Provincial Department of Education - NWP

10 E I

First Term Test - Grade 12 - 2019

Index No : ....

Combined Mathematics I

Three hours only

Instructions:

- *This question paper consists of two parts.*
  Part A (Question 1 - 10) and **Part B** (Question 11 - 17)
- **Part A**
  *Answer **all** questions. Write your answers to each question in the space provided. you may use additional sheets if more space is needed.*
- **Part B**
  Answer five questions only. Write your answers on the sheets provided.
- *At the end of the time allocated, tie the answers of the two parts together so that* **Part A** *is on top of* **part B** *before handing them over to the supervisor.*
- *You are permitted to remove only* **Part B** *of the question paper from the Examination Hall.*

For Examiner's Use only

(10) Combined Mathematics I		
Part	Question No	Marks Awarded
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
	Total	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	
Paper 1 total		
Percentage		

Paper I	
Paper II	
Total	
Final Marks	

Final Marks

In Numbers	
In Words	

Marking Examiner	
Marks Checked by 1	
2	
Supervised by	

Combined Mathematics 12 - I (Part - A)

Answer all the questions in Part A and only for five questions in Part B.

01) Solve the equation $4^{x+1} + 2^{4x+2} = 80$.

02) Solve the inequality $\frac{12}{x-3} < x+1$.

03) Solve the equation $\log_2 x = \log_4(x + 6)$.

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04) Find the two points which lie on the x axis and lie at a distance $4\sqrt{2}$ units from the point $(-2,4)$. Obtain the distance between those two points.

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05) Find a and b such that the remainder when the polynomial $2x^4 + x^3 - x^2 + ax + b$ is divided by $(x^2 - 1)$ is $2x + 3$.

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06) Find the partial fractions of the rational function, $\dfrac{3x^2 - 7}{x^3 + 2x^2 - 8x}$.

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07) Solve the inequality $|3 - 2x| \leq |4 + x|$.

08) Sketch a graph of the piecewise function $y = \begin{cases} x^2 + 1 & ; \quad x \leq 0 \\ x + 3 & ; \quad 0 < x < 5 \\ -x + 1 & ; \quad x \geq 5 \end{cases}$.

09) If $a\cos\,(\lambda+\theta) = b\,cos\,(\lambda-\theta)$, show that $\tan\lambda = \left(\frac{a-b}{a+b}\right)\cot\theta$.

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10) Find the general solutions of the equation $\sin\,7\theta \;\; - \sqrt{3}\cos 4\theta = \sin\theta.$

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Combined Mathematics 12 - I (Part - B)

<u>Answer only five questions.</u>

11) a) State and prove the remainder theorem.
$f(x)$ is a polynomial function of degree greater than three and a, b and c are real distinct values. It is given that $f(1) = a,\ f(-1) = b$ and $f(o) = C$.
Show that the remainder when $f(x)$ is divided by $(x^2 - 1)$ is $\frac{1}{2}(a-b)x + \frac{1}{2}(a+b)$.
Also obtain the remainder when $f(x)$ is divided by $(x^3 - x)$.

b) Express $\frac{2.\dot{3} \text{ x } 1.2\dot{1}}{1.\dot{2}\dot{7}}$ in the form $\frac{p}{q}$, p හා $q \in \mathrm{Z}^+$ (simplification is not necessary)

12) a) let $f(x) = \frac{x+1}{x-2}$; $x \neq 2$.

i. Find the domain and the range of $f(x)$.

ii. Show that the function $f(x)$ is one to one and on to function.

iii. Find the inverse function $f^{-1}(x)$ of $f(x)$.

iv. Show that $f^{-1}(f(x)) = f\left(f^{-1}(x)\right) = x$.

b) If, $g(x) = \log_a\left(\frac{1+x}{1-x}\right)$, obtain that $g\left(\frac{2x}{1+x^2}\right) = 2g(x)$.

13) Obtain the coordinates of the point which divides the line joining the points (x_1, y_1) and (x_2, y_2) in the ratio $m:n$ internally.
The coordinates of the points A and B are $(-3,0)$ and $(7,5)$ respectively.

i. Find the coordinates of the P and Q which divide the line AB in the ratio $3:2$ internally and externally. Hence obtain the length of the line PQ.
ii. Find the coordinates of the two points which divide the line AB in to three equal parts.
iii. Find the ratio which divide the line AB by the Y axis. Find the coordinates of that point which is on the Y axis.

14) a) Show that $\log_a b = \frac{1}{\log_b a}$. Here a and b are positive real numbers and $a, b \neq 1$. Solve the equation $\log_x 2 \log_{\frac{x}{16}} 2 = \log_{\frac{x}{64}} 2$.

b) Sketch the graphs of $|x-2|$ and $|1+2x|$ in the same coordinate plane. Hence find the set of values of x which satisfies the inequality $|x-2| < 1 + |1+2x|$.

15) a) Find the real constants A, B and C such that $x^2 - 3x + 1 \equiv A\,(x+1)^2 + \{B(x+1) + C\,\}(x-2)$.
Hence separate $\frac{x^2-3x+1}{(x-2)(x+1)^2}$ in to partial fractions.

b) Let $f(x) = ax^3 + bx^2 - 2x + c$. Find the values of $a\,,b$ and c such that the remainder when $f(x)$ is divided by $(x^2 + x)$ is $6(x+1)$ and $(x-1)$ is a factor of $f(x)$. Hence obtain the remaining factors of $f(x)$.

16) a) If A, B, C are angles of a triangle with usual notation,
show that $sin^2\left(\frac{A}{2}\right) + sin^2\left(\frac{B}{2}\right) - sin^2\left(\frac{C}{2}\right) = 1 - 2\cos\left(\frac{A}{2}\right)\cos\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right)$

b) Show that $\sec x + \tan x = \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$ and deduce a similar expression for $\sec x - \tan x$.
Hence find the values of $\tan\left(\frac{7\pi}{12}\right)$ and $\tan\left(\frac{\pi}{12}\right)$ as surds.

c) Find the general solutions of the equation $2\sin\theta\sin 3\theta - 1 = 0$.

17) a) Using the expression of $Sin\,(A + B)$, obtain an expression for $\sin 3\theta$ in terms of $\sin\theta$.
Show that $Sin\,A \sin\,(60 - A)\sin\,(60 + A) = \frac{1}{4}\sin 3A$.
Hence deduce that the value of $Sin\;20^0 \sin\;40^0 \sin\;80^0$ in $\frac{\sqrt{3}}{8}$.

b) Express $f(x) = \sqrt{3}\,cos2x + sin\,2x$ in the form $R\cos(2x - \alpha)$. Here R and α constants to be determined such that $R > 0$ and $o < \alpha < \frac{\pi}{2}$
Sketch a graph of $f(x)$ in the range $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ stating the maximum and minimum values of $f(x)$.

c) If $a\,\sec\theta = 1 - b\tan\theta$ and $a^2\,sec^2\,\theta = 5 + b^2\,tan^2\,\theta$, show that $a^2b^2 + 4a^2 = 9b^2$.

Provincial Department of Education - NWP

10 E II

First Term Test - Grade 12 - 2019

Index No : ....

Combined Mathematics II

Three hours only

Instructions:

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  Part A (Question 1 - 10) and **Part B** (Question 11 - 17)
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  Answer all questions. Write your answers to each question in the space provided. you may use additional sheets if more space is needed.
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(10) Combined Mathematics II		
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A	1	
	2	
	3	
	4	
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	6	
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	15	
	16	
	17	
	Total	
Paper 1 total		
Percentage		

Paper I	
Paper II	
Total	
Final Marks	

Final Marks

In Numbers	
In Words	

Marking Examiner	
Marks Checked by 1	
2	
Supervised by	

(Part - A)

1) Two forces of magnitudes P and $2P$ act on a point inclined at an angle 60^0 . Find the magnitude of the resultant. Also if the angle between the resultant force and the force $2P$ is α , show that $\tan\alpha = \frac{\sqrt{3}}{5}$.

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2) $ABCDEF$ is a regular hexagon with centre O. Show that $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} + \overrightarrow{OE} + \overrightarrow{OF} = \underline{0}$.

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3) Let $\underline{a} = \underline{i} - 2\underline{j}$ and $\underline{b} = -3\underline{i} + \underline{j}$. If the vector $\underline{a} + \lambda\underline{b}$ is parallel to the vector $-\underline{i} - 3\underline{j}$, find the value of λ .

4) Let $\underline{a} = \underline{i} + \sqrt{3}\underline{j}$ and $\underline{b}$ is a vector with magnitude $\sqrt{3}$. If the angle between $\underline{a}$ and $\underline{b}$ is $\frac{\pi}{3}$, express $\underline{b}$ in the form of $x + iy$. Here $x < 0$ and x and y are constants to be determined.

5) The position vectors of the points A,B and C in the triangle ABC are $\underline{a}$, $\underline{b}$ and $\underline{c}$ respectively. D and E are mid points of the sides AB and AC. Find the position vectors of D and E .
Hence, show that $DE = \frac{1}{2} BC$ and $DE // BC$.

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6) The resultant of two forces P and Q inclined at an angle θ $(\theta < \frac{\pi}{2})$ is $\sqrt{3}\,Q$. If the resultant makes an angle 30^0 with force P, find the value of θ. Also show that $P = 2Q$.

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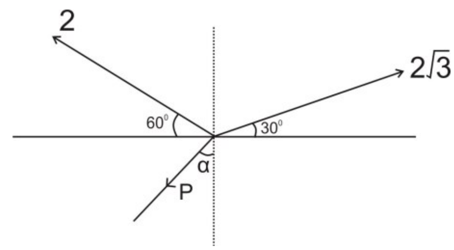
7) The forces of magnitude 2, $2\sqrt{3}$ and P Newton act on a particle as shown in the diagram. If the particle is in equilibrium, show that $\alpha = \frac{\pi}{6}$ and find the value of P.



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8) A system of forces P, $2P, 3\sqrt{3}\,P$ and $4P$ Newton act on a particle. The first force is horizontal and each other forces are inclined at an angles of 60^0, 90^0, 150^0 to each other. Find the magnitude and the direction of the resultant force.

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9) The position vectors of the points A, B, C relative to a point O are $60\underline{i} + 3\underline{j}$, $40\underline{i} - 8\underline{j}$ and $a\,\underline{i} - 52\,\underline{j}$ respectively. Find the value of a such that A, B and C are collinear. Here $a \in R$.

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10) The resultant force of two forces with magnitudes $P + Q$ and $P - Q$ is $\sqrt{P^2 + 3Q^2}$.

Here $P \neq Q$. Find the angle between the two forces.

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Combined Mathematics 12 - II (Part B)

Answer five questions only.

11) a) Let the coordinates of the point P relative to the cartesian coordinate system OXY are (a, b). Obtain the position vector of P relative to the origin O. Hence write an expression for $|\overrightarrow{OP}|$.
The coordinates of the points A and B relative to O are $(-2, -\sqrt{2})$ and $(3, 4\sqrt{2})$.
i. Find $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Hence find $\overrightarrow{AB}$.
ii. Find $|\overrightarrow{AB}|$.
iii. When C is the midpoint of AB, find $\overrightarrow{OC}$.
iv. Find the unit vector in the direction $\overrightarrow{OC}$ and find the vector with magnitude $\sqrt{19}$ units which is in the direction $\overrightarrow{OC}$.
v. Show that there are two possible points for $'D'$ such that $|\overrightarrow{OD}| = \sqrt{19}$ and OC is perpendicular to OD. Find the coordinates of them.

b) If the forces $2\underline{i} - 3\underline{j}$, $7\underline{i} + 4\underline{j}$, $-5\underline{i} - 9\underline{j}$, $P\underline{i} + 2\underline{j}$ and $\underline{i} - Q\underline{j}$ acting on a particle are in equilibrium, find the magnitudes of the forces P and Q. Here $\underline{i}$ and $\underline{j}$ are unit vectors acting along the perpendicular axes OX and OY.

12) a) $\underline{a}$ and $\underline{b}$ are any non zero, non-parallel vectors. When λ and μ are two scalars, show that $\lambda\underline{a} + \mu\underline{b} = \underline{0}$ if and only if $\lambda = \mu = 0$.

b) (i) $OACB$ is a parallelogram. The midpoint of AC is D. The intersection point of the diagonal AB and the line OD is E. If $\overrightarrow{OA} = \underline{a}$ and $\overrightarrow{OB} = \underline{b}$, show that $\overrightarrow{OD} = \underline{a} + \frac{1}{2}\underline{b}$.

(ii) Show that $\overrightarrow{OE} = \lambda\left(\underline{a} + \frac{1}{2}\underline{b}\right)$. By taking $\overrightarrow{AE} = \mu\,\overrightarrow{AB}$
show that $\overrightarrow{OE} = (1-\mu)\underline{a} + \mu\underline{b}$.
Hence, prove that $\mu = \frac{1}{3}$ and $\lambda = \frac{2}{3}$.

(iii) Also show that $\overrightarrow{AE} = \frac{\overrightarrow{AB}}{3}$ and $AE : AB = 1 : 3$.

13) a) The side BC of the parallelogram $OACB$ is produced to the point D such that $BD = 3BC$. Let $\overrightarrow{OA} = \underline{a}$ and $\overrightarrow{OB} = \underline{b}$. Express $\overrightarrow{OD}$ interms of $\underline{a}$ *and* $\underline{b}$.
Find the constant λ and μ such that $\overrightarrow{OE} = \lambda\overrightarrow{OD}$ and $\overrightarrow{AE} = \mu\,\overrightarrow{AC}$. Here E is the intersection point of the lines OD and AC.

b) In the triangle OAB, Q is a point closer to be on the line AB and it divids the side AB in the ratio 4:1. P is a point on OQ such that $OP : OQ = 1 : 2$. The produced line AP meet the side OB at R. The position vectors of the points A and B relative to the point O are $\underline{a}$ and $\underline{b}$ respectively.
i. Find $\overrightarrow{OQ}$ and show that, $\overrightarrow{OP} = \frac{1}{15}(\underline{a} + 4\underline{b})$.
ii. Express $|\overrightarrow{AP}|$ in terms of $\underline{a}$ and $\underline{b}$.
iii. Show that $\overrightarrow{OR} = \overrightarrow{OA} + \lambda\,\overrightarrow{AP}$ and find the value of λ such that the expression $\overrightarrow{OA} + \lambda\,\overrightarrow{AP}$ is independent of $\underline{a}$.
iv. Hence express the position vector of R in terms of $\underline{b}$ and show that $OR : OB = 2 : 7$.

14) a) Define the scalar product and vector product of two vectors $\underline{a}$ and $\underline{b}$.

If $\overrightarrow{OA} = \underline{a}$ and $\overrightarrow{OB} = \underline{b}$ in the triangle OAB, show that the area of the triangle OAB is giveen by $\frac{1}{2}\,|\underline{a} \times \underline{b}\,|$.

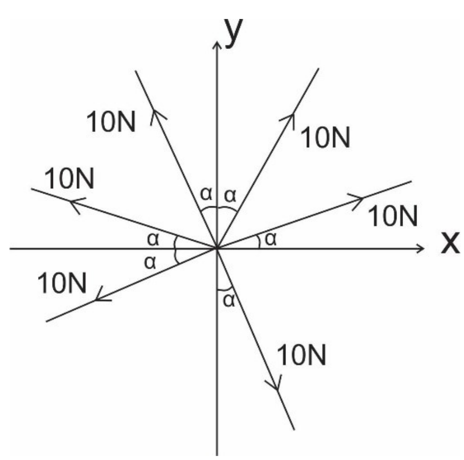
b) Let $\underline{a}$ and $\underline{b}$ are any two vectors such that the scalar product of $\underline{a} + \underline{b}$ and $\underline{a} - \underline{b}$ is zero. Show that the magnitudes of the vectors $\underline{a}$ and $\underline{b}$ are equal.

c) The position vectors of the points A and B relative to a fixed point O are defined as $\underline{a} + 2\underline{b}$ and $3\underline{a} - \underline{b}$ respectively. If OA and OB are perpendicular to each other, find $\underline{a}\,.\,\underline{b}$.
If $|\underline{a}| = 2$ and $|\underline{b}| = 1$, find the angle between $\underline{a}$ and $\underline{b}$.

d) Let $\underline{a} = 3\underline{i} + 4\underline{j}$ and $\underline{b}$ is an unit vector such that $\underline{b} = \lambda\underline{i} + \mu\underline{j}$. Here λ and μ are two scalers and $\mu > 0$. $\underline{i}$ and $\underline{j}$ are unit vectors in the usual notation. If $\underline{a}$ and $\underline{b}$ are perpendicular to each other, find the constants λ and μ.

15) a) $ABCDEF$ is a regular hexagon. The forces of $2, P, 5, Q$ and 3 newtons acting on a point act along the sides AB, CA, AD, AE and AF respectively. Find the values of P and Q such that the particle is in equilibrium.

b) Show that the magnitude of the resultant of the system of forces acting on the particle below is $10\sqrt{2}\ N$ and the direction which it makes with the positive direction of the X axis is $\frac{\tan\alpha+1}{\tan\alpha-1}$.

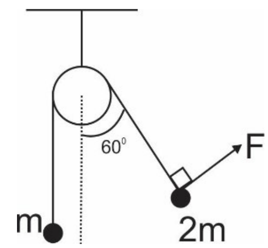


c) Three coplanar forces with magnitudes $6, 2\sqrt{3}$ and 8 newtons act on a point O in the directions OA, OB and OC respectively. If $A\hat{O}B = 30^0$, $B\hat{O}C = 90^0$ find the magnitude of the resultant of the system of forces and the angle which the resultant makes with OA.

16) a) Find the condition for a system of coplanar forces acting on a particle to be in equilibrium by condering the resolution of forces.

b) One end of a light inextensible string is attached to a fixed point A and a particle of weight W is attached to a point B on the string and a particle of weight W is attached to the other end C of the string. It is in equilibrium , by means offer horizontal force of $3w$ applied at C with the parts AB and BC of the string are taut and making the acute angles α and β respectively with the horizontal. Find the tensions of the parts of the string and the magnitudes of the angles α and β .

c) A particle of mass m is attached to one end of a light in extensible string and the string passes around a smooth light pulley and a mass $2m$ is attached to the other end. The system is kept in equilibrium by a force F applied as shown in the figure. Find the tension in the string and the magnitude of the force F.



17) a) Two forces P and Q inclined at an angle θ at a point. Obtain expressions for the resultant of the two forces and the angle which the resultant makes with the force P. Hence show that the resultant bisect the angle between the two forces when these two forces are equal.

b) Two inclined forces which are equal in magnitude act on a particle. If the magnitude of the square of the resultant of those two forces is twice as the product of the two forces, find the angle between the two forces.

Using the above result, deduce the angle between each force and the resultant.

c) When two equal forces are inclined at an angle 2α , their resultant is twice as the resultant when they are inclined at an angle 2β . Show that $\cos\alpha = 2\cos\beta$.

1\. $4^{x+1} + 2^{4x+2} = 80$

$4 \cdot 2^{2x} + 4\left(2^{2x}\right)^2 - 80 = 0$ (5)

Let

$2^{2x} = t$

(5)

$4t^2 + 4t - 80 = 0$

$t^2 + t - 20 = 0$ (5)

$(t+5)(t-4) = 0$

$t = -5$ or $t = 4$

(5)

$2^{2x} = -5$ or $2^{2x} = 4$

(no sol^n) $\underline{\underline{x = 1}}$ (5)

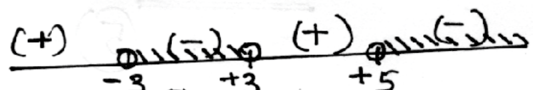
25

2\. $\dfrac{12}{x-3} < x+1$

$\dfrac{12}{x-3} - x - 1 < 0$ (5)

(5)

$\dfrac{12 - (x+1)(x-3)}{(x-3)}$ (5) $= \dfrac{(x-5)(x+3)}{(3-x)} < 0$



(5)

$\underline{\underline{x \in (-3,3) \cup (5, \infty)}}$ 25

(5)

3). $\log_2 x = \log_4 (x+6)$

$\log_2 x = \dfrac{1}{2\log_{(x+6)} 2} = \dfrac{1}{2}\log_2(x+6)$ (5)

$\log_2 x^2 = \log_2 (x+6)$ (5)

$x^2 = x+6$

$x^2 - x - 6 = 0$ (5)

$(x-3)(x+2) = 0$
(5)

$x = 3$ or $x = -2$ (5)

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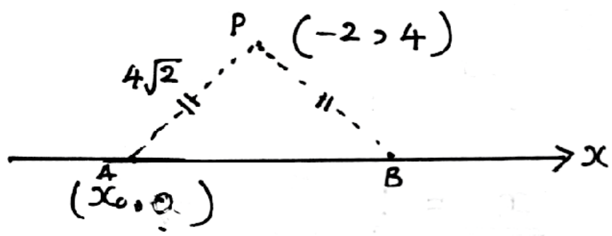
4). 

$(x_0+2)^2 + 16 = 32$ (5)

(5) $x_0 + 2 = \pm 4$

$x_0 = 2$ $\quad$ $x_0 = -6$

(5)
$A(2, 0)$ $\quad$ $B(-6, 0)$ (5)

$\therefore$ AB distance = 8 units
(5)

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5) $2x^4+x^3-x^2+ax+b \equiv (x^2-1)\,\phi(x) + 2x+3$ (10)

$x=1 \longrightarrow$

$a+b+2 = 5$ — ①

$x=-1 \longrightarrow$ (5)

$b-a = 1$ — ②

(5) $\underline{\underline{a=1}}$ $\underline{\underline{b=2}}$ (5) [25]

6) $\dfrac{3x^2-7}{x^3+2x^2-8x} = \dfrac{3x^2-7}{x(x^2+2x-8)} = \dfrac{3x^2-7}{x(x+4)(x-2)}$ (5)

$\dfrac{3x^2-7}{x(x+4)(x-2)} = \dfrac{A}{x} + \dfrac{B}{(x+4)} + \dfrac{C}{(x-2)}$ (5)

$3x^2-7 \equiv A(x^2+2x-8) + B(x-2)x + C(x+4)x$

$x^2 \longrightarrow \quad 3 = A+B+C$ — ①

$x \longrightarrow \quad 0 = 2A-2B+4C$

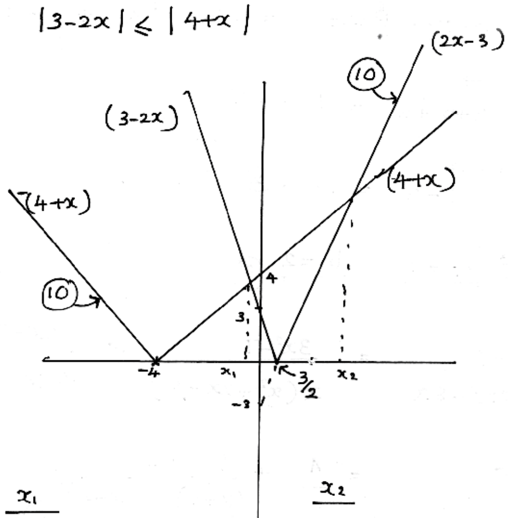
$0 = A-B+2C$ — ②

$x^0 \longrightarrow \quad -7 = -8A$ — ③ (10)

$\underline{\underline{A=\dfrac{7}{8}}}$ $\quad B=\underline{\underline{\dfrac{41}{24}}}$ $\quad C=\underline{\underline{\dfrac{5}{12}}}$ [25]

$\underline{\underline{\dfrac{3x^2-7}{x^3+2x^2-8x} = \dfrac{7}{8x} + \dfrac{41}{24(x+4)} + \dfrac{5}{12(x-2)}}}$ (5)

7) $|3-2x| \leq |4+x|$



$\underline{x_1}$

$4+x = 3-2x$

$3x=-1$

$x=-\frac{1}{3}$

$\underline{x_2}$

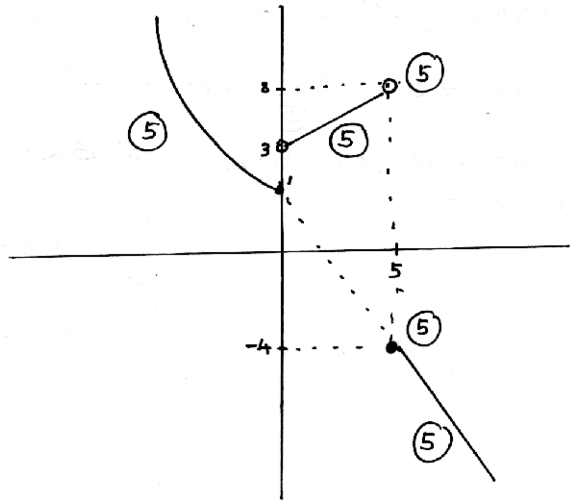
$2x-3 = 4+x$

$x = 7$

$\therefore \underline{\underline{x \in \left[-\frac{1}{3}, 7\right]}}$ (5) [25]

08)

$$y = \begin{cases} x^2+1 & ; x \le 0 \\ x+3 & ; 0 < x < 5 \\ -x+1 & ; x \ge 5 \end{cases}$$



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09) $a\cos(\lambda+\alpha) = b\cos(\lambda-\alpha)$

$a(\cos\lambda\cos\alpha - \sin\lambda\sin\alpha) = b(\cos\lambda\cos\alpha + \sin\lambda\sin\alpha)$ (5) (5)

$\cos\lambda(a\cos\alpha - b\cos\alpha) = \sin\lambda(b\sin\alpha + a\sin\alpha)$ (5)

(5) $\tan\lambda = \dfrac{\cos\alpha\,(a-b)}{\sin\alpha\,(b+a)}$

$\tan\lambda = \left(\dfrac{a-b}{a+b}\right)\cot\alpha$ (5)

25

10) $\sin 7\alpha - \sqrt{3}\cos 4\alpha = \sin\alpha$

$\sin 7\alpha - \sin\alpha = \sqrt{3}\cos 4\alpha$

(5) $2\cos 4\alpha \sin 3\alpha - \sqrt{3}\cos 4\alpha = 0$

$\cos 4\alpha\,(2\sin 3\alpha - \sqrt{3}) = 0$ (5)

$\cos 4\alpha = 0$ or (5) $2\sin 3\alpha - \sqrt{3} = 0$

(5) $4\alpha = 2n\pi \pm \pi/2 \; ; n \in \mathbb{Z}$

$\sin 3\alpha = \dfrac{\sqrt{3}}{2}$

$3\alpha = m\pi + (-1)^m(\pi/3)$ (5) $; m \in \mathbb{Z}$

25

11) Prove - Remainder Theorom. (15)

$f(x) \equiv (x^2-1)\,\phi(x) + (Ax+B)$ (10)

$f(x) = (x-1)(x+1)\,\phi(x) + (Ax+B)$ (5)

$x=1 \longrightarrow$

$f(1) = A+B$ —(1) (5)

$x=-1 \longrightarrow$

$f(-1) = B-A$ —— (2) (5)

$A = \frac{1}{2}\{f(1) - f(-1)\}$

But, $f(1) = a$, $f(-1) = b$, $f(0) = c$

Therefore, $A = \frac{1}{2}(a-b)$ (5)

$B = \frac{1}{2}(a+b)$ (5)

$\therefore$ Remainder - $(Ax+B)$

$= \frac{1}{2}(a-b)x + \frac{1}{2}(a+b)$ (10)

(45)

$f(x) = (x^3 - x)\,h(x) + (a_0x^2 + b_0x + c_0)$ ⑩

$f(x) = x(x-1)(x+1)\,h(x) + (a_0x^2 + b_0x + c_0)$ ⑤

$f(0) = c = c_0$ —— (1) ⑤

$f(1) = a_0 + b_0 + c_0$ —— (2) ⑤

$f(-1) = a_0 - b_0 + c_0$ —— (3) ⑤

$b_0 = \frac{1}{2}\{f(1) - f(-1)\}$

$b_0 = \frac{1}{2}(a - b)$ ⑤

From (2)

$a = a_0 + b_0 + c_0$

$a_0 = a - \frac{1}{2}(a - b) - c$

$a_0 = \frac{1}{2}(a + b - 2c)$ ⑤

$\therefore$ Remainder; $\frac{1}{2}(a+b-2c)x^2 + \frac{1}{2}(a-b)x + c$ ⑤ ⑤

△ 50

b). $\dfrac{2.\dot{3} \times 1.2\dot{1}}{1.\dot{2}\dot{7}} = N$

Let

$x = 2.\dot{3}$

$x = 2.3333\ldots$

$10x = 23.333\ldots$

$\Big\}\; 9x = 21$

$x = \dfrac{21}{9}$ (10)

Let $y = 1.2\dot{1}$

$y = 1.21111\ldots$

$10y = 12.11111\ldots$

$100y = 121.1111\ldots$

$\Big\}\; 90y = 109$

$y = \dfrac{109}{90}$ (10)

Let, $z = 1.\dot{2}\dot{7}$

$z = 1.27272727\ldots$

$100z = 127.272727\ldots$

$\Big\}\; 99z = 126$

$z = \dfrac{126}{99} = \dfrac{42}{33}$

$z = \dfrac{14}{11}$ (10)

$\therefore\; N = \dfrac{21}{9} \times \dfrac{109}{90} \times \dfrac{11}{14} = \dfrac{33 \times 109}{6 \times 90}$ (10) (40)

12)

(a). $f(x) = \frac{x+1}{x-2}$; $x \neq 2$

(i). Domain of f (D_f) = $\mathbb{R} \setminus \{2\}$ (5)

Range of f (R_f)

Let $y = \frac{x+1}{x-2}$

$x = \frac{2y+1}{y-1}$

$\therefore R_f = \mathbb{R} \setminus \{1\}$ (10)

15

ii). Let, any $x_1, x_2 \in D_f$

$f(x_1) = f(x_2)$ (5)

$\frac{x_1+1}{x_1-2} = \frac{x_2+1}{x_2-2}$

(10)

$(x_1+1)(x_2-2) = (x_2+1)(x_1-2)$

$x_1x_2 - 2x_1 + x_2 - 2 = x_1x_2 - 2x_2 + x_1 - 2$

$x_1 = x_2$ (5)

$\therefore$ f is one - one function. (5)

25

Let $y \in C_f$, we want to get $x \in D_f$ (6)

such that $f(x) = y$ (10)

$$\frac{x+1}{x-2} = y \Rightarrow x = \frac{2y+1}{y-1} \in D_f$$ (10)

Hence,

f is onto - function. (5) 25

iii). $f(x) = \frac{x+1}{x-2} = y.$

$$x = \frac{2y+1}{y-1}$$

$$\therefore f^{-1}(x) = \frac{2x+1}{x-1} \quad ; \quad x \neq 1$$ 15

iv). $f^{-1}f(x) = f(f^{-1}(x))$

$$f^{-1}f(x) = f^{-1}\left(\frac{x+1}{x-2}\right)$$ (5)

$$= \frac{2\left(\frac{x+1}{x-2}\right)+1}{\frac{x+1}{x-2} - 1}$$ (5)

$$= \frac{2x+2+x-2}{x+1-x+2} = \frac{3x}{3}$$ (5)

$$f^{-1}f(x) = x$$ ——— (A) (5)

$$ff^{-1}(x) = f\left(\frac{2x+1}{x-1}\right)$$ (5)

$$= \frac{\frac{2x+1}{x-1} + 1}{\frac{2x+1}{x-1} - 2}$$ (5)

$$= \frac{2x+1+x-1}{2x+1-2x+2}$$ (5)

$$= \frac{3x}{3}$$

$ff^{-1}(x) = x$ ——— (B) (5)

△40

From (A) and (B)

$$f^{-1}f(x) = ff^{-1}(x) = x$$

6). $g(x) = \log_a\left(\frac{1+x}{1-x}\right)$

$$g\left(\frac{2x}{1+x^2}\right) = \log_a\left(\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right)$$ (10)

$$g\left(\frac{2x}{1+x^2}\right) = \log_a\left(\frac{1+x^2+2x}{1+x^2-2x}\right) \quad (5)$$

$$= \log_a\left(\frac{1+x}{1-x}\right)^2 \quad (5)$$

$$= 2\log_a\left(\frac{1+x}{1-x}\right) \quad (5)$$

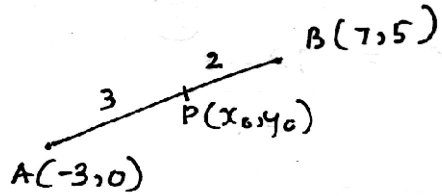
$$g\left(\frac{2x}{1+x^2}\right) = 2g(x) \quad (5)$$

(30)

B). To Proof –

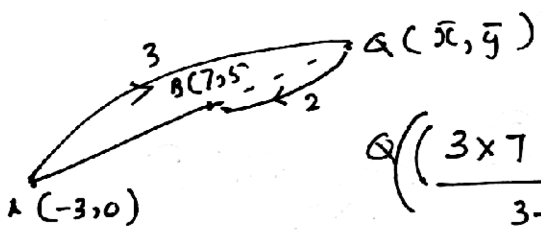
(35)

i).



$$P\left(\frac{3\times7-2\times3}{5}, \frac{3\times5+2\times0}{5}\right) \quad (10)$$

$$\underline{\underline{P(3,3)}} \quad (5)$$



$$Q\left(\frac{3\times7-2\times(-3)}{3-2}, \frac{3\times5-2\times0}{3-2}\right) \quad (10)$$

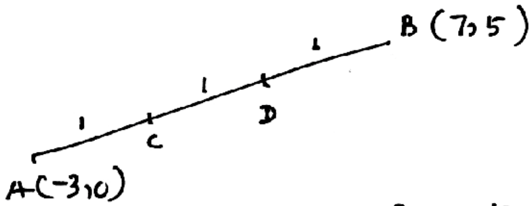
$$\underline{\underline{Q(27, 15)}} \quad (5)$$

$$\therefore PQ = \sqrt{(27-3)^2+(15-3)^2} \quad (10) \quad = \sqrt{24^2+12^2}$$

$$= \underline{\underline{12\sqrt{5} \text{ units}}} \quad (5)$$

(45)

ii).



$C\left(\frac{7+2\times -3}{3}, \frac{5+0}{3}\right)$ (10)

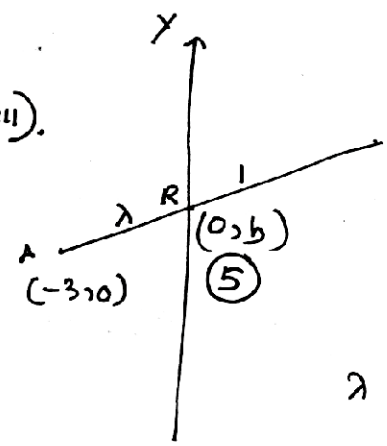
$C\left(\frac{1}{3}, \frac{5}{3}\right)$ (5)

$D\left(\frac{14-3}{3}, \frac{10+0}{3}\right)$ (10)

$D\left(\frac{11}{3}, \frac{10}{3}\right)$ (5)

30

iii).



(5)

$0 = \frac{7\lambda - 3}{\lambda + 1}$ (5)

$\lambda = 3/7$ (5)

$\frac{AR}{RB} = \frac{3}{7}$ (5)

$b = \frac{5\lambda - 0}{1+\lambda}$ (5) $= \frac{5 \times 3}{7 \times 10_2} \times 7$ (5)

$b = \frac{3}{2}$ (5)

$\therefore$ Point $R \equiv \left(0, \frac{3}{2}\right)$ (5)

40

14). To Proof [30]

$$\log_x 2 \times \log_{x/16} 2 = \log_{x/64} 2$$

$$\log_x 2 \times \frac{1}{\log_2\left(\frac{x}{16}\right)} = \frac{1}{\log_2\left(\frac{x}{64}\right)}$$ (10)

$$\log_x 2 \times \frac{1}{(\log_2 x - 4)} = \frac{1}{(\log_2 x - 6)}$$ (10)

Let,

$$\log_2 x = t$$ (5)

$$\frac{1}{t(t-4)} = \frac{1}{(t-6)}$$

$$t^2 - 4t - t + 6 = 0$$ (10)

$$t^2 - 5t + 6 = 0$$

$$(t-3)(t-2) = 0$$ (5)

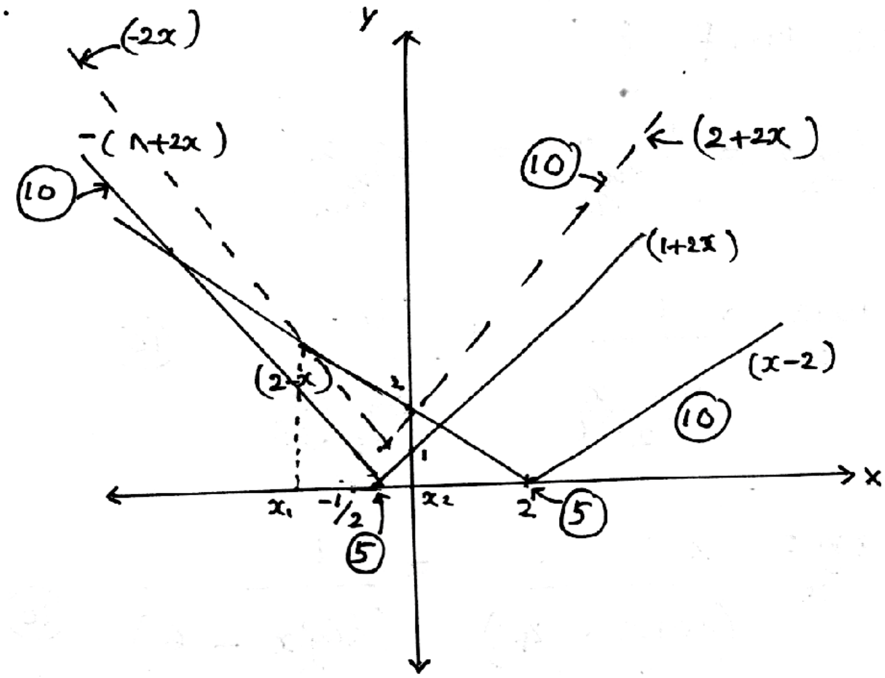
$t = 3$ (5) or $t = 2$ (5)

$\therefore \log_2 x = 3$ or $\log_2 x = 2$

$x = 8$ (5) or $x = 4$ (5)

[50]

b).



$x_1 \longrightarrow$

$2-x = -2x$ (5)

$x = -2$

$x_1 = -2$, $x_2 = 0$ (5)

$\therefore$ solution $x < -2$ or $x > 0$ (10)

50

15) (a). $x^2 - 3x + 1 \equiv A(x+1)^2 + \{B(x+1) + C\}(x-2)$

$x^2 \longrightarrow \quad 1 = A + B$ ——— (1) (5)

$x^1 \longrightarrow \quad -3 = 2A - 2B + (C + B)$

$-3 = 2A - B + C$ ——— (2) (5)

$x^0 \longrightarrow \quad 1 = A - 2(C + B)$

$1 = A - 2C - 2B$ ——— (3) (5)

(5) $A = -\frac{1}{9}$ $\quad B = \frac{10}{9}$ (5) $\quad C = -\frac{15}{9}$ (5)

$$\frac{x^2-3x+1}{(x-2)(x+1)^2} = \frac{\frac{-1}{9}(x+1)^2 + \left\{\frac{10}{9}(x+1) + \left(\frac{-15}{9}\right)\right\}(x-2)}{(x-2)(x+1)^2}$$ (20)

$$= \frac{-1}{9(x-2)} + \frac{10}{9(x+1)} - \frac{15}{9(x+1)^2}$$ (20)

(70)

b) $f(x) = ax^3 + bx^2 - 2x + c$

From the division Algorithm,

$ax^3+bx^2-2x+c \equiv (x^2+x)\,\phi(x) + 6(x+1)$ (10)

$ax^3+bx^2-2x+c \equiv x(x+1)\,\phi(x) + 6(x+1)$ (10)

$x=0$

$c = 6$ —— (1) (5)

$x=-1$

$-a+b+2+6 = 0$

$b-a = -8$ —— (2) (5)

$(x-1)$, factor of the Function $f(x)$.

$f(1) = 0$ (5)

$a+b-2+c = 0$

$a+b = -4$ —— (3) (5)

$\therefore$ $a = 2$ (5) $b = -6$ (5) $c = 6$ (5)

$$f(x) = 2x^3 - 6x^2 - 2x + 6 = (x-1)(2x^2 - 4x - 6) \quad (5) \quad (10)$$

$$= (x-1)(x-3)2(x+1)$$

$$= 2(x-1)(x+1)(x-3) \quad (10)$$

(80)

16)

a). $\sin^2\frac{A}{2} + \sin^2\frac{B}{2} - \sin^2\frac{C}{2} = 1 - 2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

L.H.S $\longrightarrow$

$$\sin^2\frac{A}{2} + \sin^2\frac{B}{2} - \sin^2\frac{C}{2}$$

$$= 1 - \cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) - \sin^2 C/2 \quad (5) \quad (5)$$

$$= 1 - \sin C/2 \cos\left(\frac{A-B}{2}\right) - \sin^2 C/2 \quad (\because A+B+C=\pi) \quad (5) \quad (5)$$

$$= 1 - \sin C/2 \left\{\cos\left(\frac{A-B}{2}\right) + \cos\left(\frac{A+B}{2}\right)\right\} \quad (10)$$

$$= 1 - \sin C/2 \times 2\cos\frac{A}{2}\cos\frac{B}{2} \quad (5)$$

$$= 1 - 2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2} \quad (5)$$

(40)

b). $\sec x + \tan x = \tan(\pi/4 + x/2)$

R.H.S.

$$\tan(\pi/4 + x/2)$$

$$= \frac{\tan\pi/4 + \tan x/2}{1 - \tan\pi/4 \tan x/2} \quad (10)$$

$$= \frac{1 + \tan x/2}{1 - \tan x/2} \quad (5) \quad = \frac{\cos x/2 + \sin x/2}{\cos x/2 - \sin x/2}$$

$$= \frac{(\cos x/2 + \sin x/2)^2}{\cos x} \quad (5)$$

$$= \frac{1 + \sin x}{\cos x} = \sec x + \tan x \quad (5)$$

$$\therefore \sec x + \tan x = \tan(\pi/4 + x/2) \quad (5)$$

(30)

Hence,

$$\sec x + \tan x = \tan(\pi/4 + x/2) \quad \text{——(A)}$$

If, $x = -x$ (5)

$$\sec x - \tan x = \tan(\pi/4 - x/2) \quad \text{——(B)}$$ (5)

(A) $\longrightarrow$ $x = 2\pi/3$ (5)

$$\tan\left(\pi/4 + \frac{2\pi}{3 \times 2}\right) = \tan\left(\frac{7\pi}{12}\right)$$ (5)

$$\therefore \tan\left(\frac{7\pi}{12}\right) = \sec\left(\frac{2\pi}{3}\right) + \tan\left(\frac{2\pi}{3}\right)$$ (5)

$$= \sec(\pi - \pi/3) + \tan(\pi - \pi/3)$$

(5)

$$= -\sec \pi/3 - \tan \pi/3$$

$$= -2 - \sqrt{3}$$

$$\tan\left(\frac{7\pi}{12}\right) = -(2 + \sqrt{3})$$ (5)

(B) $\longrightarrow$ $x = \pi/3$ (5)

$$\tan(\pi/4 - \pi/6) = \tan(\pi/12)$$ (5)

$$\therefore \tan(\pi/12) = \sec \pi/3 - \tan \pi/3$$ (5)

$$= 2 - \sqrt{3}$$

$$\tan(\pi/12) = 2 - \sqrt{3}$$ (5)

55

c) $2 \sin \alpha \sin 3\alpha - 1 = 0$

(5) $\cos 2\alpha - \cos 4\alpha - 1 = 0$

$$\cos 2\alpha - 2\cos^2 2\alpha + 1 - 1 = 0$$

(5)

$$\cos 2\alpha (1 - 2\cos 2\alpha) = 0$$ (5)

$\cos 2\alpha = 0$ or $\cos 2\alpha = 1/2$

$$\cos 2\alpha = \cos \pi/2$$ (5)

$$2\alpha = 2n\pi \pm \pi/2 \; ; \; n \in \mathbb{Z}$$ or

$$\cos 2\alpha = 1/2$$

$$\cos 2\alpha = \cos(\pi/3)$$

(5)

$$2\alpha = 2m\pi \pm \pi/3 \; ; \; m \in \mathbb{Z}$$

25

17. $\sin(A+B) = \sin A\cos B + \cos A\sin B$ (5)

$\sin(\alpha + 2\alpha) = \sin\alpha\cos 2\alpha + \cos\alpha\sin 2\alpha$ (5)

$\sin 3\alpha = \sin\alpha(1-2\sin^2\alpha) + 2\sin\alpha(1-\sin^2\alpha)$ (5)

(5)

$\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$

20

$\sin A\sin(60-A)\sin(60+A) = \frac{1}{4}\sin 3A$

L.H.S $\longrightarrow$

$= \sin A\sin(60-A)\sin(60+A)$

$= \sin A\,\frac{1}{2}\{\cos 2A - \cos 120\}$ (5)

$= \sin A\times\frac{1}{2}(\cos 2A + \frac{1}{2})$ (5)

$= \sin A\times\frac{1}{4}(2\cos 2A + 1)$ (5)

$= \sin A\times\frac{1}{4}\{2(1-2\sin^2 A)+1\}$ (5)

$= \frac{\sin A}{4}(3 - 4\sin^2 A)$

$= \frac{(3\sin A - 4\sin^3 A)}{4}$ (5)

$= \frac{1}{4}\sin 3A$. (5)

35

$$\sin 20^\circ \sin 40^\circ \sin 80^\circ = \frac{1}{4}\sin(3 \times 20^\circ) \quad (5)$$

$$= \frac{1}{4}\sin 60^\circ$$

$$= \frac{1}{4} \times \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{8} \quad (5)$$

10

b). $$f(x) = \sqrt{3}\cos 2x + \sin 2x$$

$$= 2\left(\frac{\sqrt{3}}{2}\cos 2x + \frac{1}{2}\sin 2x\right) \quad (10)$$

$$= 2\left(\cos\frac{\pi}{6}\cos 2x + \sin\frac{\pi}{6}\sin 2x\right)$$

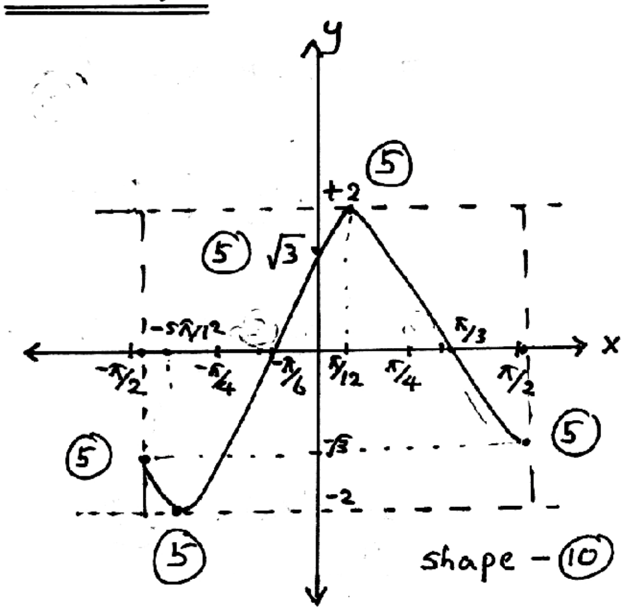
$$f(x) = 2\cos\left(2x - \frac{\pi}{6}\right) \quad (5)$$

(5) $R = 2$ $\qquad$ $\alpha = \frac{\pi}{6}$ (5)

25

$f(x)_{max} = 2$

$f(x)_{min} = -2$



shape - (10)

35

17).

c). $a\sec\alpha = 1 - b\tan\alpha$ —— (1)

$a^2\sec^2\alpha = 5 + b^2\tan^2\alpha$ —— (2)

(2) − (1)^2;

$a^2\sec^2\alpha - a^2\sec^2\alpha = 5 + b^2\tan^2\alpha - (1 - b\tan\alpha)^2$

$0 = 5 - 1 + 2b\tan\alpha$ (10)

$\tan\alpha = -\frac{2}{b}$ (5)

$\sec\alpha = \frac{3}{a}$

But,

$\sec^2\alpha - \tan^2\alpha = 1$ (5)

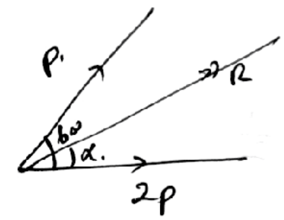
$\left(\frac{3}{a}\right)^2 - \left(\frac{-2}{b}\right)^2 = 1$

$\frac{9}{a^2} - \frac{4}{b^2} = 1$

$a^2b^2 + 4a^2 = 9b^2$ (5)

25

(1).



$R^2 = P^2 + (2P)^2 + 2(P)(2P)\cos 60$ (10)

$= P^2 + 4P^2 + 4P^2 \times \frac{1}{2}$

$R^2 = 7P^2$

$R = \sqrt{7}P$ (5)

$\tan\alpha = \frac{P\sin 60}{2P + P\cos 60}$ (5) $= \frac{P \times \sqrt{3}/2}{2P + P/2}$

$\tan\alpha = \frac{\sqrt{3}}{5}$ (5)

[25]

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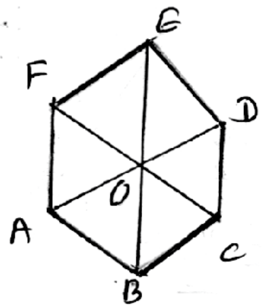
(5).



$\overrightarrow{OD} = -\overrightarrow{OA}$ (5)

$\overrightarrow{OE} = -\overrightarrow{OB}$ (5)

$\overrightarrow{OF} = -\overrightarrow{OC}$ (5)

$\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} + \overrightarrow{OE} + \overrightarrow{OF} = \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} - \overrightarrow{OA} - \overrightarrow{OB} - \overrightarrow{OC}$ (5)

$= \underline{0}$ (5)

[25]

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(3).

$\underline{a} = \underline{i} - 2\underline{j}$ $\qquad$ $\underline{b} = -3\underline{i} + \underline{j}$

$\underline{a} + \lambda\underline{b} = (\underline{i} - 2\underline{j}) + \lambda(-3\underline{i} + \underline{j})$ (5)

$\Rightarrow$ since $\parallel \rightarrow$ $\underline{a} + \lambda\underline{b} = k(-\underline{i} - 3\underline{j})$ (5)

$(\underline{i} - 2\underline{j}) + \lambda(-3\underline{i} + \underline{j}) = k(-\underline{i} - 3\underline{j})$.

$(1 - 3\lambda + k)\underline{i} + (-2 + \lambda + 3k)\underline{j} = \underline{0}$ (5)

$\therefore$ $1 - 3\lambda + k = 0 \longrightarrow 3\lambda - k = 1$ — (1) (5)

$-2 + \lambda + 3k = 0 \longrightarrow \lambda + 3k = 2$ — (2)

(1)$\times 3$ + (2). $\quad 9\lambda + \lambda = 5$

$\lambda = 1/2$ (5)

[25]

(4). $\underline{a} = \underline{i} + \sqrt{3}\,\underline{j}$ $\qquad |\underline{b}| = \sqrt{3}$ $\qquad \underline{b} = x\underline{i} + y\underline{j}$

$|\underline{a}| = 2$ $\qquad \therefore \sqrt{x^2+y^2} = \sqrt{3}$

$x^2 + y^2 = 3.$ — (1) (5)

$\underline{a} \cdot \underline{b} = |\underline{a}|\,|\underline{b}| \cos \pi/3.$ (5)

$(\underline{i} + \sqrt{3}\,\underline{j}) \cdot (x\underline{i} + y\underline{j}) = 2 \times \sqrt{3} \times \frac{1}{2}$

$x + \sqrt{3}y = \sqrt{3}.$

$x = \sqrt{3}(1-y)$ — (2) (5)

from (1). $3(1-y)^2 + y^2 = 3.$

$4y^2 - 6y + 3 = 3$ (5)

$y(4y-6) = 0 \Rightarrow y = 0$ or $y = 3/2$

when $y = 0$, $\quad x = \sqrt{3}.$ ✗.

when $y = \frac{3}{2}$ $\quad x = \sqrt{3}\left(1 - \frac{3}{2}\right) = -\sqrt{3}/2$

Since $x < 0$ $\quad \underline{b} = -\frac{\sqrt{3}}{2}\underline{i} + \frac{3}{2}\underline{j}$ (5) $\qquad$ [25]

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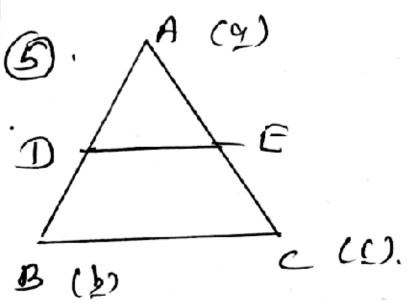
(5).



$\overrightarrow{OA} = \underline{a}$

$\overrightarrow{OB} = \underline{b}$

$\overrightarrow{OC} = \underline{c}.$

$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$

$= \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$

$= \underline{a} + \frac{1}{2}(\underline{b} - \underline{a}).$

$\overrightarrow{OD} = \left(\frac{\underline{a}+\underline{b}}{2}\right)$ (5)

$\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AE}$

$= \overrightarrow{OA} + \frac{1}{2}(\overrightarrow{AC})$

$= \underline{a} + \frac{1}{2}(\underline{c} - \underline{a})$

$\overrightarrow{OE} = \left(\frac{\underline{a}+\underline{c}}{2}\right)$ (5)

$\overrightarrow{DE} = \overrightarrow{DO} + \overrightarrow{OE}$ (5)

$= -\left(\frac{\underline{a}+\underline{c}}{2}\right) + \left(\frac{\underline{a}+\underline{c}}{2}\right)$

$= \frac{1}{2}(\underline{c} - \underline{b}).$

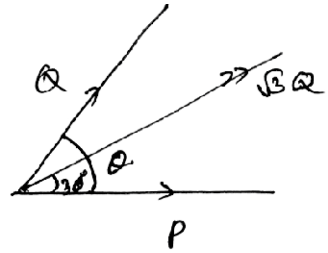
$\overrightarrow{DE} = \frac{1}{2}\overrightarrow{BC}$ (5)

$\therefore$ DE // BC and

(5) $DE = \frac{1}{2}BC.$

[25]

(6).



$\tan 30^\circ = \dfrac{Q\sin\theta}{P+Q\cos\theta}$ (5)

$\dfrac{1}{\sqrt{3}} = \dfrac{Q\sin\theta}{P+Q\cos\theta}$.

$P+Q\cos\theta = \sqrt{3}Q\sin\theta$ — (2)

$R^2 = P^2+Q^2+2PQ\cos\theta$

$3Q^2 = P^2+Q^2+2PQ\cos\theta$ — (1) (5)

(1) ⟹

$2PQ\cos\theta = 3Q^2-P^2-Q^2$.

$P^2+Q^2\cos^2\theta+2PQ\cos\theta = 3Q^2\sin^2\theta$.

$P^2+Q^2\cos^2\theta+3Q^2-P^2-Q^2 = 3Q^2\sin^2\theta$ (5)

$Q^2\cos^2\theta+2Q^2-3Q^2\sin^2\theta = 0$

$\cos^2\theta+2-3\sin^2\theta = 0$.

$4\sin^2\theta-3 = 0$

$\sin^2\theta = 3/4$

$\sin\theta = \pm\dfrac{\sqrt{3}}{2}$

since $\theta < \pi/2 \Rightarrow \theta = \sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right) = \pi/3$ (5)

from (2);

$P+\dfrac{Q}{2} = \sqrt{3}Q\times\dfrac{\sqrt{3}}{2}$ (5)

$P+\dfrac{Q}{2} = \dfrac{3Q}{2}$

$P = 2Q$

[25]

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(7).

$\rightarrow X = 2\sqrt{3}\cos 30 - 2\cos 60 - P\sin\alpha = 0$. (5)

$2\sqrt{3}\times\dfrac{\sqrt{3}}{2} - 2\times\dfrac{1}{2} - P\sin\alpha = 0$

$P\sin\alpha = 2$. — (1)

$\uparrow Y = 2\sin 60 + 2\sqrt{3}\sin 30 - P\cos\alpha = 0$ (5)

$2\times\dfrac{\sqrt{3}}{2} + 2\sqrt{3}\times\dfrac{1}{2} - P\cos\alpha = 0$

$P\cos\alpha = 2\sqrt{3}$ — (2)

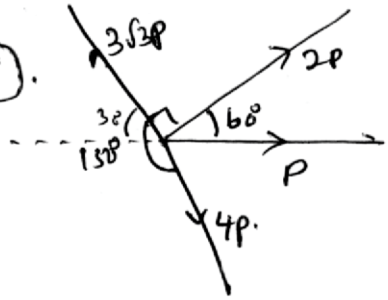
$\dfrac{(1)}{(2)}$ $\tan\alpha = \dfrac{1}{\sqrt{3}}$

$\alpha = \pi/6$. (5)

from (1); $P = \dfrac{2}{\sin\alpha}$ (5) $= \dfrac{2}{\sin\pi/6} = 4\,N$ (5)

[25]

(8)



(5)

$\rightarrow X = P + 2P\cos 60 - 3\sqrt{3}P\cos 30 + 4P\cos 60$ (5)

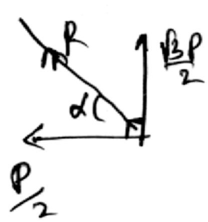
$= P + P - \frac{9}{2}P + 2P$

$X = 4P - \frac{9P}{2} = -\frac{P}{2}$

$\uparrow Y = 2P\sin 60 + 3\sqrt{3}P\sin 30 - 4P\sin 60$ (5)

$= 2P \times \frac{\sqrt{3}}{2} + 3\sqrt{3}P \times \frac{1}{2} - 4P \times \frac{\sqrt{3}}{2}$

$Y = \frac{\sqrt{3}}{2}P$



$R^2 = \frac{3P^2}{4} + \frac{P^2}{4} = P^2$

$R = P$ (5)

$\tan\alpha = \frac{\sqrt{3}P/2}{P/2}$

$\alpha = \pi/3$ (5)

25

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(9)

$\overrightarrow{AB} = \underline{b} - \underline{a}$

$= (40\underline{i} - 8\underline{j}) - (60\underline{i} + 3\underline{j})$

$= -20\underline{i} - 11\underline{j}$ (5)

$\overrightarrow{BC} = \underline{c} - \underline{b}$

$= (a\underline{i} - 52\underline{j}) - (40\underline{i} - 8\underline{j})$ (5)

$\overrightarrow{BC} = (a - 40)\underline{i} - 44\underline{j}$

If points A, B, C are colinear;

$$\vec{AB} = k\vec{BC} \quad (5)$$

$$-20\underline{i} - 11\underline{j} = k\left[(a-40)\underline{i} - 44\underline{j}\right] \quad (5)$$

$-11 = -44k$

$k = \frac{1}{4}$

$-20 = k(a-40)$

$-20 = \frac{1}{4}(a-40)$

$-80 = a - 40$

$-40 = a$

$\underline{a = -40}$ (5)

25

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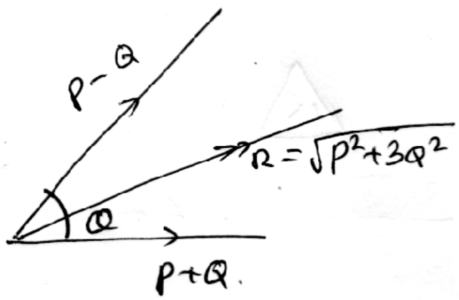
(10)



$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

$$P^2 + 3Q^2 = (P+Q)^2 + (P-Q)^2 + 2(P+Q)(P-Q)\cos\theta \quad (5)$$

$$P^2 + 3Q^2 = P^2 + Q^2 + 2PQ + P^2 + Q^2 - 2PQ + 2(P^2 - Q^2)\cos\theta$$

$$P^2 + 3Q^2 = 2P^2 + 2Q^2 + 2(P^2 - Q^2)\cos\theta$$

$$Q^2 = P^2 + 2(P^2 - Q^2)\cos\theta \quad (5)$$

$$2(P^2 - Q^2)\cos\theta = (Q^2 - P^2) \quad (5)$$

$$\cos\theta = \frac{-(P^2 - Q^2)}{2(P^2 - Q^2)}$$

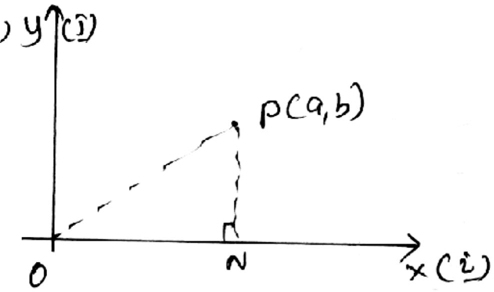
$$\cos\theta = -\frac{1}{2} \quad (5)$$

$\underline{\theta = 120^\circ}$ (5)

25

part B.

(11) (a)



$\overrightarrow{OP} = \overrightarrow{ON} + \overrightarrow{NP}$ (5)

$\overrightarrow{OP} = a\underline{i} + b\underline{j}$ (5)

$|\overrightarrow{OP}| = OP = \sqrt{a^2+b^2}$ (5) [15]

(i) $\overrightarrow{OA} = -2\underline{i} - \sqrt{2}\,\underline{j}$ (5)

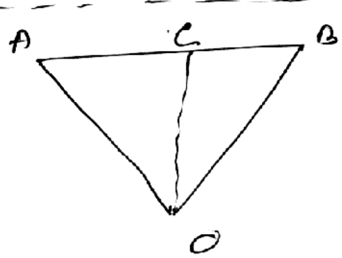
$\overrightarrow{OB} = 3\underline{i} + 4\sqrt{2}\,\underline{j}$ (5)

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = 2\underline{i} + \sqrt{2}\,\underline{j} + 3\underline{i} + 4\sqrt{2}\,\underline{j}$

$\overrightarrow{AB} = 5\underline{i} + 5\sqrt{2}\,\underline{j}$ (5)

(ii) $|\overrightarrow{AB}| = \sqrt{5^2 + (5\sqrt{2})^2} = 5\sqrt{3}.$ (5) [20]

(iii)



$\overrightarrow{OC} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$ (10)

$= \left(2\underline{i} - \sqrt{2}\,\underline{j}\right) + \frac{1}{2}\left(5\underline{i} + 5\sqrt{2}\,\underline{j}\right)$

$\overrightarrow{OC} = \frac{1}{2}\underline{i} + \frac{3\sqrt{2}}{2}\underline{j}$ (5)

(iv) $|\overrightarrow{OC}| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{3\sqrt{2}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{18}{4}} = \frac{\sqrt{19}}{2}$ (5)

Unit vector along $\overrightarrow{OC}$ $(\underline{u}) = \frac{\overrightarrow{OC}}{|\overrightarrow{OC}|}$ (5) $= \frac{1}{\left(\frac{\sqrt{19}}{2}\right)}\left[\frac{1}{2}\underline{i} + \frac{3\sqrt{2}}{2}\underline{j}\right]$

$\underline{u} = \frac{1}{\sqrt{19}}\left[\underline{i} + 3\sqrt{2}\,\underline{j}\right]$ (5)

Vector with magnitude $\sqrt{19}$; $\Rightarrow \sqrt{19}\, \underline{u}$

$= \underline{i} + 3\sqrt{2}\,\underline{j}$ (5) [35]

(v). $\overrightarrow{OC} \cdot \overrightarrow{OD} = 0$.

Let $\overrightarrow{OD} = x\underline{i} + y\underline{j} \Rightarrow \sqrt{x^2 + y^2} = \sqrt{19}$

$|\overrightarrow{OD}| = \sqrt{19}$ $\quad x^2 + y^2 = 19$ — ① (5)

$\overrightarrow{OC} \cdot \overrightarrow{OD} = 0$

$\left(\frac{1}{2}\underline{i} + \frac{3\sqrt{2}}{2}\underline{j}\right) \cdot (x\underline{i} + y\underline{j}) = 0$ (10)

$\frac{x}{2} + \frac{3\sqrt{2}}{2}y = 0$. (5)

$x + 3\sqrt{2}y = 0$.

$x = -3\sqrt{2}y$. (5)

from ①; $(-3\sqrt{2}y)^2 + y^2 = 19$.

$18y^2 + y^2 = 19$

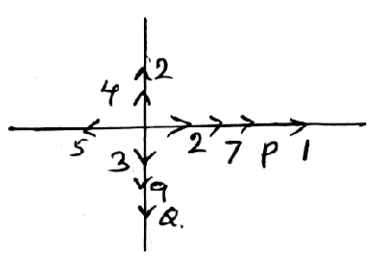
$y^2 = 1$ (5)

$y = \pm 1$ (5)

When $y = -1$; $x = 3\sqrt{2}$ (5) $\therefore \overrightarrow{OD} = (3\sqrt{2}\,\underline{i} - 1\,\underline{j})$ (5)

When $y = +1$; $x = -3\sqrt{2}$ (5) $\overrightarrow{OD} = (-3\sqrt{2}\,\underline{i} + \underline{j})$ (5) [55]

(b).



(5)

$\overrightarrow{X} = 0$.

$2 + 7 + P + 1 - 5 = 0$ (5)

$\underline{P = -5}$ (5)

$\uparrow Y = 0$. (5)

$4 + 2 - 9 - 3 - Q = 0$

$\underline{Q = -6}$ (5) [25]

(12) (a) $\lambda \underline{a} + \mu \underline{b} = \underline{0}$

Let $\lambda \neq 0$, then $\underline{a} = -\frac{\mu}{\lambda} \underline{a}$

This is of the form $\underline{a} = k\underline{b} \Rightarrow \underline{a} \parallel \underline{b}$

But $\underline{a}$ and $\underline{b}$ are non-parallel vectors.

This is impossible. That is if $\lambda = 0$ then $\mu \underline{b} = 0$

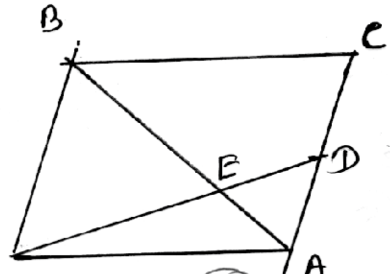
but $\underline{b} \neq 0 \Rightarrow \mu = 0$ $\therefore \lambda = \mu = 0$

Conversly; let $\lambda = 0$ and $\mu = 0$.

$\Rightarrow$ Then $\lambda \underline{a} + \mu \underline{b} = 0 + 0 = 0$

That is if $\lambda \underline{a} + \mu \underline{b} = \underline{0} \Leftrightarrow \lambda = 0$ and $\mu = 0$. (30)

(b).



(5)

$\overrightarrow{OA} = \underline{a}$
$\overrightarrow{OB} = \underline{b}$

(i)

$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$ (10)
$= \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AC}$ (5)
$= \overrightarrow{OA} + \frac{1}{2}\overrightarrow{OB}$
$\overrightarrow{OD} = \underline{a} + \frac{1}{2}\underline{b}$ (5)

(25)

(ii)

$\overrightarrow{OE} = \lambda \overrightarrow{OD}$ (5)
$\overrightarrow{OE} = \lambda \left[\underline{a} + \frac{1}{2}\underline{b}\right]$ (5)

$\overrightarrow{AE} = \mu \cdot \overrightarrow{AB}$ (5)
$\overrightarrow{AE} = \mu(\underline{b} - \underline{a})$. (5)

$\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AE}$ (10)
$= \underline{a} + \mu(\underline{b} - \underline{a})$
$\overrightarrow{OE} = (1-\mu)\underline{a} + \mu \underline{b}$ (5)

(35)

$\lambda(a + \frac{1}{2}b) = (1-\mu)a + \mu b$ (10)

$(\lambda - 1 + \mu)a + (\frac{\lambda}{2} - \mu)b = 0$ (10)

$\therefore \lambda + \mu = 1$ — ① (10)

$\frac{\lambda}{2} = \mu.$ — ② (10)

$\therefore \lambda = 2\mu.$

$\therefore 3\mu = 1$

$\mu = \frac{1}{3}.$ (5) $\lambda = \frac{2}{3}$ (5)

$\therefore \overrightarrow{AE} = \frac{1}{3}\overrightarrow{AB}$ (5)

$AE : AB = 1 : 3$ (5)

60

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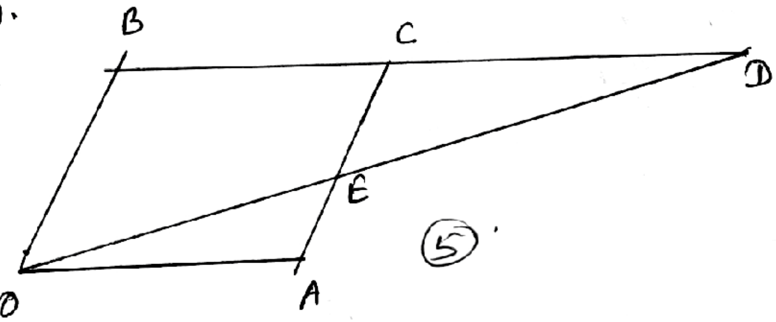
(13). (a).



(5)

$\overrightarrow{OA} = a$ $\overrightarrow{OB} = b$ $\overrightarrow{BD} = 3\overrightarrow{BC} = 3a$ (5)

$\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD}$ (5)

$\overrightarrow{OD} = b + 3a$ (5)

$\overrightarrow{OE} = \lambda\overrightarrow{OD}$

$\overrightarrow{OE} = \lambda(b + 3a)$ (5)

$\overrightarrow{AE} = \mu\overrightarrow{AC}$

$\overrightarrow{AE} = \mu[b]$

$\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AE}$

$\overrightarrow{OE} = a + \mu b$ (5)

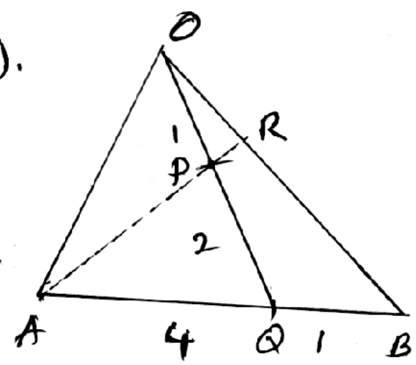
$\underline{a} + \mu \underline{b} = 3\lambda \underline{a} + \lambda \underline{b}$ (10)

$\therefore \quad 3\lambda = 1$ (5) $\qquad \mu = \lambda$ (5)

$\lambda = \frac{1}{3}$ (5)

$\therefore \lambda = \mu = \frac{1}{3}$ (5) $\qquad$ 60

(b).



$\overrightarrow{OA} = \underline{a}$

$\overrightarrow{OB} = \underline{b}$

$\overrightarrow{OQ} = \overrightarrow{OA} + \overrightarrow{AQ}$ (5)

$= \underline{a} + \frac{4}{5}\overrightarrow{AB}$

$= \underline{a} + \frac{4}{5}(\underline{b} - \underline{a})$ (5)

$\overrightarrow{OQ} = \frac{1}{5}(\underline{a} + 4\underline{b})$ (5)

$\overrightarrow{OP} = \frac{1}{3}\overrightarrow{OQ}$ (5) $= \frac{1}{3} \times \frac{1}{5}(\underline{a} + 4\underline{b})$

$\overrightarrow{OP} = \frac{1}{15}(\underline{a} + 4\underline{b})$ (5)

(ii) $\overrightarrow{AP} = \overrightarrow{AO} + \overrightarrow{OP}$ (5)

$= -\underline{a} + \frac{1}{15}(\underline{a} + 4\underline{b})$ (5)

$= \frac{1}{15}[\underline{a} + 4\underline{b} - 15\underline{a}]$

$\overrightarrow{AP} = \frac{1}{15}(4\underline{b} - 14\underline{a})$ (5) $\qquad$ 40

$\overrightarrow{OR} = \overrightarrow{OA} + \overrightarrow{AR} = \overrightarrow{OA} + \lambda\overrightarrow{AP}$. (5)

$\overrightarrow{OA} + \lambda\overrightarrow{AP} = \underline{a} + \frac{1}{15}\lambda\left[4\underline{b} - 14\underline{a}\right]$ (5)

$= \frac{1}{15}\left[15 - 14\lambda\right]\underline{a} + \frac{4\lambda}{15}\underline{b}$ (5)

for independance from $\underline{a}$;

$15 - 14\lambda = 0$ (5)

$\lambda = \frac{15}{14}$. (5)

(iv). $\overrightarrow{OR} = \overrightarrow{OA} + \lambda\overrightarrow{AP} = \frac{4}{15}\lambda\underline{b}$. (5)

$= \frac{4}{15} \times \frac{15}{14}\underline{b}$ (5)

$\overrightarrow{OR} = \frac{2}{7}\underline{b}$ (5)

$\overrightarrow{OR} = \frac{2}{7}\overrightarrow{OB}$ (5)

$\therefore OR : OB = 2 : 7$. (5)

50

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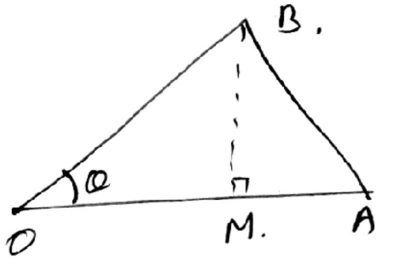
(14) (a). dot product $\Rightarrow \underline{a}\cdot\underline{b} = |a||b|\cos\theta$. (5)

θ is the angle between two vectors.

cross product $\Rightarrow \underline{a}\times\underline{b} = |\underline{a}||\underline{b}|\sin\theta\ \underline{n}$. (5)

where θ is the angle between two vectors and $\underline{n}$ is the unit vector which obey right handed screw law in the direction perpendicular to both $\underline{a}$ and $\underline{b}$.

10



$\overrightarrow{OA} = \underline{a}$

$\overrightarrow{OB} = \underline{b}$

Area of the triangle $OAB = \frac{1}{2} \times OA \times BM$ (5)

$= \frac{1}{2} \times |\underline{a}| \times OB \sin\theta$ (5)

$= \frac{1}{2} \times |\underline{a}||\underline{b}| \sin\theta$ (5)

$\triangle OAB = \frac{1}{2} |\underline{a} \times \underline{b}|$ (5)

20

(b). $(\underline{a} + \underline{b}) \cdot (\underline{a} - \underline{b}) = 0$ (10)

$\underline{a} \cdot \underline{a} - \underline{a} \cdot \underline{b} + \underline{a} \cdot \underline{b} - b \cdot b = 0$ (10)

$|a|^2 - |b|^2 = 0$

$|a| = |b|$ (10)

30

(c). $\overrightarrow{OA} = \underline{a} + 2\underline{b}$

$\overrightarrow{OB} = 3\underline{a} - \underline{b}$ (5)

$\overrightarrow{OA} \cdot \overrightarrow{OB} = 0$ (5)

$(\underline{a} + 2\underline{b}) \cdot (3\underline{a} - \underline{b}) = 0$

$3\underline{a} \cdot \underline{a} - \underline{a} \cdot \underline{b} + 6\, a \cdot \underline{b} - 2\, b \cdot \underline{b} = 0$

$3|a|^2 + 5\, \underline{a} \cdot \underline{b} - 2|b|^2 = 0$ (5)

$\therefore \underline{a} \cdot \underline{b} = \frac{2}{5}|b|^2 - \frac{3}{5}|a|^2$ (5)

$|a| = 2 \qquad |b| = 1$. Let θ be the angle between vectors.

$\underline{a} \cdot \underline{b} = |a||b|\cos\theta = \frac{2}{5}|b|^2 - \frac{3}{5}|a|^2$ (5)

(5)

$= 2 \times 1 \cos\theta. \quad = \frac{2}{5}|1|^2 - \frac{3}{5}|2|^2$

$2\cos\theta = \frac{2}{5} - \frac{12}{5}$ (5)

$2\cos\theta = -2$

$\cos\theta = -1$ (5)

$\underline{\underline{\theta = \pi}}$. (5)

45

6b. $\underline{a} = 3\underline{i} + 4\underline{j}$

$\underline{b} = \lambda\underline{i} + \mu\underline{j} \qquad |b| = 1.$

$\lambda^2 + \mu^2 = 1.$ —— (1).

(5)

$\underline{a} \cdot \underline{b} = 0.$ (5)

$(3\underline{i} + 4\underline{j}) \cdot (\lambda\underline{i} + \mu\underline{j}) = 0.$ (10)

$3\lambda + 4\mu = 0.$ —— (2)

$\lambda = -\frac{4\mu}{3}.$ (5)

from (1), $\frac{16\mu^2}{9} + \mu^2 = 1.$ (5)

(5)

$25\mu^2 = 9.$

(5)

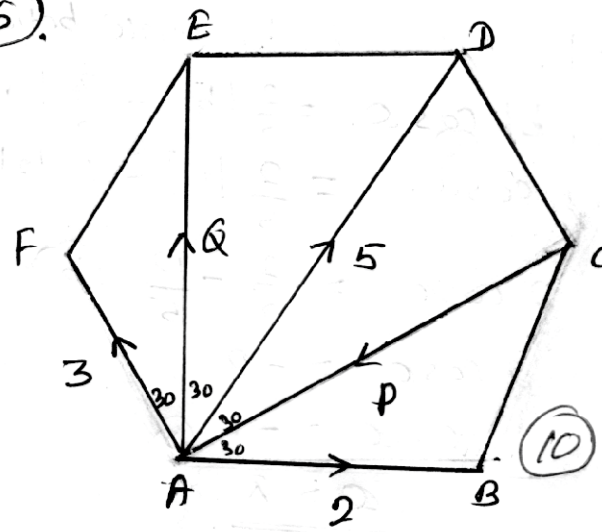
$\underline{\underline{\mu = \frac{3}{5}}} \quad (\mu > 0).$

$\therefore \quad \lambda = -\frac{4}{3} \times \frac{3}{5}$

$\underline{\underline{\lambda = -\frac{4}{5}}}$ (5)

45

(15)



(10)

$$\overrightarrow{X} = 2 - P\cos 30 + 5\cos 60 - 3\cos 60 = 0 \quad (10)$$

$$2 - P \times \frac{\sqrt{3}}{2} + \frac{5}{2} - 3 \times \frac{1}{2} = 0 \quad (5)$$

$$4 - \sqrt{3}P + 5 - 3 = 0$$

$$-\sqrt{3}P + 6 = 0$$

$$P = \frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{3}$$

$$P = 2\sqrt{3}\ N \quad (5)$$

$$\uparrow Y = 3\cos 30 + Q + 5\cos 30 - P\cos 60^\circ = 0 \quad (10)$$

$$3 \times \frac{\sqrt{3}}{2} + Q + 5 \times \frac{\sqrt{3}}{2} - P \times \frac{1}{2} = 0 \quad (5)$$

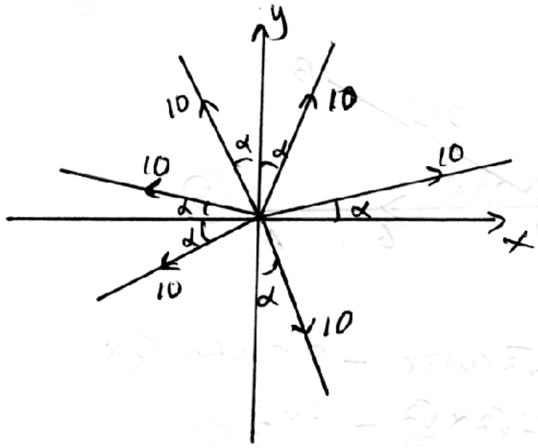
$$3\sqrt{3} + 2Q + 5\sqrt{3} - P = 0.$$

$$3\sqrt{3} + 2Q + 5\sqrt{3} - 2\sqrt{3} = 0$$

$$2Q + 6\sqrt{3} = 0$$

$$Q = -3\sqrt{3}\ N \quad (5)$$

(50)



$\rightarrow X = 10\cos\alpha + 10\sin\alpha - 10\sin\alpha - 10\cos\alpha - 10\cos\alpha + 10\sin\alpha$ (5)

$= 10\sin\alpha - 10\cos\alpha.$

$= 10(\sin\alpha - \cos\alpha)$ (5)

$\uparrow Y = 10\sin\alpha + 10\cos\alpha + 10\cos\alpha + 10\sin\alpha - 10\sin\alpha - 10\cos\alpha$ (5)

$= 10(\sin\alpha + \cos\alpha)$ (5)

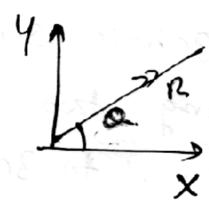
$R^2 = X^2 + Y^2$

$= 10^2(\sin\alpha - \cos\alpha)^2 + 10^2(\sin\alpha + \cos\alpha)^2$ (5)

$= 10^2\left[\sin^2\alpha + \cos^2\alpha - 2\sin\alpha\cos\alpha + \sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha\right]$ (5) (5)

$= 10^2[2]$

$R = 10\sqrt{2}$ N. (5)



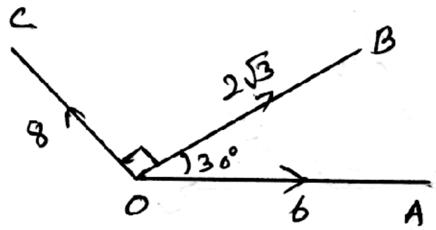
$\tan\theta = \dfrac{Y}{X}$

$= \dfrac{10(\sin\alpha + \cos\alpha)}{10(\sin\alpha - \cos\alpha)}$ (5)

$\tan\theta = \dfrac{\tan\alpha + 1}{\tan\alpha - 1}$ (5)

50

(e).



(5)

$\rightarrow X = 6 + 2\sqrt{3}\cos 30 - 8\cos 60$ (5)

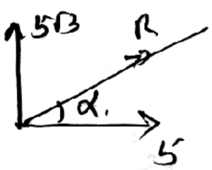
$= 6 + 2\sqrt{3} \times \frac{\sqrt{3}}{2} - 8 \times \frac{1}{2}$ (5)

$= 5$ (5)

$\uparrow Y = 2\sqrt{3}\sin 30 + 8\sin 60$ (5)

$= 2\sqrt{3} \times \frac{1}{2} + 8 \times \frac{\sqrt{3}}{2}$ (5)

$Y = 5\sqrt{3}$ (5)



$R = \sqrt{5^2 + (5\sqrt{3})^2}$

$= 5\sqrt{4}$

$R = 10N$ (5)

$\tan\alpha = \frac{5\sqrt{3}}{5} = \sqrt{3}$ (5)

$\alpha = \pi/3$ (5)

[50]

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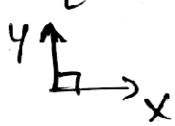
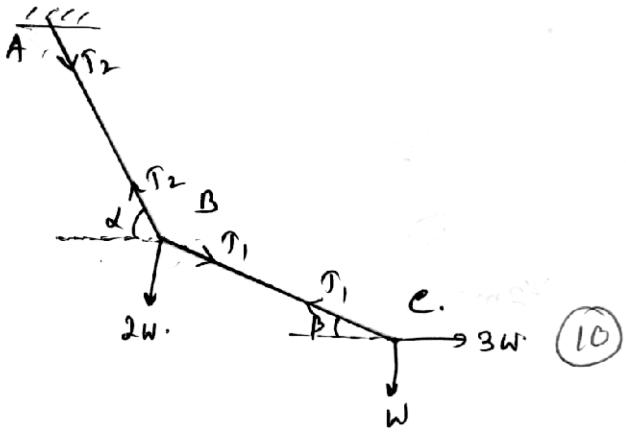
(16).

(a) Algebric sum of resolved components of forces along two perpendicular directions, should seperatly equal to zero.



$X = 0$ and $Y = 0$. [10]



(10)

at B

$\uparrow$ $T_2 \sin\alpha - T_1 \sin\beta - 2W = 0$ — (1) (10)

$\rightarrow$ $T_1 \cos\beta - T_2 \cos\alpha = 0$ — (2) (10)

at C

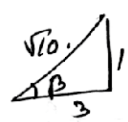
$\uparrow$ $T_1 \sin\beta - W = 0$ — (3) (10)

$\rightarrow$ $3W - T_1 \cos\beta = 0$ — (4) (10)

$\therefore T_1 \cos\beta = 3W$

$T_1 \sin\beta = W$

$\frac{(3)}{(4)} \Rightarrow \tan\beta = \frac{1}{3}$ (5)



$\therefore T_1 \sin\beta = W$ (5)

$T_1 = \frac{W}{\sin\beta} = \frac{W}{\frac{1}{\sqrt{10}}} = \sqrt{10}\,W$ (5)

$T_1 = \sqrt{10}\,W$ (5)

From (1) and (3):

$T_2 \sin\alpha = 2W + T_1 \sin\beta$

$= 2W + \sqrt{10}\,W \times \frac{1}{\sqrt{10}} = 3W$ (5)

$T_2 \sin\alpha = 3W$ — (5) (5)

$T_2 \cos\alpha = T_1 \cos\beta = \sqrt{10}\,W \times \frac{3}{\sqrt{10}}$ (5)

$T_2 \cos\alpha = 3W$ — (6) (5)

$\frac{(5)}{(6)}$ $\tan\alpha = 1$ (5)

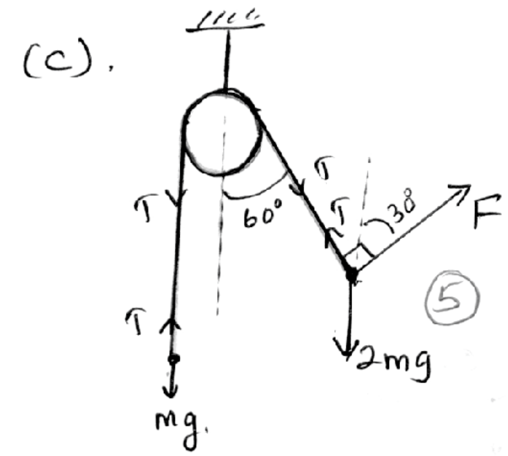
$\alpha = \pi/4$ (5)

From (5): (5)

$T_2 = \frac{3W}{\sin\alpha} = \frac{3W}{\sin\pi/4} = 3\sqrt{2}\,W$ (5)

110

(c).



for (m);

↑ $T - mg = 0$

$T = mg$ (5)

for (2m)

↑ $T\cos 60 - 2mg + F\cos 30 = 0$. (5)

$mg \times \frac{1}{2} - 2mg - F \times \frac{\sqrt{3}}{2} = 0$ (5)

(5)

$-\frac{3mg}{2} - \frac{\sqrt{3}}{2} F = 0$

$\sqrt{3} F = 3mg$

$F = \sqrt{3}\, mg$ (5)

(30)

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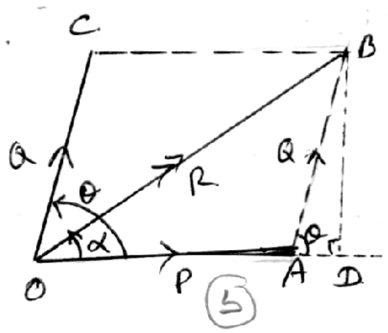
(17)



$OB^2 = (OD)^2 + BD^2$

$R^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2$ (5)

$R^2 = P^2 + Q^2 + 2PQ\cos\theta$. (10)

$\tan\alpha = \frac{BD}{OD} = \frac{Q\sin\theta}{P + Q\cos\theta}$ (5) (5)

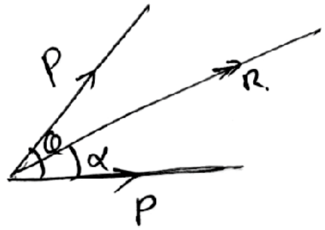
When $P = Q$;

$\tan\alpha = \frac{P\sin\theta}{P + P\cos\theta} = \frac{\sin\theta}{1 + \cos\theta} = \frac{2\sin\theta/2\,\cos\theta/2}{1 + 2\cos^2\frac{\theta}{2} - 1}$ (5) (5)

$= \frac{2\sin\theta/2\,\cos\theta/2}{2\cos^2\theta/2} = \tan\theta/2$ (5)

$\tan\alpha = \tan\theta/2 \implies \alpha = \theta/2$. (5)

(50)



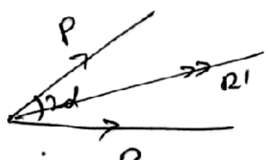
$$R^2 = 2(P)(P)$$
$$R^2 = 2P^2 \quad (10)$$

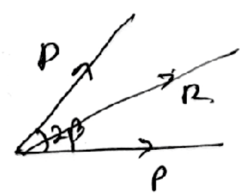
using $R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$$R^2 = P^2 + P^2 + 2P^2\cos\theta \quad (10)$$
$$2P^2 = 2P^2 + 2P^2\cos\theta$$
$$2P^2\cos\theta = 0 \quad (10)$$
$$\therefore \cos\theta = 0 \quad (5)$$
$$\theta = \pi/2 \quad (5)$$

from the above result $\alpha = \frac{\theta}{2} = \frac{\pi}{4} = 45^\circ$ (10) [50]

(e).





Given that $R' = 2R$ (5)

$$(R')^2 = P^2 + P^2 + 2P^2\cos 2\alpha \quad (10)$$
$$(2R)^2 = 2P^2 + 2P^2\cos 2\alpha$$
$$4R^2 = 2P^2[1 + \cos 2\alpha] \quad (5)$$
$$4R^2 = 2P^2[1 + 2\cos^2\alpha - 1] \quad (5)$$
$$4R^2 = 4P^2\cos^2\alpha$$
$$R^2 = P^2\cos^2\alpha \quad \text{— ①}$$

$$R^2 = P^2 + P^2 + 2P^2\cos 2\beta \quad (10)$$
$$R^2 = 2P^2 + 2P^2\cos 2\beta$$
$$= 2P^2(1 + \cos 2\beta) \quad (5)$$
$$= 2P^2[2\cos^2\beta]$$
$$R^2 = 4P^2\cos^2\beta \quad \text{— ②}$$

from ① and ②; $4P^2\cos^2\beta = P^2\cos^2\alpha$ (5)

$$2\cos\beta = \cos\alpha \quad (5)$$

[50]