2x+1)^2(x-2)^2}$$

$$= \frac{-2(3x-1)(x+3)}{(2x+1)^2(x-2)^2}$$

$\Rightarrow \dfrac{dy}{dx} = 0$ When $\underline{\underline{x = \frac{1}{3}}}$ and $\underline{\underline{x = -3}}$ 5

10

(iii) $\frac{dy}{dx} = \frac{2}{(2x+1)^2} - \frac{2}{(x-2)^2}$

$\frac{d^2y}{dx} = -4(2x+1)^{-3} \times 2 - 2 \times (-2)(x-2)^{-3}$

$= \frac{-8}{(2x+1)^3} + \frac{4}{(x-2)^3}$ 5

When $x = -3$

$\frac{d^2y}{dx^2} = \frac{-8}{(2x+1)^3} + \frac{4}{(x-2)^3}$

$\left(\frac{d^2y}{dx^2}\right)_{x=-3} = \frac{4}{(-3-2)^3} - \frac{8}{(-6+1)^3}$

$= \frac{8}{125} - \frac{4}{125}$

$= \frac{4}{125}$

$\therefore \left(\frac{d^2y}{dx^2}\right)_{x=-3} > 0$ 5 Local minimum point 5

15

(iv) Vertical asymptote ; $x = -\frac{1}{2}$ and $x = 2$. 5

Horizontal asymptote ;

$x \xrightarrow{\lim} -\infty \quad y \xrightarrow{\lim} 0$ 5

$x \xrightarrow{\lim} +\infty \quad y \xrightarrow{\lim} 0$

	$-\infty < x < -3$	$x = -3$	$-3 < x < -\frac{1}{2}$	$-\frac{1}{2} < x < \frac{1}{3}$	$x = \frac{1}{3}$	$\frac{1}{3} < x < 2$	$2 < x < \infty$
Sign of $f'(x)$	$-$	0	$+$	$+$	0	$-$	$-$

When $x = -3$ When $x = \frac{1}{3}$ (25)

$y = \frac{-1}{5}$ $y = \frac{-9}{5}$

There are two turning points

$\left(-3, \frac{-1}{5}\right)$ – Local min.

$\left(\frac{1}{3}, -\frac{9}{5}\right)$ – Local max (5)

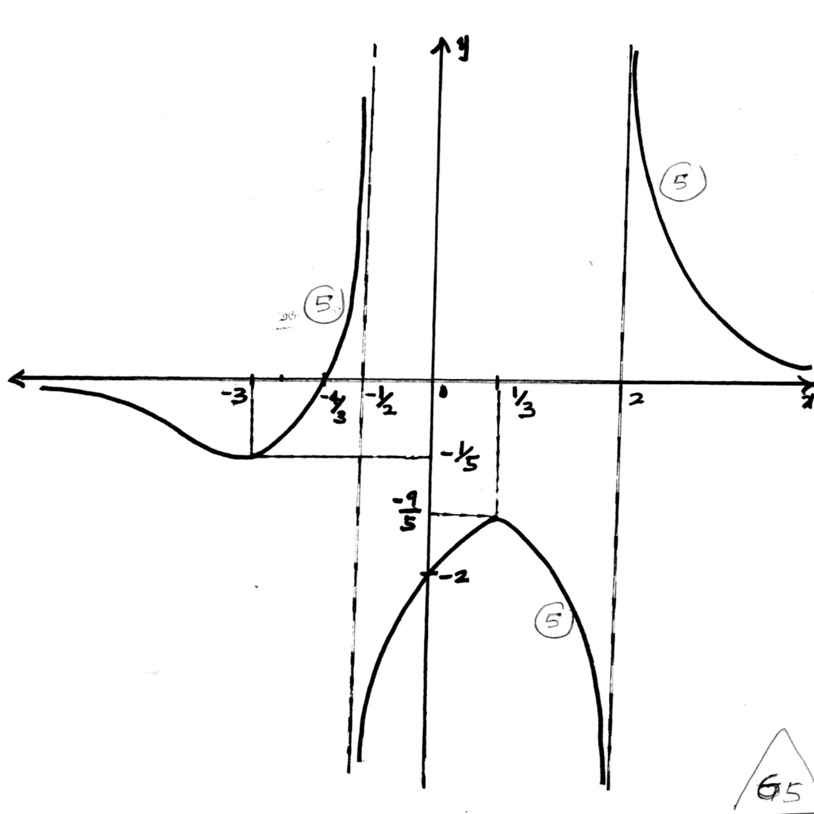
$x \xrightarrow{\lim} -\frac{1}{2}^{-} \quad y \xrightarrow{\lim} \infty$

$x \xrightarrow{\lim} -\frac{1}{2}^{+} \quad y \xrightarrow{\lim} -\infty$

$x \xrightarrow{\lim} 2^{-} \quad y \xrightarrow{\lim} -\infty$ (5)

$x \xrightarrow{\lim} 2^{+} \quad y \xrightarrow{\lim} +\infty.$

When $x = 0$ $y = \frac{4}{1 \times (-2)} = -2. \rightarrow (0, -2)$ (5)



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v. $\text{Area} = \int_4^{12} \frac{3x+4}{(2x+1)(x-2)}\,dx$ 5

$= \int_4^{12} \frac{2}{x-2} - \frac{1}{2x+1}\,dx$

$= 2\int_4^{12} \frac{1}{x-2}\,dx - \frac{1}{2}\int_4^{12} \frac{2}{2x+1}\,dx$ 5

$= 2\left\{\ln|x-2|\right\}_4^{12} - \frac{1}{2}\left\{\ln|2x+1|\right\}_4^{12}$ 5

$= 2\ln\left|\frac{10}{2}\right| - \frac{1}{2}\ln\left|\frac{25}{9}\right|$

$= 2\ln|5| - \ln\left|\frac{5}{3}\right|$

$= \ln\left|25 \times \frac{3}{5}\right| = \underline{\underline{\ln|15|}}$ 5

20

$\text{Volume} = \int_4^{12} \pi \cdot \frac{(3x+4)^2}{(2x+1)^2(x-2)^2}\,dx$ 5

$= \pi\int_4^{12} \frac{8}{5(2x+1)} - \frac{4}{5(x-2)} + \frac{4}{(x-2)^2} + \frac{1}{(2x+1)^2}\,dx$

$= \pi\left\{\frac{4}{5}\int_4^{12}\frac{2}{2x+1}\,dx - \frac{4}{5}\int_4^{12}\frac{1}{x-2}\,dx + 4\int_4^{12}(x-2)^{-2}dx\right.$ 5

$\left. + \frac{1}{2}\int_4^{12}(2x+1)^{-2}\times 2\,dx\right.$

$= \pi\left\{\frac{4}{5}\left(\ln|2x+1|\right)_4^{12} - \frac{4}{5}\left(\ln|x-2|\right)_4^{12}\right.$

$\left. + 4\left[\frac{-1}{x-2}\right]_4^{12} + \frac{1}{2}\left[\frac{-1}{2x+1}\right]_4^{12}\right\}$ 5

$= \pi\left\{\frac{4}{5}\ln\left|\frac{25}{9}\right| - \frac{4}{5}\ln\left|\frac{10}{6}\right| + 4\left[\frac{1}{2} - \frac{1}{10}\right]\right.$

$\left. + \frac{1}{2}\left[\frac{1}{9} - \frac{1}{25}\right]\right\}$

$= \pi\left\{\frac{4}{5}\ln\left|\frac{25}{9} \times \frac{6}{10}\right| + 4\left[\frac{5-1}{10}\right] + \frac{1}{2}\left[\frac{25-9}{225}\right]\right.$ 5

$= \pi\left\{\frac{4}{5}\ln\left|\frac{5}{3}\right| + 4 \times \frac{4}{10} + \frac{1}{2} \times \frac{16}{225}\right\}$

$= \pi\left\{\frac{4}{5}\ln\left|\frac{5}{3}\right| + \frac{16}{10} + \frac{8}{225}\right\}$

$= \frac{\pi}{5}\left\{4\ln\left|\frac{5}{3}\right| + 8 + \frac{8}{45}\right\}$

$= \frac{4\pi}{5}\left\{\ln\left|\frac{5}{3}\right| + 2 + \frac{2}{45}\right\}$

25

$= \frac{4\pi}{5}\left\{\underline{\underline{\frac{92}{45}}} + \ln\left(\frac{5}{3}\right)\right\}$ 5

(14) (a) $f(x) = \dfrac{(x-1)(x-5)}{(x-4)(x-2)} = \dfrac{x^2-6x+5}{x^2-6x+8}$

$$f'(x) = \frac{(x^2-6x+8)(2x-6)-(x^2-6x+5)(2x-6)}{(x-4)^2(x-2)^2}$$

$$= \frac{(2x-6)(3)}{(x-4)^2(x-2)^2}$$

$$= \frac{6(x-3)}{(x-4)^2(x-2)^2}$$ 25

For turning points

$f'(x) = 0$ 5 $x = 3$.

$$f(x) = \frac{1 - 6/x + 5/x^2}{1 - 6/x + 8/x^2}$$

for $x \longrightarrow -\infty$; $f(x) \to 1$
$x \longrightarrow +\infty$; $f(x) \to 1$ $\Big\} \Rightarrow$ 5 $y=1$ is a horizontal asymptotes.

$x=4$ and $x=2$ are vertical asymptotes. 10

x	$-\infty < x < 2$	$2 < x \leq 3$	$3 < x < 4$	$4 < x < \infty$
sign of $f'(x)$	$(-)$ ↘	$(-)$ ↘	$(+)$ ↗	$(+)$ ↗

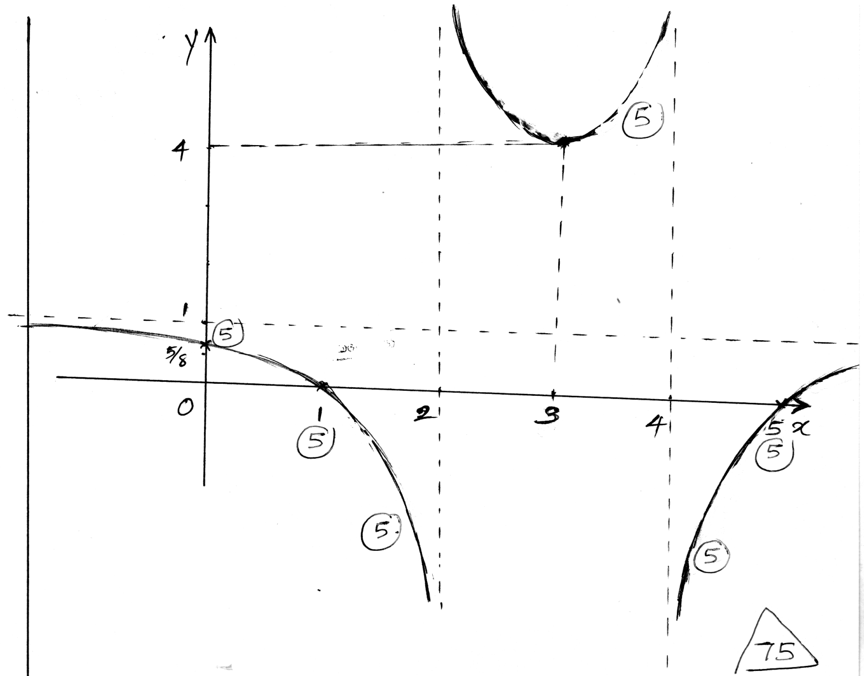
20

When $x = 3$, $f(x) = 4$

when $x = 1$, $x = 5$, $f(x) = 0$

when $x = 0$ $f(x) = 5/8$

$(3, 4)$ is a local minimum point. 5



(b) (i) $V = \pi x^2 h$; h is the height of the

But $A = 80\pi = 2\pi x^2 + (h+1)2\pi x$

$$\Rightarrow \frac{40 - x^2}{x} - 1 = h$$

$$\Rightarrow h = \frac{40 - x^2 - x}{x} \quad (10)$$

$$\therefore V = \pi x^2 \frac{(40 - x^2 - x)}{x}$$

$$= \underline{\underline{\pi(40x - x^2 - x^3)}} \quad (10)$$

(ii) $$\frac{dV}{dx} = \pi(40 - 2x - 3x^2) \quad (5)$$

$$= \pi(10 - 3x)(4 + x) \quad (5)$$

When $\frac{dV}{dx} = 0$; $x = 10/3$ or $\cancel{x = -4}$

$\therefore x = 10/3$ (5)

x	$0 < x < 10/3$	$x = 10/3$	$10/3 < x$
$\frac{dV}{dx}$	+	0	− (10)

$\therefore$ When $x = 10/3$ cm, V is maximum. (5)

50

(15) a. $\dfrac{1}{x(x-1)^2} = \dfrac{A}{x} + \dfrac{B}{x-1} + \dfrac{C}{(x-1)^2}$ (10)

$1 \equiv A(x-1)^2 + Bx(x-1) + Cx$

$x^2 \rightsquigarrow \quad A+B=0 \quad -(1)$

$x \rightsquigarrow \quad 0 = -2A-B+C \quad -(2)$

constant $\rightsquigarrow \quad \underline{\underline{1 = A}}$ (5)

$\underline{\underline{B=-1}}$ (5) $\underline{\underline{C=1}}$ (5)

$$\frac{1}{x(x-1)^2} = \underline{\underline{\frac{1}{x} - \frac{1}{x-1} + \frac{1}{(x-1)^2}}}$$ (25)

$$\int f(x)\,dx = \int \frac{1}{x} - \frac{1}{x-1} + \frac{1}{(x-1)^2}\,dx$$

$$= \int \frac{1}{x}\,dx - \int \frac{1}{x-1}\,dx + \int (x-1)^{-2}\,dx$$ (10)

$$= \underline{\underline{\ln|x| - \ln|x-1| - \frac{1}{x-1} + C}} \,;$$ (20)

(30)

Let $x = e^t$

$\dfrac{dx}{dt} = e^t$

$$\int \frac{1}{x(x-1)^2}\,dx = \ln|x| - \ln|x-1| - \frac{1}{x-1} + C$$

$$\int \frac{1}{e^t(e^t-1)^2} \times e^t\,dt = \ln|e^t| - \ln|e^t-1| - \frac{1}{e^t-1} + C$$

$$\int \frac{1}{(e^t-1)^2}\,dt = \underline{\underline{t - \ln|e^t-1| - \frac{1}{e^t-1} + C}}$$ (10)

$$(b)\ \int x e^{-2x}dx = \int x \frac{d}{dx}\left(\frac{-e^{-2x}}{2}\right) dx \quad (5)$$

$$= -\frac{xe^{-2x}}{2} + \int \frac{1}{2}e^{-2x} \times 1\,dx \quad (10)$$

$$= -\frac{xe^{-2x}}{2} - \frac{1}{4}\int e^{-2x} \times (-2)\,dx \quad (5)$$

$$= \underline{\underline{-\frac{xe^{-2x}}{2} - \frac{1}{4}e^{-2x} + c}} \quad (5) \qquad 25$$

$$(c)\ \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx \qquad 10$$

$$I = \int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x}\,dx \rightsquigarrow (1)$$

$$= \int_0^{\pi/2} \frac{\sin^3(\frac{\pi}{2}-x)}{\sin(\frac{\pi}{2}-x) + \cos(\frac{\pi}{2}-x)}\,dx \quad (5)$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos^3(x)}{\cos x + \sin x}\,dx \rightsquigarrow (2) \quad (5)$$

$$(1)+(2) \quad 2I = \int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x}\,dx + \int_0^{\pi/2} \frac{\cos^3 x}{\sin x + \cos x}\,dx \quad (5)$$

$$= \int_0^{\pi/2} \frac{\sin^3 x + \cos^3 x}{\sin x + \cos x}\,dx$$

$$= \int_0^{\pi/2} \frac{(\sin x + \cos x)(\sin^2 x - \sin x \cos x + \cos^2 x)}{(\sin x + \cos x)}\,dx \quad (10)$$

$$= \int_0^{\pi/2} 1 - \sin x \cos x\,dx \quad (5)$$

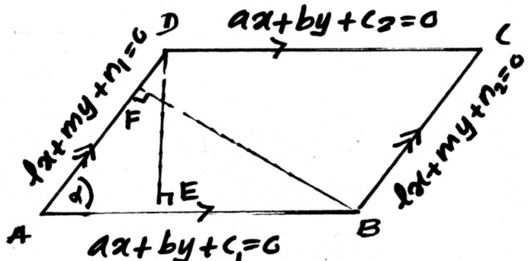
$$= \int_0^{\pi/2} 1\,dx - \int_0^{\pi/2} \sin x \cos x\,dx.$$

$$= (x)_0^{\pi/2} - \left\{\frac{\sin^2 x}{2}\right\}_0^{\pi/2} \quad (5)$$

$$= \frac{\pi}{2} - 0 \quad (5) \quad - \left\{\frac{\sin^2 \pi/2}{2} - \frac{\sin^2(0)}{2}\right\} \quad (5)$$

$$= \underline{\underline{\frac{\pi}{2} - 1}} \quad (5) \qquad 50$$

(16)



$$\text{Area} = |AB \times DE|$$

$$= \left|\frac{DE \times BF}{\sin\alpha}\right| \rightsquigarrow (A)$$

$$\sin\alpha = \frac{BF}{AB}$$

$$AB = \frac{BF}{\sin\alpha}$$

$$BF = \left|\frac{n_2-n_1}{\sqrt{l^2+m^2}}\right| \rightsquigarrow (1) \qquad DE = \left|\frac{c_2-c_1}{\sqrt{a^2+b^2}}\right| \rightsquigarrow (3)$$

$AB \rightsquigarrow m_1$

$$ax+by+c_1=0$$

$$y = -\frac{a}{b}x - \frac{c_1}{b}$$

$$m_1 = -\frac{a}{b}$$

$AD \rightsquigarrow m_2$

$$lx+my+n_1=0$$

$$y = -\frac{l}{m}x - \frac{n_1}{m}$$

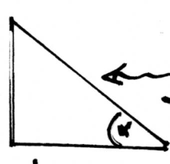
$$m_2 = -\frac{l}{m}$$

$$\tan\alpha = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

$$= \left|\frac{-\frac{a}{b}+\frac{l}{m}}{1+\frac{a}{b}\times\frac{l}{m}}\right|$$

$$= \left|\frac{bl-ma}{bm+al}\right|$$

$bl-ma$



$bm+al$

$$\leftarrow \sqrt{(bl-ma)^2+(bm+al)^2}$$

$$= \sqrt{(a^2+b^2)(m^2+l^2)}$$

$$\sin\alpha = \frac{bl - ma}{\sqrt{(a^2+b^2)(m^2+l^2)}} \rightsquigarrow (3)$$

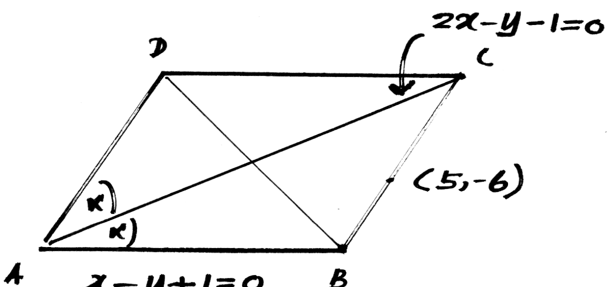
Substitute (1), (2) and (3) in (A)

$$Area = \frac{\left|\frac{n_2-n_1}{\sqrt{l^2+m^2}}\right|\left|\frac{c_2-c_1}{\sqrt{a^2+b^2}}\right|}{\left|\frac{bl-ma}{\sqrt{(a^2+b^2)(m^2+l^2)}}\right|}$$

$$= \left|\frac{(n_2-n_1)(c_2-c_1)}{am-bl}\right|$$

$$= \frac{|n_2-n_1||c_2-c_1|}{|am-bl|}$$

60



Let the gradient of AD is m_1

$$\tan D\hat{A}C = \tan B\hat{A}C$$

$$\left|\frac{m-2}{1+m.2}\right| = \left|\frac{1-2}{1+2}\right|$$

$$\frac{m-2}{1+2m} = \pm\frac{1}{3}$$

$\ominus \rightarrow \underline{m=1} \leftarrow$ Gradient of AD. 10

$\oplus \rightsquigarrow \; m = -7$

$\therefore$ The gradient of AD and BC is -7

$\therefore$ The eq^n of BC

$$\frac{y+6}{x-5} = -7 \implies \underline{\underline{y + 7x - 29 = 0}} \quad 15$$

$$\left.\begin{array}{l} 2x - y - 1 = 0 \\ y + 7x - 29 = 0 \end{array}\right\} \Rightarrow C \equiv \left(\frac{10}{3}, \frac{17}{3}\right) \quad 5$$

$\therefore$ The eq^n of DC ;

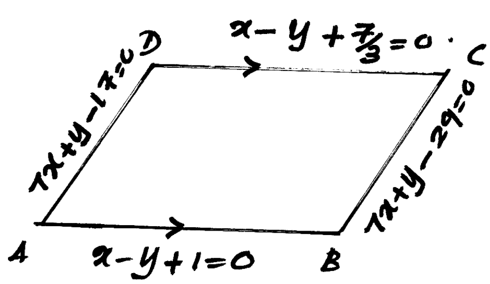
$$\frac{y - \frac{17}{3}}{x - \frac{10}{3}} = 1 \implies \underline{\underline{3y - 3x - 7 = 0}} \quad 15$$

$$\left.\begin{array}{l} 2x - y - 1 = 0 \\ x - y + 1 = 0 \end{array}\right\} \Rightarrow A \equiv (2, 3) \quad 5$$

$\therefore$ The eq^n of AD :

$$\frac{y-3}{x-2} = -7 \Rightarrow \underline{\underline{y + 7x - 17 = 0}} \quad 15$$

65



$$\text{Area} = \frac{|29 - 17|\,|\frac{7}{3} - 1|}{|1 \times 1 + 7 \times 1|}$$

$$= \left(12 \times \frac{4}{3}\right) \times \frac{1}{8} = \frac{16}{8} \times 1 = \underline{\underline{2}}. \quad 25$$

17) $\theta = 36^\circ$

$$5\theta = 180^\circ$$

$$3\theta + 2\theta = 180^\circ$$

$$3\theta = 180^\circ - 2\theta$$

$$\sin 3\theta = \sin(180 - 2\theta)$$

$$\underline{\underline{\sin 3\theta = \sin 2\theta}} \quad 10$$

5 5

$$3\sin\theta - 4\sin^3\theta = 2\sin\theta\cos\theta$$

$$4\sin^3\theta + 2\sin\theta\cos\theta - 3\sin\theta = 0$$

5

$$\sin\theta\{4\sin^2\theta + 2\cos\theta - 3\} = 0$$

$$\sin\theta\{4(1-\cos^2\theta) + 2\cos\theta - 3\} = 0$$

$$\sin\theta\{4 - 4\cos^2\theta + 2\cos\theta - 3\} = 0$$

5

$$\sin\theta\{4\cos^2\theta - 2\cos\theta - 1\} = 0$$

5

$$\sin\theta = 0 \quad \text{or} \quad 4\cos^2\theta - 2\cos\theta - 1 = 0$$

$$\sin 36 = 0 \quad \text{or} \quad \cos\theta = \frac{2 \pm \sqrt{4 + 4\times4\times1}}{2\times4}$$

$\neq$ 5

$$= \frac{1 \pm \sqrt{5}}{4}$$

$$\cos 36 = \frac{1+\sqrt{5}}{4} \quad 5 \quad \text{or} \quad \cos 36 = \frac{1-\sqrt{5}}{4}$$

$\neq$ 5

$(0 < \cos 36 < 1)$

$$\therefore \underline{\underline{\cos 36 = \frac{\sqrt{5}+1}{4}}}$$

50

$$\sin 12^\circ \sin 48^\circ \sin 54^\circ = -\frac{1}{2}\left\{-2\sin 12 \sin 48\right\} \sin 54$$

$$= -\frac{1}{2}\left\{\cos 60^\circ - \cos 36\right\} \sin 54 \quad 5$$

$$= -\frac{1}{2}\left\{\frac{1}{2} - \frac{\sqrt{5}+1}{4}\right\} \sin(90-36) \quad 5$$

$$= -\frac{1}{2}\left\{\frac{2-\sqrt{5}-1}{4}\right\} \sin(90-36)$$

$$= -\frac{1}{2}\left\{\frac{1-\sqrt{5}}{4}\right\} \cos 36 \quad 5$$

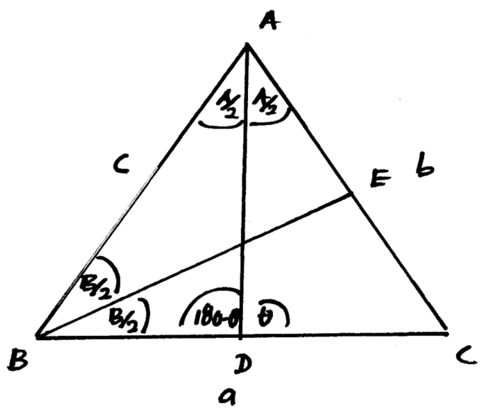
$$= \frac{\sqrt{5}-1}{8} \times \frac{\sqrt{5}+1}{4}$$

$$= \frac{5-1}{32}$$

$$= \frac{1}{8} \quad 5$$

/20

b. Sin Rule



$$\frac{AD}{\sin B} = \frac{BD}{\sin(A/2)} \Rightarrow BD = \frac{AD \sin(A/2)}{\sin B} \quad 5$$

$$\frac{AD}{\sin C} = \frac{DC}{\sin(A/2)} \Rightarrow DC = \frac{AD \sin(A/2)}{\sin C} \quad 5$$

$$BD + DC = a$$

$$\frac{AD\sin(A/2)}{\sin B} + \frac{AD\sin(A/2)}{\sin C} = a \qquad 5$$

$$AD\sin(A/2)\left\{\frac{\sin B + \sin C}{\sin B\sin C}\right\} = a \qquad 5$$

$$AD\sin(A/2)\left\{\frac{kb + kc}{k^2bc}\right\} = a$$

$$AD = \frac{kabc}{(b+c)\sin(A/2)}$$

$$= \frac{bc\sin A}{(b+c)\sin(A/2)} \qquad 5$$

$$= \frac{2bc\sin(A/2)\cos(A/2)}{(b+c)\sin(A/2)}$$

$$= \frac{2bc\cos(A/2)}{(b+c)} \qquad 5$$

$$BE = \frac{2ac\cos\left(\frac{B}{2}\right)}{a+c} \qquad 5$$

If $AD = BE$

$$\frac{2bc\cos\left(\frac{A}{2}\right)}{b+c} = \frac{2ac\cos\left(\frac{B}{2}\right)}{a+c}$$

$$\frac{b\cos\left(\frac{A}{2}\right)}{b+c} = \frac{a\cos\left(\frac{B}{2}\right)}{a+c}$$

$$\frac{k\sin B\cos\left(\frac{A}{2}\right)}{k\sin B + k\sin C} = \frac{k\sin A\cos\left(\frac{B}{2}\right)}{k\sin A + k\sin C} \qquad 5$$

$$\frac{\sin B\cos\left(\frac{A}{2}\right)}{2\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)} = \frac{\sin A\cos\left(\frac{B}{2}\right)}{2\sin\left(\frac{A+C}{2}\right)\cos\left(\frac{A-C}{2}\right)} \qquad 5$$

$$\frac{\sin B\cos\left(\frac{A}{2}\right)}{2\sin\left(\frac{\pi}{2}-\frac{A}{2}\right)\cos\left(\frac{B-C}{2}\right)} = \frac{\sin A\cos\left(\frac{B}{2}\right)}{2\sin\left(\frac{\pi}{2}-\frac{B}{2}\right)\cos\left(\frac{A-C}{2}\right)}$$

$$\frac{\sin B\cos\left(\frac{A}{2}\right)}{2\cos\left(\frac{A}{2}\right)\cos\left(\frac{B-C}{2}\right)} = \frac{\sin A\cos\left(\frac{B}{2}\right)}{2\cos\left(\frac{B}{2}\right)\cos\left(\frac{A-C}{2}\right)}$$

5

$$\sin B\cos\left(\frac{A-C}{2}\right) = \sin A\cos\left(\frac{B-C}{2}\right)$$

50

c) $f(x) = 2\cos^2 x + 4\sin x\cos x - 2\sin^2 x$

5

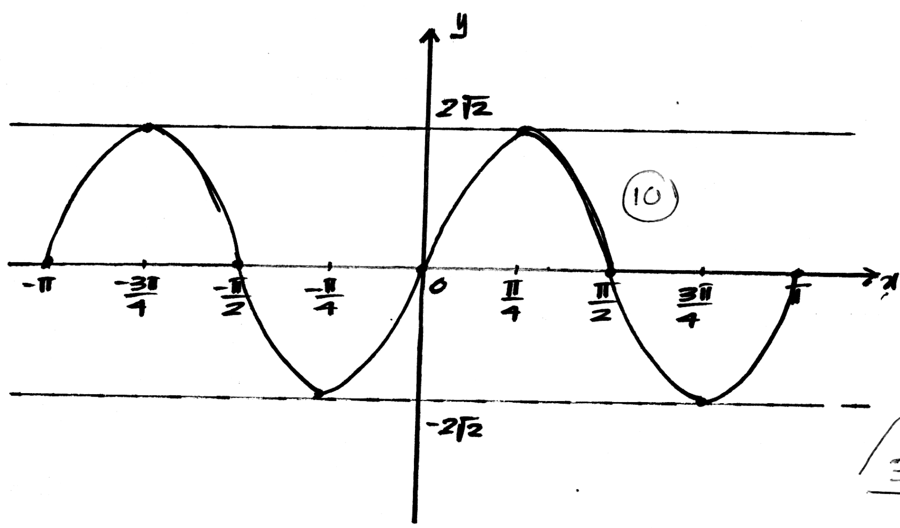
$= 2\cos 2x + 2\sin 2x$.

$= 2\sqrt{2}\left\{\frac{1}{\sqrt{2}}\cos 2x + \frac{1}{\sqrt{2}}\sin 2x\right\}$ 5

$= 2\sqrt{2}\left\{\cos 2x\sin\frac{\pi}{4} + \sin 2x\cos\frac{\pi}{4}\right\}$

$= 2\sqrt{2}\sin\left(2x + \frac{\pi}{4}\right)$ 5

$A = 2\sqrt{2}\ ,\ \alpha = \frac{\pi}{4}$ 5



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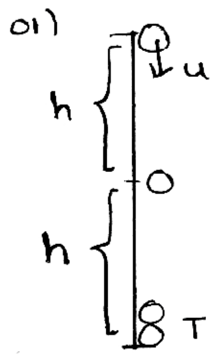
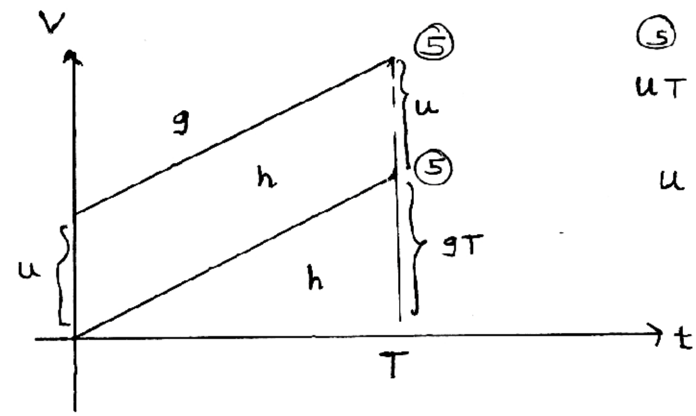
Part A

01) 

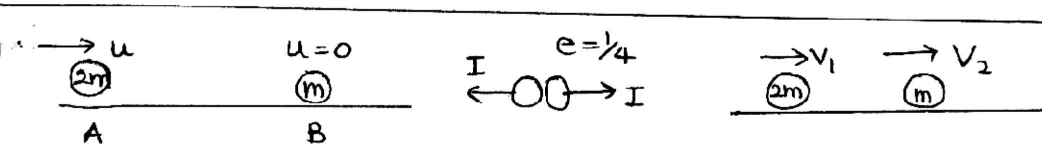


5 5

$uT = \frac{1}{2} T \cdot gT$

$u = \frac{gT}{2}$ 5

25

02) 

Conservation of momentum

$2mV_1 + mV_2 = 2mu$

$2V_1 + V_2 = 2u$ —① 5

Newton's law of restitution.

$V_1 - V_2 = -eu$ —② 5

$3V_2 = 2u(1+e)$

$V_2 = \frac{2u(1+e)}{3}$ 5

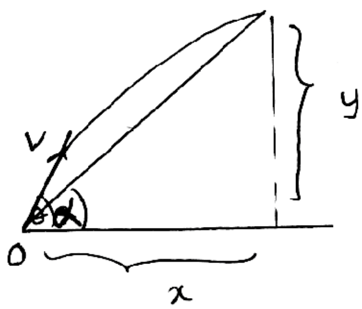
$I = \frac{2mu(1+e)}{3}$ 5

$= \frac{2mu}{3} \times \frac{5}{4}$

$= \frac{5mu}{6}$ 5

25

3)



$\uparrow v^2 = u^2 + 2as$

$0 = (V\sin\theta)^2 - 2gy$

$y = \dfrac{V^2\sin^2\theta}{2g}$ 5

$\uparrow v = u + at$

$0 = V\sin\theta - gt$

$t = \dfrac{V\sin\theta}{g}$ 5

$\rightarrow s = ut$

$x = V\cos\theta \times \dfrac{V\sin\theta}{g}$ 5

$\tan\alpha = \dfrac{y}{x}$

$= \dfrac{\dfrac{V^2\sin^2\theta}{2g}}{\dfrac{V^2\sin\theta\cos\theta}{g}}$ 5

$\tan\alpha = \dfrac{\tan\theta}{2}$

$2\tan\alpha = \tan\theta$ 5

25

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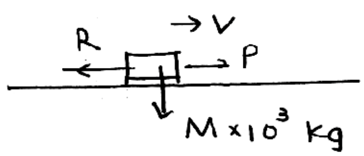
4)



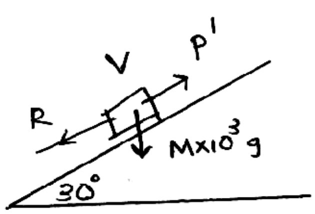
$P = R$ 5

$H = PV$

$H \times 10^3 = PV$

$P = \dfrac{H \times 10^3}{V}$

$\therefore R = \dfrac{H \times 10^3}{V}$ 5



$\nearrow P' = R' + M \times 10^3 g \sin 30$

$P' = \dfrac{H \times 10^3}{V} + \dfrac{M \times 10^3 g}{2}$ 5

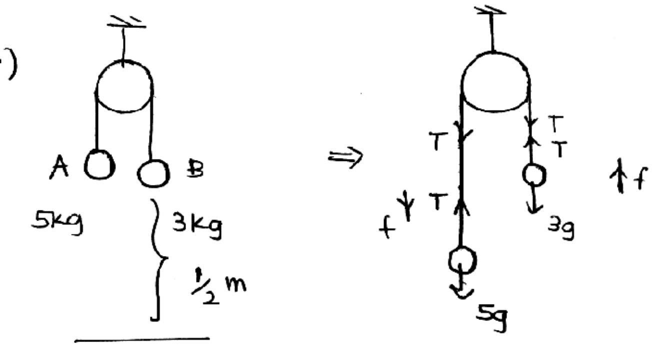
$H = PV$

$H' \times 10^3 = P'V = H \times 10^3 + \dfrac{M \times 10^3 g}{2} \times V$ 5

$H' = H + \dfrac{MgV}{2}$ 5

25

5)



A ↓ $F = ma$

$5g - T = 5f$ — ① 5

B ↑ $T - 3g = 3f$ — ② 5

$2g = 8f$

$f = \frac{g}{4}$ 5

When A hits the ground velocities of A and B are equal.

A ↓ $v^2 = u^2 + 2as$

$v^2 = 0 + 2 \times \frac{g}{4} \times \frac{1}{2}$

$v = \frac{\sqrt{g}}{2}$ 5

A ↑ $v^2 = u^2 + 2as$

$0 = \frac{g}{4} - 2gh$

$h = \frac{1}{8}$ m

∴ the height reached by B $= 1 + \frac{1}{8} = \frac{9}{8}$ m 5
from the ground level

25

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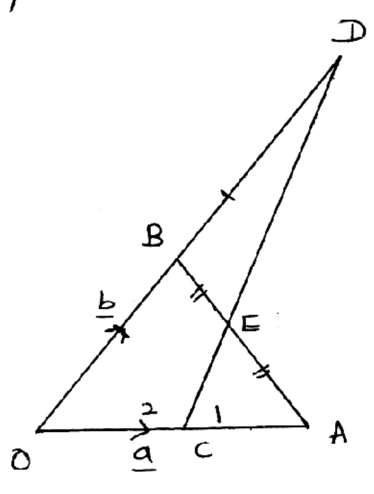
6)



$\overrightarrow{CE} = \overrightarrow{CA} + \overrightarrow{AE}$ 5

$= \frac{1}{3}\overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$

$= \frac{1}{3}\underline{a} + \frac{1}{2}(\underline{b} - \underline{a})$

$= -\frac{\underline{a}}{6} + \frac{\underline{b}}{2}$

$= \frac{3\underline{b} - \underline{a}}{6}$ 5

$\overrightarrow{ED} = \overrightarrow{EB} + \overrightarrow{BD}$

$= \frac{1}{2}(\underline{b} - \underline{a}) + \underline{b}$

$= \frac{1}{2}(3\underline{b} - \underline{a})$ 5

$\overrightarrow{CE} = \frac{1}{3} \times \frac{1}{2}(3\underline{b} - \underline{a})$

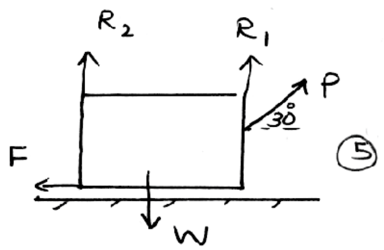
$\overrightarrow{CE} = \frac{1}{3}\overrightarrow{ED}$ 5

∴ C, E, D are collinear.

CE : ED = 1 : 3 5

25

7



5

$\uparrow \quad R_1 + R_2 + \frac{P}{2} = W \quad 5$

$R_1 + R_2 = W - \frac{P}{2}$

$\rightarrow \quad F = \frac{\sqrt{3}}{2} P \quad 5$

For the equilibrium

$\mu \geq \frac{|F|}{R_1 + R_2} \quad 5$

$\mu \geq \frac{\sqrt{3}P/_2}{2W - P/_2}$

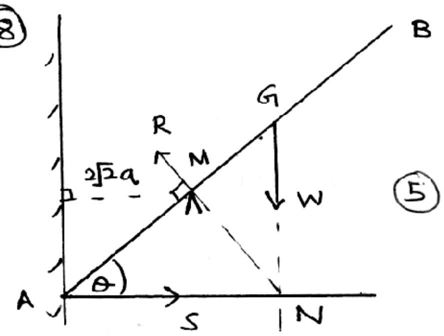
$\mu \geq \frac{\sqrt{3}P}{2W - P}$

$P \leq \frac{2W\mu}{\sqrt{3} + \mu} \quad 5$

$P_{max} = \frac{2\mu W}{\sqrt{3} + \mu}$

25

8



5

$AN = 8a \cos\theta$

$AM = 8a \cos^2\theta$

$8a \cos^3\theta = 2\sqrt{2}a \quad 5$

$\cos^3\theta = \left(\frac{1}{\sqrt{2}}\right)^3$

$\cos\theta = \frac{1}{\sqrt{2}}$

$\theta = \pi/4 \quad 5$

For the rod AB

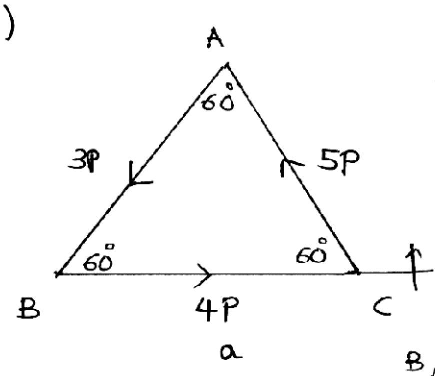
$\overset{\curvearrowleft}{A}) \quad R \times 8a \cos^2\theta = W \times 8a \cos\theta \quad 5$

$R \times \cos\frac{\pi}{4} = W$

$R = \sqrt{2}W \quad 5$

25

9)



$\rightarrow X = 4P - (5P + 3P)\frac{1}{2} = 0$ (5)

$\uparrow Y = (5P - 3P)\frac{\sqrt{3}}{2} = \sqrt{3}P$ (5)

$R = \sqrt{3}P \uparrow$ (5)

$B, \curvearrowleft G = \sqrt{3}P\left(a + \frac{3a}{2}\right)$ (5)

$= \frac{5\sqrt{3}\,aP}{2}$ (5)

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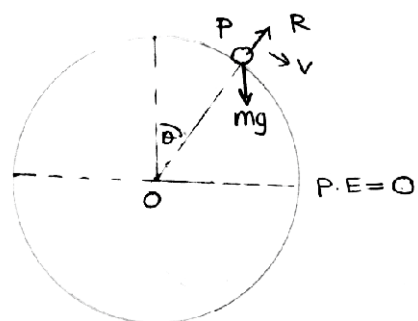
---

10)



Applying the principle of Conservation of energy for P,

$\frac{1}{2}mv^2 + mg\,a\cos\theta = mga$ (5)

$v^2 = 2ga(1 - \cos\theta)$

$\swarrow F = ma$

$mg\cos\theta - R = \frac{mv^2}{a}$ (5)

$R = mg\cos\theta - \frac{m}{a}(1 - \cos\theta)2ga$

$R = mg(3\cos\theta - 2)$

When the particle leaves the sphere, $R = 0$ (5)

$\cos\theta = \frac{2}{3}$

$\theta = \cos^{-1}\left(\frac{2}{3}\right)$ (5)

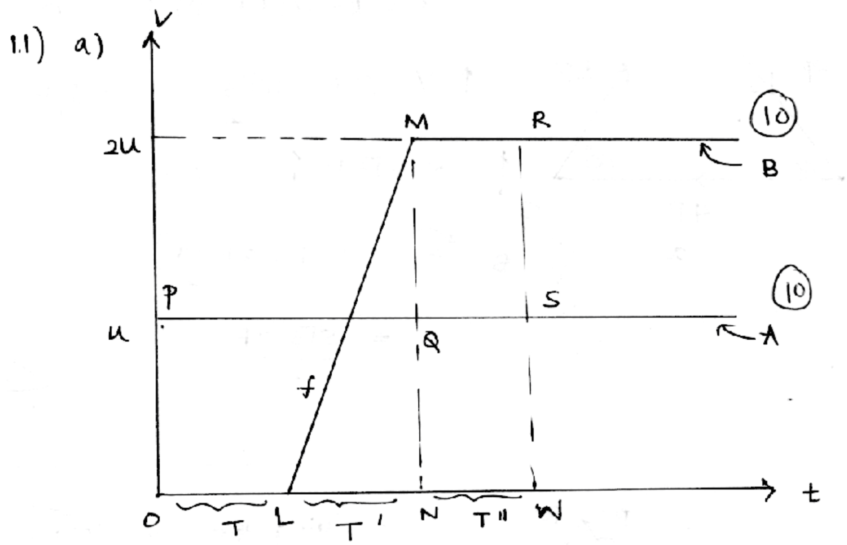
$v^2 = 2ga\left(1 - \frac{2}{3}\right)$

$v = \sqrt{\frac{2ga}{3}}$ (5)

25

Part B

11) a)



$\frac{2u}{T'} = f$ (5)

distance between the cars when B reaches the maximum velocity

Area of OPQN — Area of LMN (5)

$= (T+T')u - \frac{1}{2}T' \times 2u$ (5) (5)

$= \left(T + \frac{2u}{f}\right)u - \frac{1}{2} \times \frac{2u}{f} \times 2u$

$= uT + \frac{2u^2}{f} - \frac{2u^2}{f}$

$= uT$ (5)

when B passes A,

Area of OPSW = Area of LMRW (5)

$(T + T' + T'')u = \left(\frac{1}{2} \times T' \times 2u\right) + 2uT''$ (5) (5)

$Tu = T''u$

$T = T''$ (5)

$\therefore$ Total time $= T + \frac{2u}{f} + T = 2T + \frac{2u}{f} = 2\left(T + \frac{u}{f}\right)$ (5)

70

b) $\underline{V}_{P_1,E} = \uparrow$ $\quad \underline{V}_{P_1,W} = u$ $\quad \underline{V}_{W,E} = \underline{\theta}\swarrow v$

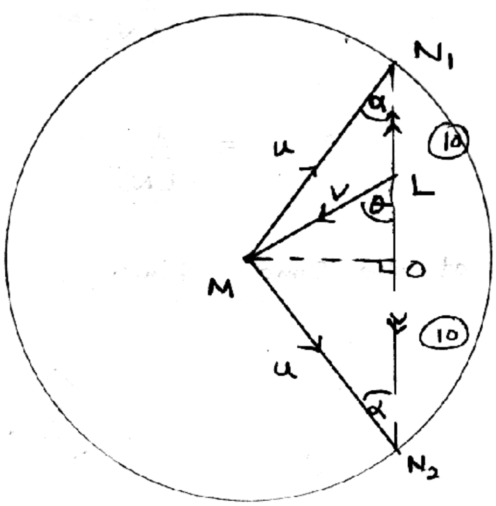
$\underline{V}_{P_1,E} = \underline{V}_{P_1,W} + \underline{V}_{W,E}$

$\overset{N_1}{\underset{L}{\uparrow}} = u_{MN_1} + \underline{\theta}\swarrow{}^{L}_{M}\, v$ (5)

$V_{P_2,E} \downarrow$ $\quad V_{P_2,W} = u$ $\quad \underline{V}_{W,E} = \underline{\theta}\swarrow v$

$\underline{V}_{P_2,E} = V_{P_2,W} + V_{W,E}$

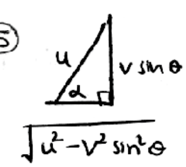
$\overset{L}{\underset{N_2}{\downarrow}} = u_{MN_2} + \underline{\theta}\swarrow{}^{L}_{M}$ (5)



for $\underline{OMN_1\ \Delta}$

$u \sin\alpha = v \sin\theta$

$\sin\alpha = \dfrac{v\sin\theta}{u}$ (5)



If the time taken for the first plane is t_1,

$$t_1 = \frac{d}{LN_1} \quad (5)$$

If the time taken for the second plane is t_2,

$$t_2 = \frac{d}{LN_2} \quad (5)$$

$$t_1 - t_2 = \frac{d}{LN_1} - \frac{d}{LN_2}$$

$$= d\left\{ \frac{1}{(u\cos\alpha - v\cos\theta)} - \frac{1}{(u\cos\alpha + v\cos\theta)} \right\} \quad (5)\ (5)$$

$$t_1 - t_2 = d\left\{ \frac{u\cos\alpha + v\cos\theta - (u\cos\alpha - v\cos\theta)}{u^2\cos^2\alpha - v^2\cos^2\theta} \right\}$$

$$= \frac{2dv\cos\theta}{u^2\cos^2\alpha - v^2\cos^2\theta} \quad (5)$$

$$= \frac{2dv\cos\theta}{u^2\left(\frac{u^2 - v^2\sin^2\theta}{u^2}\right) - v^2\cos^2\theta} \quad (5)$$

$$= \frac{2dv\cos\theta}{u^2 - v^2}$$

65

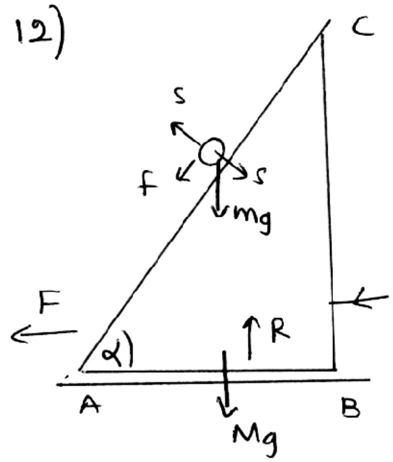
Time taken for P_1 to come back to the point A $= \dfrac{d}{LN_2}$

" " " P_2 " " $= \dfrac{d}{LN_1}$ (10)

$\therefore$ both planes reaches A at the same time. (5)

15

12)



$\underline{a}_{M,E} = \overleftarrow{F}$ $\quad\quad$ $\underline{a}_{m,E} = \underset{\alpha}{f\swarrow} + \overleftarrow{F}$

(10) $\quad\quad$ 10

for the system,

$F = ma$

$\leftarrow \quad P = MF + m\,(F + f\cos\alpha)$ (10)

$P = (M+m)F + m\cos\alpha\, f$ —— ① (5)

(m) $\underset{\alpha}{\swarrow}$ $\quad mg\sin\alpha = m\,(f + F\cos\alpha)$ (10)

$g\sin\alpha = f + F\cos\alpha$ —— ② (5)

30

① − ② × $m\cos\alpha$

$P - mg\sin\alpha\cos\alpha = F\,(M + m - m\cos^2\alpha)$ (5)

$$F = \frac{P - mg\sin\alpha\cos\alpha}{M + m\sin^2\alpha}$$ (5) (5)

by ②,

$g\sin\alpha - F\cos\alpha = f$

$$g\sin\alpha - \left(\frac{P - mg\sin\alpha\cos\alpha}{M + m\sin^2\alpha}\right)\cos\alpha = f$$ (5)

$$f = \frac{Mg\sin\alpha + mg\sin^3\alpha - P\cos\alpha + mg\sin\alpha\cos^2\alpha}{M + m\sin^2\alpha}$$ (5)

$$= \frac{Mg\sin\alpha + mg\sin^3\alpha - P\cos\alpha + mg\sin\alpha(1 - \sin^2\alpha)}{M + m\sin^2\alpha}$$

$$= \frac{(M+m)g\sin\alpha - P\cos\alpha}{M + m\sin^2\alpha}$$ (5)

30

III for the particle to move upwards along the plane or to come to rest

$f = 0$ or $f > 0$ (10)

$$\frac{(M+m)\, g \sin\alpha - P \cos\alpha}{M + m \sin^2\alpha} \geq 0 \quad (10)$$

$$\frac{(M+m)\, g \sin\alpha}{\cos\alpha} \geq P \quad (5)$$

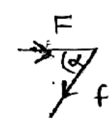
25

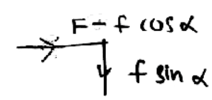
IV when $P = 0$,

$$F = \frac{-mg \sin\alpha \cos\alpha}{M + m \sin^2\alpha} \quad (10)$$

$$f = \frac{(M+m)\, g \sin\alpha}{M + m \sin^2\alpha} \quad (10)$$

20

$\therefore$ The acceleration of the particle relative to the earth $\Big\} =$  (10)

$=$ 

$$= \sqrt{(F + f\cos\alpha)^2 + (f \sin\alpha)^2} \quad (10)$$

$$= \sqrt{F^2 + 2fF \cos\alpha + f^2}$$

(5)

$$= \sqrt{\left(\frac{mg \sin\alpha \cos\alpha}{M + m \sin^2\alpha}\right)^2 + \frac{2\, mg \sin\alpha \cos\alpha}{M + m \sin^2\alpha} \times \frac{(M+m)\, g \sin\alpha}{M + m \sin^2\alpha} + \frac{(M+m)^2 g^2 \sin^2\alpha}{(M + m \sin^2\alpha)^2}} \quad (5)$$

$$= \frac{g \sin\alpha}{M + m \sin^2\alpha} \sqrt{m^2\cos^2\alpha + (M+m)^2 + 2m(M+m)\cos^2\alpha}$$

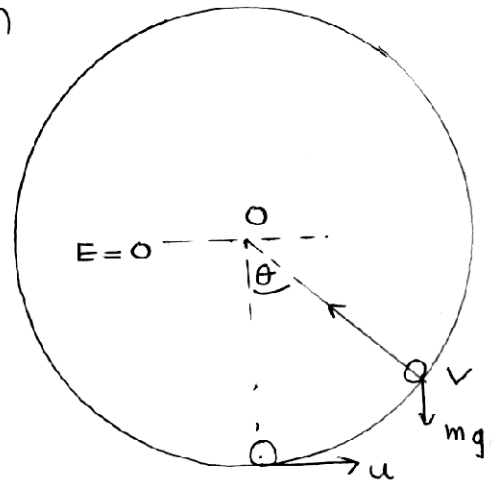
$$= \frac{g \sin\alpha}{M + m \sin^2\alpha} \sqrt{M^2 \cancel{+ m^2} - m^2(1 - \sin^2\alpha) + 2mM(1 - \cos^2\alpha)}$$

$$= \frac{g \sin\alpha}{M + m \sin^2\alpha} \sqrt{M^2 + m^2 \sin^2\alpha + 2mM \sin^2\alpha}$$

$$= \frac{g \sin\alpha \sqrt{[M^2 + m(\sin^2\alpha)(m + 2M)]}}{M + m \sin^2\alpha} \quad (5)$$

35

13) a)



5

Principle of Conservation of energy,

$$-mga + \frac{1}{2}mu^2 = -mga\cos\theta + \frac{1}{2}mv^2 \quad 10$$

$$-2ga + u^2 = -2ga\cos\theta + v^2$$

$$v^2 = u^2 + 2ga\cos\theta - 2ga \quad 5$$

$F = ma$

$$R - mg\cos\theta = \frac{mv^2}{a} \quad 10$$

$$R = mg\cos\theta + \frac{m}{a}(u^2 + 2ga\cos\theta - 2ga)$$

$$= \frac{m}{a}(u^2 + 3ga\cos\theta - 2ga) \quad 5$$

35

for the particle to leave the circular path $R = 0$. 5

$$\frac{m}{a}(u^2 + 3ga\cos\theta - 2ga) = 0$$

$$3ga\cos\theta = 2ga - u^2$$

$$\cos\theta = \frac{2ga - u^2}{3ga} \quad 5$$

when it leaves the circular path $\frac{\pi}{2} < \theta < \pi$ 5

$$\cos\frac{\pi}{2} < \cos\theta < \cos\pi \quad 5$$

$$0 > \frac{2ga - u^2}{3ga} > -1 \quad 5$$

$$2ga < u^2 < 5ga \quad 5$$

30

to complete the circular path when $\theta=\pi$,

$V>0$, $R>0$ (5)

$$V^2_{\theta=\pi} = u^2+2ga\,(\cos\pi)-2ga$$
$$= u^2-2ga-2ga$$
$$= u^2-4ga$$

$V_{\theta=\pi}>0$

$$u^2-4ga \geq 0$$
$$u^2 > 4ga \quad (5)$$

$$R_{\theta=\pi} = \frac{m}{a}\left(u^2+3ga\cos\pi-2ga\right)$$
$$= \frac{m}{a}\left(u^2-5ga\right)$$

$R_{\theta=\pi}>0$

$$\frac{m}{a}(u^2-5ga)>0$$
$$u^2 > 5ga \quad (5)$$

(5)

$4ga$ $\quad$ $5ga$

To satisfy both conditions $u^2>5ga$ (5)

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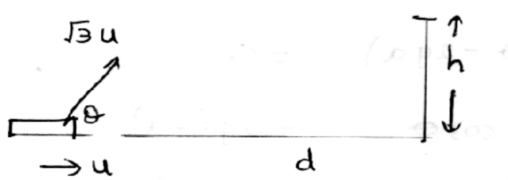
b)



Ship – S

Earth – E

Shell – B

(5)

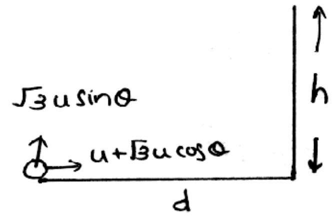
$\underline{V}_{S,E} = \overrightarrow{u}$ $\qquad$ $\underline{V}_{B,S} = u\sqrt{3}$ at θ

$$\underline{V}_{B,E} = \underline{V}_{B,S} + \underline{V}_{S,E}$$
$$= \sqrt{3}u\ (\theta) + \overrightarrow{u} \quad (10)$$

$\uparrow \sqrt{3}u\sin\theta$

$\rightarrow u+\sqrt{3}u\cos\theta$ (5)

20



$\rightarrow$ $s = ut$.

$d = (u+\sqrt{3}u\cos\theta)\,t$ (10)

$t = \dfrac{d}{u+\sqrt{3}u\cos\theta}$ (5)

$\uparrow$ $s = ut + \frac{1}{2}at^2$

$h = \sqrt{3}u\sin\theta\, t - \frac{g}{2}t^2$ (10)

$h = \sqrt{3}u\sin\theta \times \left(\dfrac{d}{u+\sqrt{3}u\cos\theta}\right) - \dfrac{g}{2}\left(\dfrac{d}{u+\sqrt{3}u\cos\theta}\right)^2$ (10)

$2h(u+\sqrt{3}u\cos\theta)^2 = 2(u+\sqrt{3}u\cos\theta)\sqrt{3}u\sin\theta\, d - gd^2$

$gd^2 + 2u^2h(1+\sqrt{3}\cos\theta)^2 - 2\sqrt{3}\sin\theta\, u^2 d(1+\sqrt{3}\cos\theta) = 0$ 40

$gd^2 + 2u^2(1+\sqrt{3}\cos\theta)^2 h - 2u^2\sqrt{3}\, d\sin\theta(1+\sqrt{3}\cos\theta) = 0$ (5)

---

14) a) $\alpha \underline{a} + \beta \underline{b} = 0$

let $\alpha \neq 0$.

$\underline{a} + \dfrac{\beta}{\alpha}\underline{b} = \underline{0}$

$\underline{a} = -\dfrac{\beta}{\alpha}\underline{b}$ #

Since $|\underline{a}| \neq 0$, $|\underline{b}| \neq 0$ and $\underline{a}$ and $\underline{b}$ are non parallel vectors.

$\therefore \alpha = 0$. (5)

let $\beta \neq 0$

$\dfrac{\alpha}{\beta}\underline{a} + \underline{b} = \underline{0}$

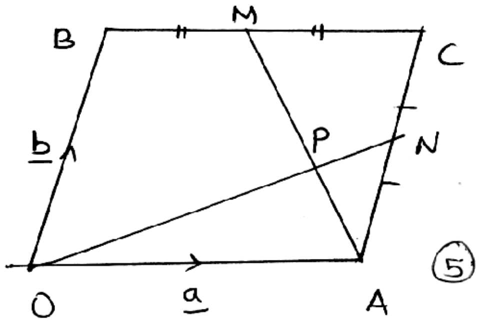
$\underline{b} = -\dfrac{\alpha}{\beta}\underline{a}$

#

$\therefore \beta = 0$ (5)

for $\alpha\underline{a} + \beta\underline{b} = \underline{0}$, $\alpha = 0$ and $\beta = 0$ (5)

15



$\overrightarrow{OP} = \lambda \overrightarrow{ON}$

$\overrightarrow{AP} = \mu \overrightarrow{AM}$

<u>OAN Δ</u>

$\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AN}$ 5

$\overrightarrow{OP} = \lambda \overrightarrow{ON}$

$\overrightarrow{OP} = \lambda\left(\overrightarrow{OA} + \overrightarrow{AN}\right)$

$= \lambda\left(\underline{a} + \dfrac{\underline{b}}{2}\right)$ 5

<u>OAP Δ</u>

$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$ 5

$= \overrightarrow{OA} + \mu \overrightarrow{AM}$

$= \underline{a} + \mu\left(\overrightarrow{AC} + \overrightarrow{CM}\right)$

$= \underline{a} + \mu\left(\underline{b} - \dfrac{\underline{a}}{2}\right)$ 5

$\boxed{25}$

$\overrightarrow{OP} = \lambda\left(\underline{a} + \dfrac{\underline{b}}{2}\right) = \underline{a} + \mu\left(\underline{b} - \dfrac{1}{2}\underline{a}\right)$ 5

$\left(\lambda - 1 + \dfrac{\mu}{2}\right)\underline{a} + \left(\dfrac{\lambda}{2} - \mu\right)\underline{b} = \underline{0}$ 5

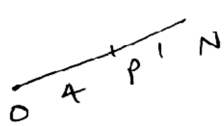
$\lambda - 1 + \dfrac{\mu}{2} = 0$ 5 or $\dfrac{\lambda}{2} - \mu = 0$ 5 (since $|\underline{a}| \neq 0$, $|\underline{b}| \neq 0$, $\underline{a}$ and $\underline{b}$ are non parallel vectors)

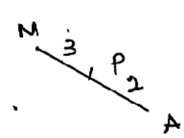
$2\mu + \dfrac{\mu}{2} = 1$ $\qquad \lambda = 2\mu$

$\dfrac{5\mu}{2} = 1$

$\mu = \dfrac{2}{5}$ 5 , $\lambda = \dfrac{4}{5}$ 5

$OP = \dfrac{4}{5} ON$ $\qquad AP = \dfrac{2}{5} AM$

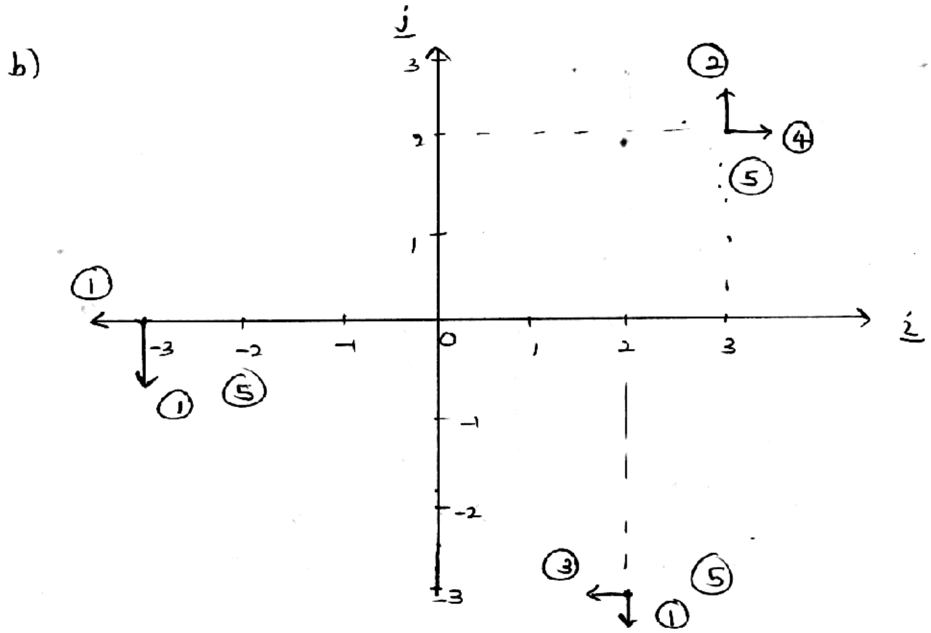




$OP : PN = \underline{\underline{4 : 1}}$ 5 $\qquad AP : PM = \underline{\underline{2 : 3}}$ 5

$\boxed{40}$

b)



15

for the system to reduce to a couple only $R = 0$, $G \neq 0$ 5

$X = 0$ and $Y = 0$.

$\rightarrow X = 4 - 1 - 3 = 0$ 5

$\uparrow Y = 2 - 1 - 1 = 0$ 5

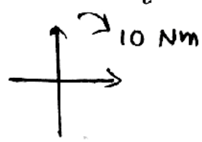
$\curvearrowleft G = (4 \times 2) - (2 \times 3) - (1 \times 3) + (3 \times 3) + (1 \times 2)$ 10

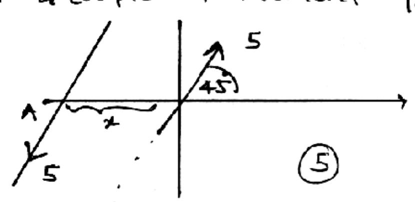
$= 8 - 6 - 3 + 9 + 2$

$= \underline{\underline{10 \text{ Nm}}}$ 5

30

For the equilibrium a couple of moment 10 Nm should be added. 5



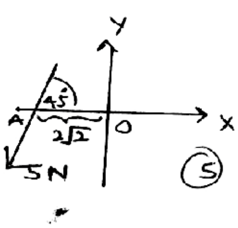


5

$x \times 5 \sin 45 = 10$ 5

$x = \frac{10\sqrt{2}}{5}$

$x = 2\sqrt{2}$ m 5

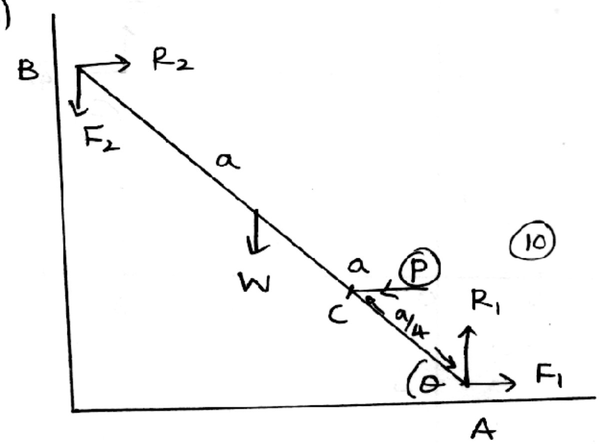


5

For equilibrium a force of 5N should be added at a distance $2\sqrt{2}$ m to the left of O. in the given direction.

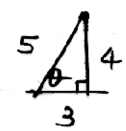
25

15) a)



(10)

$\tan\theta = \frac{4}{3}$



$F_1 = \frac{1}{2} R_1$ (5)

$F_2 = \frac{1}{2} R_2$ (5)

$\uparrow \quad R_1 - F_2 = W$ (5)

$R_1 = W + \frac{R_2}{2}$

$\rightarrow \quad P = F_1 + R_2$ (5)

$P = \frac{R_1}{2} + R_2$

$P = \frac{1}{2}\left(W + \frac{R_2}{2}\right) + R_2$

$P = \frac{W}{2} + \frac{5R_2}{4}$ (5)

$\overset{\curvearrowleft}{A}) \quad P \times \frac{a}{4} \sin\theta + W \times a\cos\theta + F_2 \times 2a\cos\theta - R_2 \times 2a\sin\theta = 0$

$P \times \frac{1}{4} \times \frac{4}{5} + W \times \frac{3}{5} + \frac{R_2}{2} \times 2 \times \frac{3}{5} - R_2 \times 2 \times \frac{4}{5} = 0$ (15)

$P + 3W + 3R_2 - 8R_2 = 0$

$R_2 = \frac{P + 3W}{5}$ (5)

$P = \frac{W}{2} + \frac{5}{4}\left(\frac{P + 3W}{5}\right)$

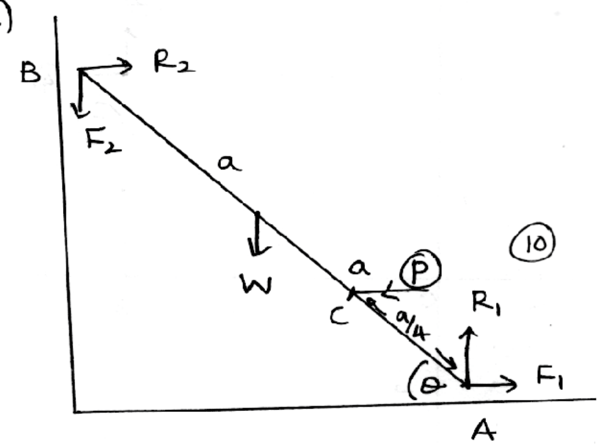
$= \frac{W}{2} + \frac{P}{4} + \frac{3W}{4}$

$\frac{3P}{4} = \frac{5W}{4}$

$P = \frac{5W}{3}$ (5)

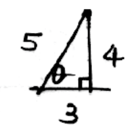
60

15) a)



(10)

$\tan\theta = \dfrac{4}{3}$



$F_1 = \frac{1}{2} R_1$ (5)

$F_2 = \frac{1}{2} R_2$ (5)

$\uparrow \quad R_1 - F_2 = W$ (5)

$R_1 = W + \dfrac{R_2}{2}$

$\rightarrow \quad P = F_1 + R_2$ (5)

$P = \dfrac{R_1}{2} + R_2$

$P = \dfrac{1}{2}\left(W + \dfrac{R_2}{2}\right) + R_2$

$P = \dfrac{W}{2} + \dfrac{5R_2}{4}$ (5)

$\curvearrowleft A)\quad P \times \dfrac{a}{4} \sin\theta + W \times a\cos\theta + F_2 \times 2a\cos\theta - R_2 \times 2a\sin\theta = 0$

(15)

$P \times \dfrac{1}{4} \times \dfrac{4}{5} + W \times \dfrac{3}{5} + \dfrac{R_2}{2} \times 2 \times \dfrac{3}{5} - R_2 \times 2 \times \dfrac{4}{5} = 0$

$P + 3W + 3R_2 - 8R_2 = 0$

$R_2 = \dfrac{P + 3W}{5}$ (5)

$P = \dfrac{W}{2} + \dfrac{5}{4}\left(\dfrac{P + 3W}{5}\right)$

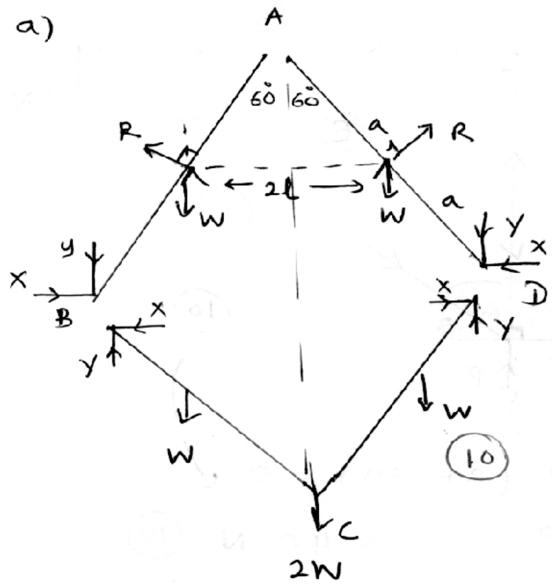
$= \dfrac{W}{2} + \dfrac{P}{4} + \dfrac{3W}{4}$

$\dfrac{3P}{4} = \dfrac{5W}{4}$

$P = \dfrac{5W}{3}$ (5)

60

16) a)



$\sin 60^\circ = \frac{l}{b}$

$\frac{\sqrt{3}}{2} = \frac{l}{b}$

$b = \frac{2l}{\sqrt{3}}$

for the system

$\uparrow \quad 2R\cos 30 = 6W$ (5)

$R = \frac{6W}{\sqrt{3}}$ (5)

for the rods BC and CD $\uparrow \quad 2Y = 4W$

$Y = 2W$ (5)

rod BC ,

$\curvearrowleft C$

$X \times 2a\cos 60^\circ - Y \times 2a\sin 60^\circ + W \times a\cos 30^\circ = 0$ (15)

$X \times 2 \times \frac{1}{2} - Y \times 2 \times \frac{\sqrt{3}}{2} + W \times \frac{\sqrt{3}}{2} = 0$

$X = 2\sqrt{3}W - \frac{\sqrt{3}W}{2} = \frac{3\sqrt{3}W}{2}$ (5)

$R' = \sqrt{4W^2 + \frac{27}{4}W^2}$

$= \frac{\sqrt{43}\,W}{2}$ (5)

$\tan\theta = \frac{2W}{3\sqrt{3}W/2}$

$\theta = \tan^{-1}\left(\frac{4}{3\sqrt{3}}\right)$ (5)

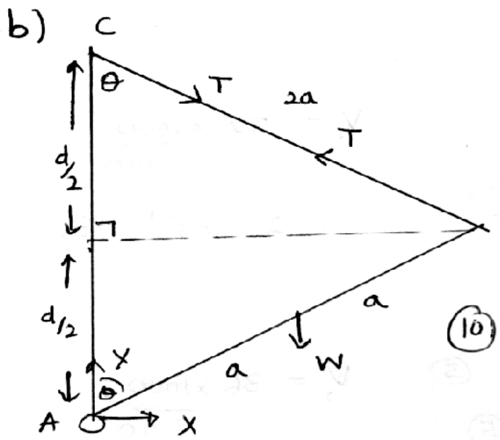
for the rod AB $\curvearrowright A$

$R \times b - X \times 2a\cos 60^\circ - Y \times 2a\sin 60^\circ - W \times a\sin 60^\circ = 0$ (15)

$\frac{6W}{\sqrt{3}} \times \frac{2l}{\sqrt{3}} - \frac{3\sqrt{3}W}{2} \times 2a \times \frac{1}{2} - 2W \times 2a \times \frac{\sqrt{3}}{2} - Wa \times \frac{\sqrt{3}}{2} = 0$

$l = \sqrt{3}a$ (10)

b)



$$\cos\theta = \frac{\frac{d}{2}}{2a} = \frac{d}{4a}$$

$$\sin\theta = \frac{\sqrt{16a^2-d^2}}{4a}$$

i) AB , A ↺ $\quad T \times d \sin\theta = W \times a \sin\theta$ 10

$$T = \frac{Wa}{d} \quad 5$$

ii) $\rightarrow \quad X = T\sin\theta$ 5

$$= \frac{Wa}{d} \times \frac{\sqrt{16a^2-d^2}}{4a} = \frac{\sqrt{16a^2-d^2}\,W}{4d} \quad 5$$

$\uparrow \quad Y = -T\cos\theta + W$ 10

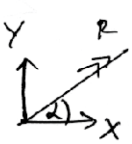
$$Y = W - \frac{Wa}{d} \times \frac{d}{4a} = \frac{3W}{4} \quad 5$$

The reaction at the hinge A, $R = \sqrt{\frac{(16a^2-d^2)}{16d^2} + \frac{9}{16}}\; W$

$$R = \sqrt{\frac{16a^2+8d^2}{16d^2}}\; W$$

$$= \left(\frac{2a^2+d^2}{2}\right)^{\frac{1}{2}} \frac{W}{d} \quad 10$$

iii)



$$\tan\alpha = \frac{\frac{3W}{4}}{\frac{\sqrt{16a^2-d^2}\,W}{4d}}$$

$$= \frac{3d}{\sqrt{16a^2-d^2}}$$

$$\alpha = \tan^{-1}\left(\frac{3d}{\sqrt{16a^2-d^2}}\right) \quad 10$$

70

(17) a)

$\rightarrow F - R = 0$

$F = 30000\,N$ (5)

$V_1 = 72 \times \frac{1000}{3600}$

$= 20\ ms^{-1}$ (5)

$P = FV_1$

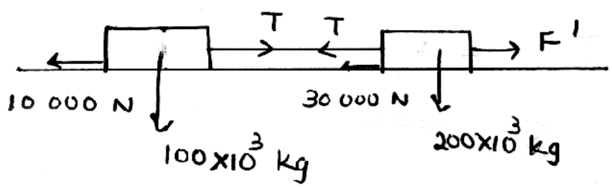
$= 30000 \times 20$

$= 600000\ W$ (5)

$= \underline{\underline{600\ KW}}$ (5)

$V_2 = 36 \times \frac{1000}{3600}$

$= 10\ ms^{-1}$



$P = FV_2$

$600 \times 10^3 = F' \times 10$

$F' = 60000\ N$ (5)

$\rightarrow F' - 40000 = 300 \times 10^3\, a$ (10)

$60000 - 40000 = 300 \times 10^3\, a$

$a = \frac{2}{30} = \underline{\underline{\frac{1}{15}\ ms^{-2}}}$ (5)

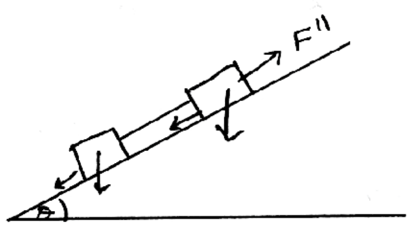
for the cabin, $\rightarrow F = ma$

$T - 10000 = 100 \times 10^3 \times \frac{1}{15}$ (5)

$T = 10000 + \frac{20000}{3}$

$= \frac{50000}{3}\ N$ (5)

50



$\sin\theta = \frac{1}{30}$

$\nearrow F'' - 30000 - 300 \times 10^3 \times 10 \times \frac{1}{30} = 300 \times 10^3 \times \frac{1}{10}$ (10)

$F'' = 30000 + 100000 + 30000$

$= 160000\ N$ (5)

$P = FV$

$600 \times 10^3 = 160 \times 10^3 \, V$ (5)

$V = \frac{60}{16} = \underline{\underline{\frac{15}{4}}} \; ms^{-1}$ (5)

25

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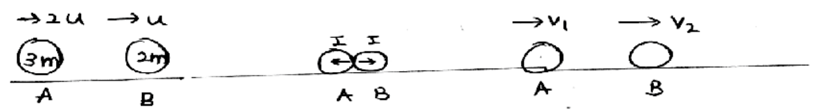
17) b)



For the system, $\vec{I} = \Delta mV$

$3mv_1 + 2mv_2 - 3m(2u) - 2m(u) = 0$ (10)

$3v_1 + 2v_2 = 8u$ —① (5)

Newton's Law of restitution,

$v_2 - v_1 = e(2u - u)$ (10)

$v_2 - v_1 = eu$ —②

$5v_1 = 8u - 2eu$

$v_1 = \underline{\underline{\frac{2u(4-e)}{5}}}$ (5)

$5v_2 = 8u + 3eu$

$v_2 = \underline{\underline{\frac{u(8+3e)}{5}}}$ (5)

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For sphere A, $\rightarrow I = \Delta mV$

$-I = 3m(v_1 - 2u)$ (5)

$I = 3m\left[2u - \frac{2u}{5}(4-e)\right]$

$I = \frac{6mu}{5}(1+e)$ (5)

for sphere B, $\rightarrow I = \Delta mV$

$I = 2m(v_2 - u)$ (5)

loss of Kinetic energy

$E = \frac{1}{2} 3m(2u)^2 + \frac{1}{2} 2mu^2 - \frac{1}{2}(3m)v_1^2 - \frac{1}{2}(2m)v_2^2$ (10)

$= \frac{1}{2} 3m (2u - v_1)(2u + v_1) + \frac{1}{2}(2m)(u - v_2)(u + v_2)$ (5)

$= \frac{1}{2} I (2u + v_1) - \frac{1}{2} I (u + v_2)$ (5)

$= \frac{1}{2} I (u + v_1 - v_2)$

$= \frac{1}{2} I (u - eu)$

$= \underline{\frac{1}{2} I (1-e) u}$ (5)

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