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Provincial Department of Education - NWP

10 E I

Third Term Test - Grade 13 - 2018

Index No : ....

Combined Mathematics I

Three hours only

Instructions:

* *This question paper consists of two parts.*
  Part A (Question 1 - 10) and **Part B** (Question 11 - 17)
* **Part A**
  *Answer **all** questions. Write your answers to each question in the space provided. you may use additional sheets if more space is needed.*
* **Part B**
  Answer five questions only. Write your answers on the sheets provided.
* *At the end of the time allocated, tie the answers of the two parts together so that* **Part A** *is on top of* **part B** *before handing them over to the supervisor.*
* *You are permitted to remove only* **Part B** *of the question paper from the Examination Hall.*

For Examiner's Use only

(10) Combined Mathematics I		
Part	Question No	Marks Awarded
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
	Total	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	
Paper 1 total		
Percentage		

Paper I	
Paper II	
Total	
Final Marks	

Final Marks

In Numbers	
In Words	

Marking Examiner	
Marks Checked by 1	
2	
Supervised by	

Combined Maths 13 - I (Part A)

01. Using Mathematical Induction prove that $\sum_{r=1}^{n}\frac{1}{(3r-1)(3r+2)}=\frac{n}{6n+4}$ for all $n\in\mathbb{Z}^{+}$.

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02. Find the set of solution satisfying the inequality $|x-2|<4-|x|$.

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03. Express $\dfrac{1}{(2+i)^2} - \dfrac{1}{(2-i)^2}$ in the form of $a+ib$, where a and $b \in \mathbb{R}$. Find modulus and amplitude of it.

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04. If x^p exists in the expansion of $\left(x^2 + \dfrac{1}{x}\right)^{2n}$, Show that its coefficient is $\dfrac{(2n)!}{\left(\dfrac{4n-p}{3}\right)!\left(\dfrac{2n+p}{3}\right)!}$

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05. Show that $\lim_{x \to \frac{\pi}{4}} \dfrac{4\sqrt{2}-(\cos x+\sin x)^5}{1-\sin 2x} = 5\sqrt{2}$.

06. Find the area enclosed by the curve $y = 2x - x^2$ and the straight line $y = -x$.

07. If $x = 2\cos t - \cos 2t,\ y = 2\sin t - \sin 2t$, find $\dfrac{dy}{dx}$ when $t = \dfrac{\pi}{2}$. Hence find the equation of the normal drawn to the curve at the point corresponding to $t = \dfrac{\pi}{2}$.

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08. Points A and B are on the line $y = x$ and $y = x - 2$ respectively. If the equation of the perpendicular bisector of AB is $4x + 3y - 32 = 0$, Find the coordinates of A and B.

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09. Show that the equations $x^2+y^2+2x-4y-8=0$ and $x^2+y^2-6x+8y+12=0$ represent two circles with same area and touching each other, Find the equation of the tangent drawn at the touching point of two circles.

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10. Solve the equation $3\cos^2\theta-2\sqrt{3}\cos\theta\sin\theta-3\sin^2\theta=0$

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Combined Mathematics 13 - I (Part B)

❖ **Answer Five questions Only.**

11. *a.* The roots of the equation $x^2+px+q=0$ are α and β and the roots of the equation $x^2+ax+b=0$ are $\dfrac{1}{\alpha}$ and γ. Prove that $(p-aq)(a-pb)=(1-bq)^2$.

Further, show that equation whose roots are β and γ is $x^2(1-bq)-x[(a+p)bq-(aq+bp)]+bq(1-bq)=0.$

b. Let $g(x)=x^2+x-k$, where $k\in\mathbb{R}$. If roots of the equation $g(x)=0$ are α and α^2, show that $k=-1$.

Let $f(x)=g(x)+a-10$, $a\in\mathbb{R}$. If roots of the equation $f(x)=0$ are real, find the range of values of a such that,

i. having at least one positive root.

ii. do not having at least one positive root.

c. When $f(x)\equiv x^4-(k+1)x^3+x^2+3x-5$, is divided by $(x-1)$ remainder is -3. Find the value of k.

For this value of k , when $f(x)$ is divided by x^2-3x+2 remainder is $px+q$, find the value of p and q. Hence solve the equation $f(x)=-3$ completely.

12. *a.* Find how many permutations can be prepared using the letter of the word "INDEPENDENCE" Out of these permutations.

i. How many words starts from the letter P.

ii. How many words contains all vowels together.

iii. How many words never keep vowels together.

iv. How many words starts from the letter I and ends with the letter P.

b. If $x_1, x_2, ..\ x_n$ are terms of an arithmetic series for all $x_i>0$, show that,

i. $$\frac{1}{\sqrt{x_1}+\sqrt{x_2}}+\frac{1}{\sqrt{x_2}+\sqrt{x_3}}+\dots\ \frac{1}{\sqrt{x_{n-1}}+\sqrt{x_n}}=\frac{n-1}{\sqrt{x_1}+\sqrt{x_n}}$$

ii. $$\frac{1}{x_1x_2}+\frac{1}{x_2x_3}+\frac{1}{x_3x_4}\dots\ \frac{1}{x_{n-1}x_n}=\frac{n-1}{x_1x_n}$$

c. Write down r^{th} term U_r of the series $\dfrac{3}{1^2\cdot 2^2}+\dfrac{5}{2^2\cdot 3^2}+\dfrac{7}{3^2\cdot 4^2}+\dots$

Using the identity $(r+1)^2-r^2\equiv 2r+1$, find $f(r)$ such that $f(r)-f(r+1)\equiv U_r$.

Hence, evaluate $\sum\limits_{r=1}^{n} Ur$.

13. *a.* Let $A\equiv\begin{pmatrix}3 & 1\\ 7 & 5\end{pmatrix}$. Find x and y such that $A^2+xI=yA$. Hence, find A' .

b. i. Find the amplitude of the complex number z given by $z=\dfrac{(1+\sqrt{3}i)^2}{4i(1-\sqrt{3}i)}$.

ii. z_1, z_2 and z_3 are complex numbers corresponding to the points A, B and C respectively.

If $\dfrac{z_1 - z_3}{z_2 - z_3} = \dfrac{1-\sqrt{3}i}{2}$, show that ABC is an equilateral triangle. i.

iii. Describe the locus of the complex number at each of the following stages.

$|z+3-2i| = \sqrt{14}$

$\arg(z+3) = \dfrac{\pi}{2}$

Sketch the locus of points of both of the above in the same diagram and hence find the complex number corresponding to the common point, of the two locus.

14. *i.* It is given that $y = \dfrac{3(x+3)}{(x-1)(x+2)}$ Show that $\dfrac{dy}{dx} = -\dfrac{3(x+5)(x+1)}{(x-2)^2(x-1)^2}$.

Indicating the asymptotes, turning points and intercepts on x and y axis, sketch the graph of the function $y = \dfrac{3(x+1)}{(x-1)(x+2)}$.

ii. Circle $x^2 + y^2 = 1$ intersects the x axis at P and Q. Another circle with a variable radius and center at Q intersects the previous circle at R, above the x axis and the line segment PQ at S. Find the maximum area that can be taken by the triangle SQR.

15. *a.* Using integration by parts, evaluate $\int x^2 \cos 2x \, dx$.

b. Express $\dfrac{1}{(x+1)(x+2)}$ in partial fractions. Hence evaluate, $\int \dfrac{1}{(x+1)(x+2)} dx$.

Using the above result and the substitution $x = \cos\theta$, evaluate $\int \dfrac{\sin\theta\cos\theta}{\cos^2\theta + 3\cos\theta + 2} d\theta$.

c. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$.

Hence, show that $I = J = \dfrac{\pi}{4}$, where $I = \displaystyle\int_0^{\frac{\pi}{2}} \dfrac{(\sin x)^{2018}}{(\cos x)^{2018} + (\sin x)^{2018}} dx$ and $J = \displaystyle\int_0^{\frac{\pi}{2}} \dfrac{(\cos x)^{2018}}{(\cos x)^{2018} + (\sin x)^{2018}} dx$.

Find the value of $\displaystyle\int_0^{\frac{\pi}{2}} \dfrac{7(\sin x)^{2018} - 3(\cos x)^{2018}}{(\cos x)^{2018} + (\sin x)^{2018}} dx$.

16. (a) Two straight lines OA and OB drawn through the origin O, intersect the line $ax+by+c=0$ at A and B, such that $O\hat{A}B = O\hat{B}A = 60^{\circ}$. Find the equations of the straight lines OA and OB.

Further show that area of the triangle OAB is given by $\dfrac{c^2}{\sqrt{3}\left(a^2+b^2\right)}$.

(b) Find the equations of the circle passing through points A, B, C where, $A \equiv (0, 3)$, $B \equiv \left(\sqrt{3}, 0\right)$, $C \equiv \left(-\sqrt{3}, 0\right)$

Straight line $y = mx - 3$ where m is the variable gradient intersects the above circle at L and M . Find the equation of the locus of the mid point of LM .

17. *a*. State and prove the Sine rule for any triangle ABC with usual notation.

Using the same notation for a triangle ABC, prove that $\dfrac{\cos^2\left(\dfrac{B-C}{2}\right)}{(b+c)^2} + \dfrac{\sin^2\left(\dfrac{B-C}{2}\right)}{(b-c)^2} = \dfrac{1}{a^2}$.

b. Let $f(x) = \sin x \cos\left(x + \dfrac{\pi}{4}\right)$.

Express $f(x)$ in the from of $a\cos(bx-\alpha)+c$, where a, b, c and $\alpha \left(0 < \alpha < \dfrac{\pi}{2}\right)$ are constants to be determined.

Let $g(x) = 4f(x) + \sqrt{2}$. Sketch the graph of $y = g(x)$ for $\dfrac{\pi}{8} \le x \le \dfrac{9\pi}{8}$.

c. A ladder is kept inside a well in contact with the wall of the well, by making an angle α with the horizontal. When the foot of the ladder moves a horizontal distance a away from the wall and the top of the ladder moves distance b vertically downwards, the inclination of the ladder to the horizontal is β. Prove that $a = b\tan\left(\dfrac{\alpha+\beta}{2}\right)$.

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Provincial Department of Education - NWP

10 | E | II

Third Term Test - Grade 13 - 2018

Index No : ....

Combined Mathematics II

Three hours only

Instructions:

* *This question paper consists of two parts.*
  Part A (Question 1 - 10) and **Part B** (Question 11 - 17)
* **Part A**
  Answer all questions. Write your answers to each question in the space provided. you may use additional sheets if more space is needed.
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For Examiner's Use only

(10) Combined Mathematics II		
Part	Question No	Marks Awarded
A	1	
	2	
	3	
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	7	
	8	
	9	
	10	
	Total	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	
Paper 1 total		
Percentage		

Paper I	
Paper II	
Total	
Final Marks	

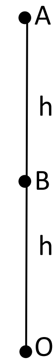
Final Marks

In Numbers	
In Words	

Marking Examiner	
Marks Checked by 1	
2	
Supervised by	

(Part B)

01) A particle P of mass m is projected vertically upwards form the bottom of a post OA of height $2h$, with a velocity of $2\sqrt{5gh}$. At the same time another particle Q of mass m is released from A. Two particles P and Q are collide and coalesce at B, where the middle of the post OA. Show that this combined object becomes instantaneous rest after reaching the highest point of the tower A.



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02) A frog of mass m is on the wooden plank of mass M. The plank is placed on a smooth floor and frog jumps forward horizontally with a velocity v relative to the plank. Applying $I = \Delta\, mV$ for the frog and the plank, find the velocity of the plank and the impluse on the plank by the frog

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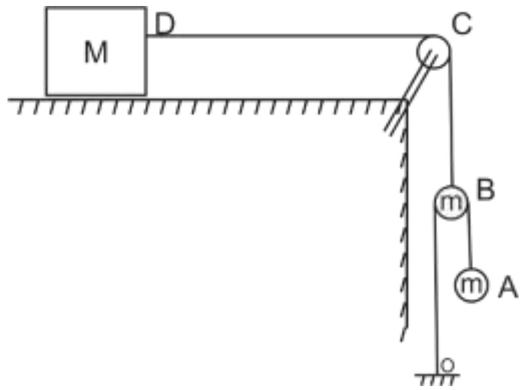
03) One end of light inextensible string attached to an object of mass M and the string passes over a smooth pulley C. Other end is attached to a smooth movable pulley B as shown in the diagram. Another light intextesible sring attracted to a fixed point O passes over the pulley B by hanging an object A of mass m at the other end. If the system is realesed write down the equations of motion needed to find the acceleration of object A.



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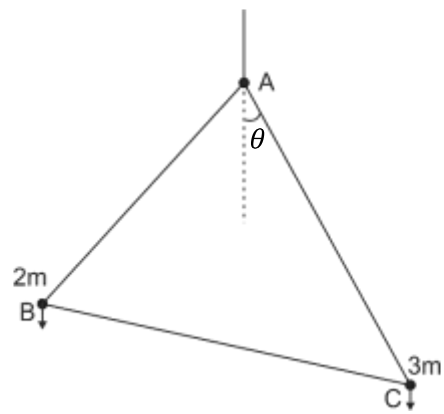
04) ABC is a light equilateral triangular lamina. Two masses $2m$ and $3m$ are attached to the vertices B and C respectively. The lamina is hung freely by A. If the system is in equilibrium in a vertical plane, show that the angle made by the side AC with the downward vertical is,

$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{4}\right)$



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05) The power of the engine of a vehicle of mass $M\ kg$ is $H\ Kw$. Vehicle moves up along the line of greatest slope of a hill inclined at angle θ to the horizontal with a uniform acceleration $a\ ms^{-2}$ against a ressistance of $2R\ N$. Show that $1000\,H = (2R + Ma + Mg\sin\theta)V$, when the velocity of the vehicle is $V\ ms^{-1}$. Here g is the gravitational acceleration.

If, $H = 7$, $R = 400$, $g = 9.8$, $M = 1200$ and $\theta = \sin^{-1}(\frac{1}{19.6})$ find the maximum velocity of the vehicle.

06) Given that $\underline{p} = 3\underline{i} + 5\underline{j}$ and $\underline{q} = \alpha\,\underline{i} + \beta\,\underline{j}$. Here α and β are two real constants and $\alpha < 0$. $\underline{i}$ and $\underline{j}$ are the unit vectors along the axes OX and OY respectively. If $\underline{p}$ and $\underline{q}$ are perpendicular to each other and $\left|\underline{q}\right| = 1$,find the values of α and β .

07) Let A and B are two independent events of a sample space Ω. Given that $P\,(A \cup B) = \frac{11}{12}$, $P\,(B) = \frac{2}{3}$ with the usual notation. Find $P\,(A)$ and $P\,(A \mid B^{|})$.

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08) Bag X contains 5 white and 3 black balls and bag Y contains 2 white and 4 black balls in same size. A ball is taken out at random from bag X and put it in to bag Y. Again a ball is taken out from bag Y.

(i) Find the probability that the second ball is being black.

(ii) If it is given that the second ball is black, find the probability that the ball taken from bag X is being white.

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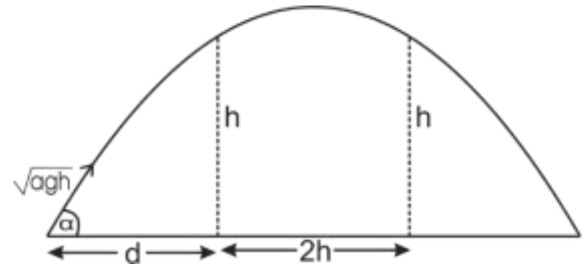
09) $3, 4, 5, a, b, c, d$ and 8 are eight positive integral observations arranged in ascending order. Here $a, b, c, d > 5$. The median and the mean are 6 and 8 respectively. Find the values of a, b, c and d.

10) Standard deviation of n observations is $S = 2\sqrt{3}$. Further given that $\sum_{i=1}^{n} x_i = 8$ and $\sum_{i=1}^{n} x_i^2 = 104$. Find the number of observations in the distribution.

11) (a) A particle is projected from O with a velocity of $\sqrt{agh}$ inclined at angle α to the horizontal, at $t = 0$. It takes time t_1 and t_2 to pass over two vertical walls of height h at a distance $2h$ apart.



Draw velocity time curves for the horizontal and vertical velocity components of the particle separately. Using the curve of horizontal component show that,

$t_2 - t_1 = \frac{2h}{\sqrt{agh}\ \cos\alpha}$.

Using the curve of vertical component, show that $t_2 + t_1 = 2\sqrt{\frac{ah}{g}}\ \text{Sin}\,\alpha$.

Hence, show that the distance from the point of projection to the nearest wall is $d = h\,[\,a\sin\alpha\ \ Cos\,\alpha - 1\,]$.

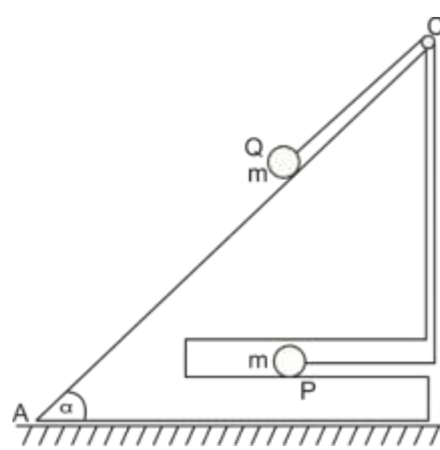
(b) A boat A sailing due North with a velocity of $3u\ ms^{-1}$ observes that another boat B at a distance a meters in West sailing in the direction 30^0 south of east at a velocity of $2u\ ms^{-1}$.

By considering the motion of B relative to the earth, draw a velocity triangle and show that the velocity of B relative to the earth is, $\sqrt{7}\,u\ ms^{-1}$.

If the direction of B is an angle θ East of North, show that $\theta = \sin^{-1}\sqrt{\frac{3}{7}}$.

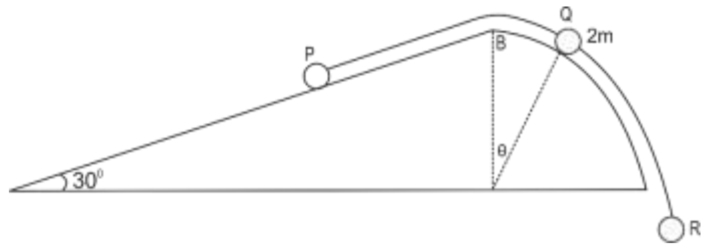
Show that boat A should sail to the direction $\sin^{-1}\frac{\sqrt{7}}{3}$ west of north of to meet the boat B such that distance from A to B is minimum.

12) (a) ABC is a smooth triangular light wedge with $A\hat{B}C = 90^0$. Side AB is horizontal and it is at rest on a smooth horizontal table. Here $B\hat{A}C = \alpha$. There is a thin groove parallel to AB. Particles P and Q each of mass m placed on the groove and the smooth inclined face CA respectively are connected by a light inextensible string which passes over a smooth pulley fixed at C. The system is released gently from rest. Assuming that particle Q moves along $\overrightarrow{CA}$, write down equations of motion for P in the direction $\overrightarrow{AB}$, for Q in the direction $\overrightarrow{CA}$ and for the system in horizontal direction. Hence, show that the acceleration of the wedge as long as particle P is in the groove is $\frac{g\sin\alpha\,(1-\cos\,\alpha)}{[\,4-(1-\cos\alpha)^2]}$.



Also find the acceleration of particle Q.

(b) Two particles P and R, each of mass m are attached to the two ends of light inextensible string of length $2l$ ($\pi a < 2l$). A particle Q of mass $2m$ is attached to the midpoint of the string. A circular part of radius a is attached to the upper part of the plane inclined at 30^0 to the horizontal. Particles P, Q and R are placed in the same vertical plane such that Q is at B string is taut. R can be moved freely vertically downwards. A small displacement is given to Q towards R. Show that angular velocity of Q is given by $4a\theta^2 = g\,[\theta + 4\,(1 - \cos\theta)]$ where θ is the angle made by Q with the upward vertical. Also find the reaction on particle $2m$ is, $\frac{mg}{2}\,[4\,(2\cos\theta - 1) - \theta]$.



13) One end of a light elastic string of natural length a and moducles of elasticity mg is attached to a fixed point A on a rough horizontal floor and the other end B is attached to a particle of mass m. Mass m is placed at B such that $AB = a$, and the particle is projected with a velocity of $2\sqrt{ag}$ in the direction $\overrightarrow{AB}$ along the horizontal floor. Show that the extension x of the string satisfies the equation. $\ddot{x} + g\left(x + \frac{g}{\sqrt{2}}\right) = 0$ for $0 < x < \sqrt{2}\,a$, when the coefficient of friction between the rough floor and the mass m is $\frac{1}{\sqrt{2}}$. Further show that the motion of the particle is simple harmonic. Find the center and the altitude of the simple harmonic motion.
After reaching the maximum extension, mass m returns back in the direction $\overrightarrow{BA}$, when y is the extension of the string in the direction $\overrightarrow{BA}$ using the law of conservation of energy show that,

$$\dot{y}^2 + \frac{gy^2}{a} - \sqrt{2}g\,y = 0 \text{ for } 0 < y < \sqrt{2}\,a.$$

Hence show that particle m is definitely comes to rest at B in its motion along $\overrightarrow{BA}$.

Show that the total time taken by the particle to move from B to D and D to B is $\sqrt{\frac{a}{g}}\left[\pi + \cos^{-1}\left(\frac{1}{3}\right)\right]$, where D is the point reached by the particle m at the maximum extension.

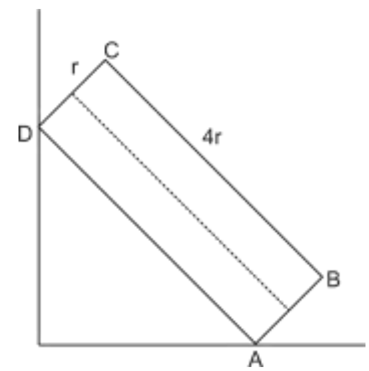
14) (a) O, A and B are three non collinear distinct points and the position vectors of A and B relative to O are $\underline{a}$ and $\underline{b}$ respectively. The position vector of C with respect to origin O is given by $\underline{c} = \frac{1}{3}\,\underline{a} + \frac{2}{3}\,\underline{b}$. Show that A, B and C are collinear and also show that $AC : CB = 2:1$. D is another point on OC, with a position vector with respect to O, $\underline{d} = \lambda\left(\underline{a} + 2\,\underline{b}\right)$. Line drawn parallel to $\overrightarrow{OA}$ through B and OC extended meet at D. Find the value of λ and the ratio OC:CD.
If OA and OD are perpendicular to each other and $|\underline{a}| = |\underline{b}|$, show that the angle between $\underline{a}$ and $\underline{b}$ is $\frac{2\pi}{3}$.

(b) $ABCDEF$ is a regular hexagon of side $2m$. Forces of magnitude $8, 2, 4, 4, 6$ and 8 act along the sides AB, CB, DC, DE, FE and AF respectively. Find the magnitude and the direction of the resultant. Also find the point where the line of action of the resultant cuts the line AB.

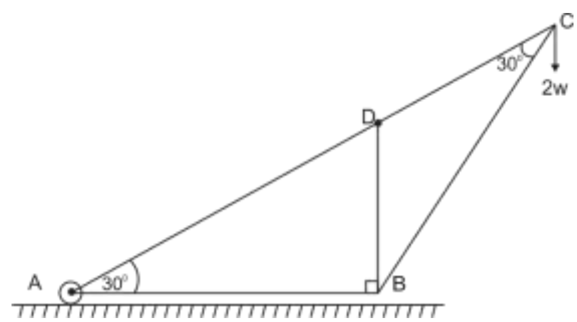
i. When a force of P passing through A is introduced to the system is reduced to a couple. Find the magnitude and the direction of P. Also find the magnitude and the sense of the couple created by the system.

ii. Find the magnitudes and the direction of force that should be added to the system in order to reduce the initial system to a couple of magnitude $18\sqrt{3}\,a$ in the direction CBA. Find the point where it cuts line AB.

15) (a) $ABCD$ is a cylindrical pipe of radius r and height $4r$. The pipe is in equilibrium in a vertical plane through its center of gravity with the point A touching a rough horizontal floor and point D touching a smooth vertical wall. The inclination of the pipe to the horizontal is θ, where $\theta = \tan^{-1}\left(\frac{4}{3}\right)$.
If the coefficient of friction between A and the floor is μ, show that $\frac{1}{8} \leq \mu$.

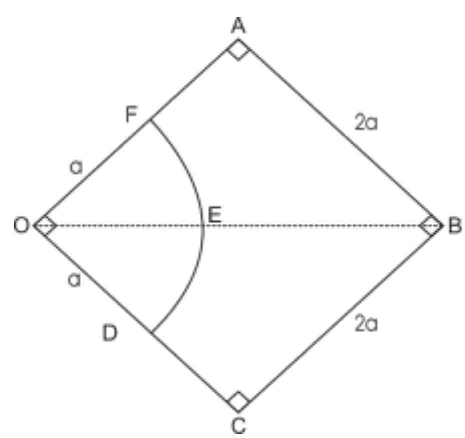


(b) A framework shown in the figure consists of five light rods AB, BC, CD, DA and DB. Here $AB = BC = a$ meters and $B\hat{A}D = B\hat{C}D = 30^0$. It is hinged freely to fixed point A and a weight $2w$ is hung at C and the frame work is in equilibrium in a vertical plane with AB horizontal. Show that the normal reaction at B is $3w$.
Draw stress diagram using Bow's notation and hence find the stresses in all rods. State whether they are tension on trusts.

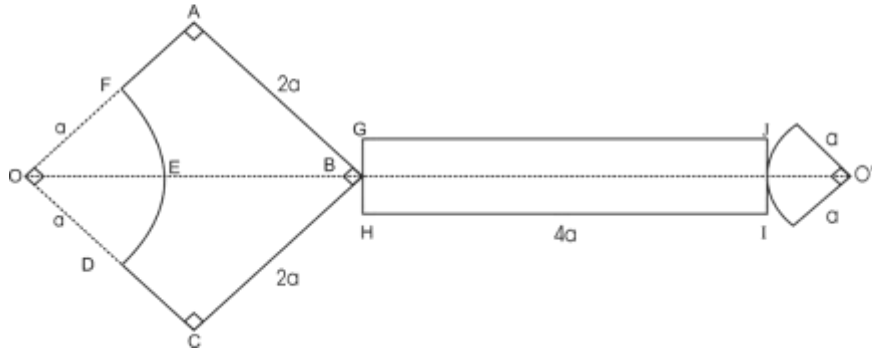


16) Show that the center of mass of a lamina in the shape of a sector of radius a subtending an angle $\frac{\pi}{2}$ at the center is on it's axis of symmetry at distance $\frac{4\sqrt{2}a}{3\pi}$ from the center O.

A sector ODEF of radius a and center O is removed from a square shaped uniform lamina of side $2a$ as shown in the figure. Show that the center of mass of the remaining part is at a distance of $\frac{44\sqrt{2}a}{3(16-\pi)}$ from O on the axis of symmetry.



Then the sector ODEF and the remaining part of square shaped lamina are connected to a rectangular lamina GHIJ of length $4a$ and width $\frac{a}{4}$ which is made by same material as shown in the figure.



Show that the center of mass of the composite body lies on the axis of symmetry at a distance $\frac{a}{60}\left[64\sqrt{2}\ + 24 + 3\left(5 + 2\sqrt{2}\right)\pi\right]$ from O.

The composite body is hung freely in a vertical plane by a light inextensible string, fixed to a ceiling and the other end to a point A on the body. Show that 00^1 line makes an angle $\tan^{-1}\dfrac{60\sqrt{2}}{4\sqrt{2} +\ 24 +\ 3(5{+}2\sqrt{2}\,)\pi}$ with the downward vertical.

17) (a) The box A contains 2 blue balls and 3 red balls of equal in size. The box B contains 1 blue ball and 2 red balls of the same type. A unbiased dice numbered from 1 to 6 is tossed once. If the result of tossing is 5 or above 5, 2 ball are taken out randomly from the box A, if the result is below 5, 2 balls are taken out randomly from the box B, one at a time without replacement.

i. Find the probability that the first ball is being red.
ii. Find the probability that two ball are being red.
iii. If it is given that two balls taken out are red, find the probability that they are taken out from the box A.

(b) Marks obtained for five subjects by a student A given below.
76 , 78, 67, 87, 92
Find the mean and the standard deviation of above 5 marks. (Take $\sqrt{76.4}\ \approx 8.8$)
Statistics of marks obtained for above 5 subjects by a student B is given below.

$$\sum_{i=1}^{5} x_i = 375, \quad \sum_{i=1}^{5} x_i^{\,2} = 28445$$

Find the mean $(\overline{x}_B)$ and the standard deviation of the marks obtained by B.
Find the coefficient of variance of marks obtained by A and B and compare the variance.

(Coefficient of variance $= \dfrac{\text{Standard deviation}}{mean}$ x 100)

01. $$\sum_{r=1}^{n} \frac{1}{(3r-1)(3r+2)} = \frac{n}{6n+4}$$

For $n=1$ $$LHS = \sum_{r=1}^{1} \frac{1}{(3r-1)(3r+2)} = \frac{1}{(3\times1-1)(3\times1+2)}$$

$$= \frac{1}{2\times5} = \frac{1}{10}$$

$$RHS = \frac{1}{6\times1+4} = \frac{1}{10}$$

$\therefore$ LHS = RHS.

$\therefore$ The result is true for $n=1$. (5)

Take any $p \in \mathbb{Z}^+$ and assume that the result is true for $n=p$.

$$\sum_{r=1}^{p} \frac{1}{(3r-1)(3r+2)} = \frac{p}{6p+4} \sim (A). \quad (5)$$

Now $$\sum_{r=1}^{p+1} \frac{1}{(3r-1)(3r+2)} = \sum_{r=1}^{p} \frac{1}{(3r-1)(3r+2)} + \frac{1}{(3p+2)(3p+5)}$$

(A) (5)

$$= \frac{p}{6p+4} + \frac{1}{(3p+2)(3p+5)}$$

$$= \frac{1}{2(3p+2)} \left\{ p + \frac{2}{3p+5} \right\}$$

$$= \frac{1}{2(3p+2)} \left\{ \frac{3p^2+5p+2}{3p+5} \right\}$$

$$= \frac{(3p+2)(p+1)}{2(3p+2)(3p+5)}$$

$$= \frac{p+1}{6p+10}. = \frac{\overline{p+1}}{6(\overline{p+1})+4}.$$ (5)

Hence if the result is true for $n=p$, then it is also true for $n=p+1$, We have already proved that the result is true for $n=1$.

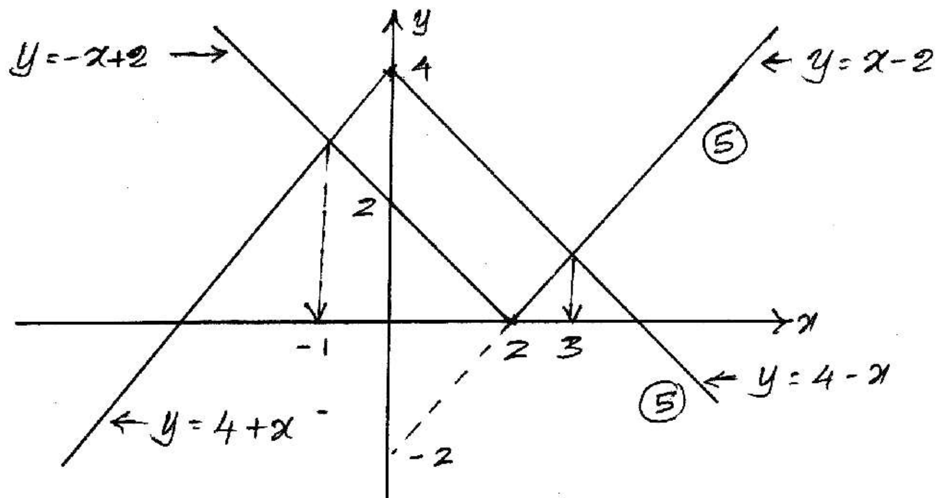
(5)

Hence by the Principle of Mathematical Induction, the result is true for all $n \in \mathbb{Z}^+$

02)

$$|x-2| = \begin{cases} x-2 \; ; & x \geq 2 \\ -x+2 \; ; & x < 2 \end{cases}$$

$$4-|x| = \begin{cases} 4-x \; ; & x \geq 0 \\ 4+x \; ; & x < 0 \end{cases}$$



$$-x+2 = 4+x \qquad\qquad x-2 = 4-x$$

$$-2 = 2x \qquad\qquad 2x = 6$$

$$\underline{\underline{x = -1}} \quad 5 \qquad\qquad \underline{\underline{x = 3}} \quad 5$$

$$\underline{\underline{-1 < x < 3}} \quad 5$$

03)

$$\frac{1}{(2+i)^2} - \frac{1}{(2-i)^2} = \frac{(2-i)^2 - (2+i)^2}{\{(2+i)(2-i)\}^2}$$

$$= \frac{4-4i+i^2-4-4i-i^2}{(4-i^2)^2}$$

$$= \frac{-8i}{25} = \underline{\underline{0 - \frac{8}{25} i}} \quad 15$$

$$\text{modulus} = \underline{\underline{\frac{8}{25}}} \quad 5 \qquad \text{Argument} = \underline{\underline{\frac{3\pi}{2}}} \quad 5$$

04) $\left(x^2+\frac{1}{x}\right)^{2n}$.

$$T_{r+1} = {}^{2n}C_r (x^2)^{2n-r}\left(\frac{1}{x}\right)^r \quad (5)$$

$$= {}^{2n}C_r\, x^{4n-3r}$$

$$4n-3r = p \Rightarrow r = \frac{4n-p}{3} \quad (5)$$

$$\text{Coefficient} = \frac{(2n)!}{\left(2n-\frac{4n-p}{3}\right)!\left(\frac{4n-p}{3}\right)!} \quad (10)$$

$$= \frac{(2n)!}{\left(\frac{2n+p}{3}\right)!\left(\frac{4n-p}{3}\right)!} \quad (5)$$

05) $\lim_{x \to \frac{\pi}{4}} \left\{\frac{4\sqrt{2}-(\cos x+\sin x)^5}{1-\sin 2x}\right\}$

let $x-\frac{\pi}{4} = \theta$

$x = \frac{\pi}{4}+\theta$. (5)

$$\lim_{\theta \to 0} \left\{\frac{4\sqrt{2}-\left(\cos\left(\frac{\pi}{4}+\theta\right)+\sin\left(\frac{\pi}{4}+\theta\right)\right)^5}{1-\sin\left(\frac{\pi}{2}-2\theta\right)}\right\}$$

$$\lim_{\theta \to 0} \left\{\frac{4\sqrt{2}-\left(\frac{1}{\sqrt{2}}\cos\theta-\frac{1}{\sqrt{2}}\sin\theta+\frac{1}{\sqrt{2}}\sin\theta+\frac{1}{\sqrt{2}}\cos\theta\right)^5}{1-\cos 2\theta}\right\}$$

$$\lim_{\theta \to 0} \left\{\frac{4\sqrt{2}-(\sqrt{2}\cos\theta)^5}{\sin^2\theta}\right\} \quad (5)$$

$$\lim_{\theta \to 0} \left\{ \frac{4\sqrt{2} - 4\sqrt{2}\cos^5\theta}{\sin^2\theta} \right\}$$

⑤

$$\lim_{\theta \to 0} \frac{4\sqrt{2}\,(1-\cos\theta)(\cos^4\theta + \cos^3\theta + \cdots + 1)}{\sin^2\theta}$$

$$\lim_{\theta \to 0} \frac{4\sqrt{2}\,\sin^2\left(\frac{\theta}{2}\right)(\cos^4\theta + \cos^3\theta + \cdots + 1)}{\sin^2\theta}$$

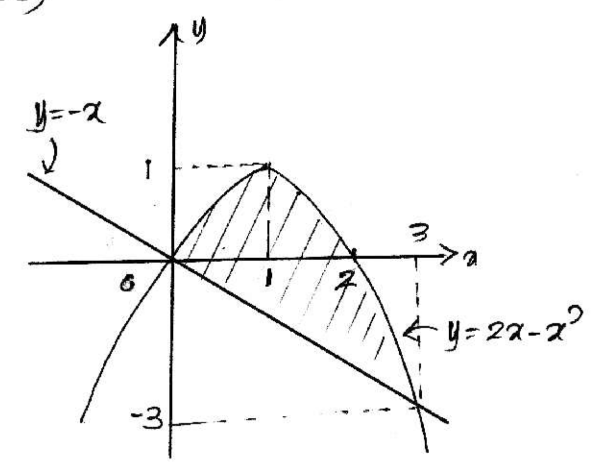
$$\lim_{\theta \to 0} \frac{\frac{4\sqrt{2}}{4} \cdot \left\{ \frac{\sin\frac{\theta}{2}}{\frac{\theta}{2}} \right\}^2 (\cos^4\theta + \cos^3\theta + \cdots)}{\left(\frac{\sin\theta}{\theta}\right)^2}$$

$$\frac{\sqrt{2} \times \left\{ \lim_{\theta \to 0} \left(\frac{\sin\frac{\theta}{2}}{\frac{\theta}{2}} \right) \right\}^2 \times \lim_{\theta \to 0} (\cos^4\theta + \cos^3\theta + \cdots + 1)}{\left(\lim_{\theta \to 0} \frac{\sin\theta}{\theta} \right)^2}$$ ⑤

⑤

$$\frac{\sqrt{2} \times 1^2 \times 5}{1} = \underline{\underline{5\sqrt{2}}}$$

06)



$2x - x^2 = -x$

$x^2 - 2x - x = 0$

$x = 0$ or $x = 3$

⑤

5 5

$$\text{Area} = \int_0^2 2x - x^2\,dx + \frac{1}{2}\times 3\times 3 + \int_2^3 2x - x^2\,dx.$$

$$= \left\{\frac{2x^2}{2} - \frac{x^3}{3}\right\}_0^2 + \frac{9}{2} + \left\{\frac{2x^2}{2} - \frac{x^3}{3}\right\}_2^3 \quad 5$$

$$= \left(\frac{8}{2} - \frac{8}{3}\right) + \frac{9}{2} + \left\{(9-9) - \left(4 - \frac{8}{3}\right)\right\} \quad 5$$

$$= 8\times\frac{1}{6} + \frac{9}{2} + \left\{-\left(\frac{12-8}{3}\right)\right\}$$

$$= \frac{4}{3} + \frac{9}{2} - \frac{4}{3}$$

$$= \underline{\underline{\frac{9}{2}}}$$

07) $x = 2\cos t - \cos 2t$

$$\frac{dx}{dt} = -2\sin t + 2\sin 2t$$

$$= \underline{\underline{2\sin t\,(2\cos t - 1)}}$$

$$y = 2\sin t - \sin 2t$$

$$\frac{dy}{dt} = 2\cos t - 2\cos 2t$$

$$= 2\cos t - 4\cos^2 t + 2. \quad 5$$

$$= \cancel{2 - 2co}$$

$$\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx} \Rightarrow \frac{dy}{dx} = \underline{\underline{\frac{2\sin t\,(2\cos t - 1)}{2\cos t - 2\cos 2t}}}$$

$$\left(\frac{dy}{dx}\right)_{t=\pi/2} = \frac{2\cos\frac{\pi}{2} - 2\cos\pi}{2\sin\frac{\pi}{2}\left(2\cos\frac{\pi}{2} - 1\right)} = \frac{2}{2(-1)} = \underline{\underline{-1}} \quad 5$$

let m_2 is the gradient of the normal

$$(-1)\, m_2 = -1$$

$$m_2 = 1 \quad (5)$$

when $x = \pi/2$; $x = 2\cos \pi/2 - \cos\pi = 1$

$$y = 2\sin \pi/2 - \sin\pi = 2$$

$$\Rightarrow (1,2) \quad (5)$$

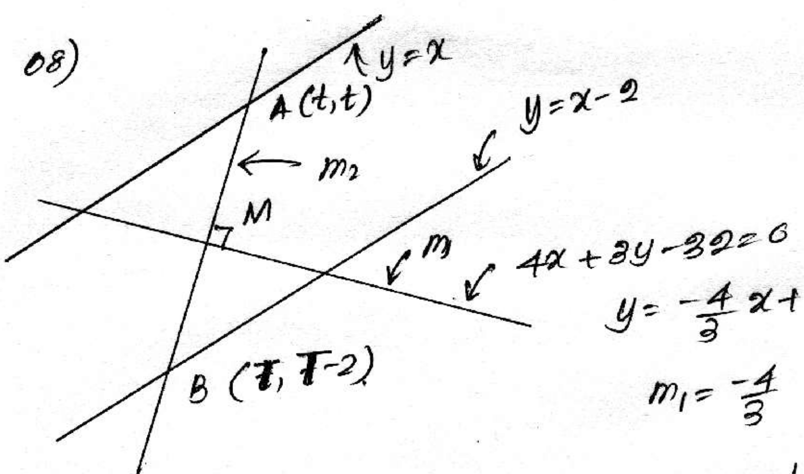
$\therefore$ The eq^n of the normal

$$\frac{y-2}{x-1} = 1 \quad (5)$$

$$y - x - 2 + 1 = 0$$

$$y - x - 1 = 0$$

08)



$$y = -\frac{4}{3}x + \frac{32}{3}$$

$$m_1 = -\frac{4}{3}$$

$$m_1 m_2 = -1$$

$$m_2 = \frac{3}{4} \quad - (1)$$

But $m_2 = \dfrac{t - T + 2}{t - T} \quad - (2)$

$(1) = (2) \quad \dfrac{t - T + 2}{t - T} = \dfrac{3}{4}$

$$4t - 4T + 8 = 3t - 3T$$

$$t - T + 8 = 0.$$

$$T - t = 8 \quad \text{— (A)} \quad (5)$$

$$M \equiv \left\{ \frac{t+T}{2}, \frac{t+T-2}{2} \right\}$$

$$4\left(\frac{t+T}{2}\right) + 3\left(\frac{t+T-2}{2}\right) - 32 = 0$$

$$4t + 4T + 3t + 3T - 6 - 64 = 0$$

$$7t + 7T - 70 = 0$$

$$t + T = 10 \quad \text{— (B)} \quad (5).$$

(A) + (B) $\quad 2T = 18 \qquad$ (B) $\quad \underline{t = 1} \qquad (5)$

$$\underline{T = 9}$$

$\therefore\ \underline{A \equiv (1, 1)}$ (5) $\quad \underline{B(9, 7)}$ (5).

09) let $S_1 \equiv x^2 + y^2 + 2x - 4y - 8 = 0$

Center $\equiv (-1, 2)$

$$r_1 = \sqrt{(-1)^2 + 2^2 + 8}$$

$$= \sqrt{1 + 4 + 8}$$

$$= \sqrt{13} \qquad (5)$$

and $S_2 \equiv x^2 + y^2 - 6x + 8y + 12 = 0$

Center $\equiv (3, -4)$

$$r_2 = \sqrt{3^2 + (-4)^2 - 12}$$

$$= \sqrt{9 + 16 - 12} = \sqrt{13}. \quad (5)$$

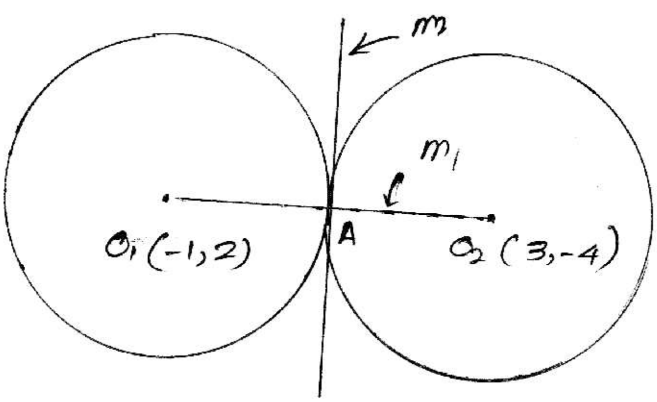
$\therefore \quad r_1 = r_2$

$\therefore$ Area of the Circle S_1 = Area of the Circle S_2. 5

$$O_1O_2 = \sqrt{(3+1)^2 + (-4-2)^2}$$
$$= \sqrt{4^2 + 6^2}$$
$$= \sqrt{16+36}$$
$$= 2\sqrt{4+9}$$
$$= 2\sqrt{13}$$

$\therefore \quad O_1O_2 = r_1 + r_2$

$\therefore$ the circles ~~are~~ touch each other. 5



$$m_1 = \frac{2+4}{-1-3} = \frac{6}{-4} = -\frac{3}{2}$$

$m_1 m_2 = -1$

$m_2 = \frac{2}{3}$

$$A \equiv \left(\frac{3-1}{2}, \frac{2-4}{2}\right)$$
$$\equiv (1,-1)$$

$\therefore$ The eq^n $\frac{y+1}{x-1} = \frac{2}{3} \Rightarrow 3y+3 = 2x-2$ 5

$3y - 2x + 5 = 0$

10) $3\cos^2\theta - 2\sqrt{3}\cos\theta\sin\theta - 3\sin^2\theta = 0$

$3(\cos^2\theta - \sin^2\theta) - \sqrt{3}\cdot 2\sin\theta\cos\theta = 0$

5 5

$3\cos 2\theta - \sqrt{3}\sin 2\theta = 0$

$\sqrt{3}\cos 2\theta - \sin\theta = 0$

$\frac{\sqrt{3}}{2}\cos 2\theta - \frac{1}{2}\sin\theta = 0$

5

$\cos 2\theta \cos\frac{\pi}{6} - \sin\theta\sin\frac{\pi}{6} = 0$

$\cos(2\theta + \pi/6) = \cos\pi/2$ 5

$2\theta + \frac{\pi}{6} = 2n\pi \pm \frac{\pi}{2}$

5

$\theta = n\pi \pm \frac{\pi}{4} - \frac{\pi}{12}$; $n \in \mathbb{Z}$.

11. a. $x^2+px+q=0 \begin{matrix}\nearrow \alpha \\ \searrow \beta\end{matrix}$ $\qquad \alpha+\beta=-p$ —— ① $\quad$ ⑤

$\alpha\beta = q$ —— ②

$x^2+ax+b=0 \begin{matrix}\nearrow 1/\alpha \\ \searrow \gamma\end{matrix}$ $\qquad \frac{1}{\alpha}+\gamma=-a$ —— ③ $\quad$ ⑤

$\frac{1}{\alpha}\gamma = b$ —— ④

$L.H.S = (p-aq)(a-pb)$

$= \left\{-\alpha-\beta+\alpha\beta\left(\frac{1}{\alpha}+\gamma\right)\right\}\left\{-\frac{1}{\alpha}-\gamma+\frac{\gamma}{\alpha}(\alpha+\beta)\right\}$ ⑩

$= \{-\alpha-\beta+\beta+\alpha\beta\gamma\}\left\{-\frac{1}{\alpha}-\gamma+\gamma+\frac{\gamma\beta}{\alpha}\right\}$

$= -\alpha\{1-\beta\gamma\}\cdot\left(-\frac{1}{\alpha}\right)\{1-\gamma\beta\}$

$= \{1-\beta\gamma\}^2$ ⑤

$= \{1-bq\}^2$ ⑤

$\therefore$ LHS = RHS.

But.

② × ④ ;

$\alpha\beta\cdot\frac{1}{\alpha}\gamma = qb$

$\beta\gamma = qb$

$$\left.\begin{aligned} &\alpha^2 + p\alpha + q = 0 \rightsquigarrow (A) \\ &\left(\frac{1}{\alpha}\right)^2 + a\left(\frac{1}{\alpha}\right) + b = 0 \\ &b\alpha^2 + a\alpha + 1 = 0 \rightsquigarrow (B) \end{aligned}\right\}$$

$(A) \times b - (B).$

$$bp\alpha + bq - a\alpha - 1 = 0$$

$$\alpha = \left(\frac{1-bq}{bp-a}\right)$$

$(2) \Rightarrow \alpha\beta = q$

$$\beta = \frac{q(bp-a)}{(1-bq)} \quad (5)$$

$(4) \Rightarrow \frac{\gamma}{\alpha} = b$

$$\gamma = \frac{b(1-bq)}{(bp-a)} \quad (5)$$

$$\gamma + \beta = \frac{q(bp-a)}{(1-bq)} + \frac{b(1-bq)}{(bp-a)}$$

$$= \frac{q(bp-a)(bp-a) + b(1-bq)(1-bq)}{(1-bq)(bp-a)}$$

$$= \frac{-q(a-bp)(bp-a) + b(p-aq)(a-pb)}{-(1-bq)(a-bp)}$$

$$= \frac{(a-pb)\{q(a-bp) + b(p-aq)\}}{-(1-bq)(a-pb)}$$

$$= \frac{aq - bpq + bp - abq}{-(1-bq)}$$

(15)

$$= \frac{(aq+bp) - b(pq+aq)}{-(1-bq)} = \underline{\underline{\frac{bq(p+a) - (aq+bp)}{(1-bq)}}}$$

$\alpha\beta = bq.$

$\therefore$ The eq^{n} is.

$$x^2 - \left\{\frac{bq(a+p) - (aq+bp)}{(1-bq)}\right\}x + bq = 0$$

$$x^2(1-bq) - x\{(a+p)bq - (aq+bp)\} + bq(1-bq) = 0.$$ (10)

$\boxed{65}$

b. $g(x) = x^2 + x - k \;;\; k \in \mathbb{R}$

$g(x) = 0$

$x^2 + x - k = 0 \begin{matrix} \nearrow \alpha \\ \rightarrow \alpha^2 \end{matrix}$

$\alpha + \alpha^2 = -1 \rightsquigarrow (1)$

(5) $\alpha \cdot \alpha^2 = -k \rightsquigarrow (2)$

$(2) \Rightarrow \alpha^3 = -k$

$1 = -k$

(5) $\underline{\underline{k = -1}}$

$\alpha^2 + \alpha - k = 0 \rightsquigarrow (3)$

$\alpha^4 + \alpha^2 - k = 0 \rightsquigarrow (4)$ (5)

$(4) - (3) \quad \alpha^4 - \alpha = 0$

$\alpha(\alpha^3 - 1) = 0$

$\alpha = 0$ or $\alpha^3 = 1$

$\#$.

$(\alpha + \alpha^2 = -1)$

$\therefore \underline{\underline{\alpha^3 = 1}}$ (5)

i. $f(x) = g(x) + a - 10$

$= x^2 + x - k + a - 10$

$= x^2 + x - k + a - 10$

But $k = -1$.

$\therefore f(x) = x^2 + x + 1 + a - 10$

$= x^2 + x + a - 9$ (5)

Let the roots of the equation $f(x) = 0$

are β, γ

for having real roots. $\Delta \geq 0$.

$1 - 4 \times 1(a - 9) \geq 0$ (5)

$\cancel{4 - 4a + 36 \geq 0}$

$\cancel{4a \leq 40.}$

$1-4a+36 \geq 0$

$4a \leq 37$

$a \leq 9\frac{1}{4}$ 5

Also $\beta+\gamma=-1$ 5

$\beta\gamma = a-9$

Only 1 root is positive i.e let $\beta>0$

then $\gamma<0$

$\Rightarrow \beta\gamma<0$ 5

$a-9<0$

$\Rightarrow a<9$.

$\therefore a<9$

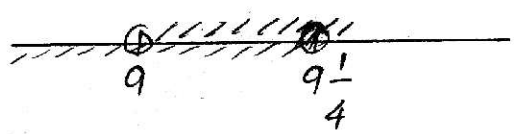
$a \in (-\infty, 9)$ 5

ii. If the two roots are negative

5 $\beta\gamma>0$

$a-9>0$

$a>9$



$a \in (9, 9\frac{1}{4}]$ 5

$\boxed{60}$

c. $f(x) = x^4 - (k+1)x^3 + x^2 + 3x - 5$

By using remainder theorem.

$f(1) = -3$

$1 - (k+1) \times 1^3 + 1 + 3 \times 1 - 5 = -3$

$k = 2$ (5)

when $k = 2$

$f(x) = x^4 - 3x^3 + x^2 + 3x - 5$

By using division algorithm.

$f(x) \equiv (x^2 - 3x + 2) Q(x) + px + q$ (5)

$f(x) = (x-1)(x-2) Q(x) + px + q$

$x = 1$;

$f(1) = p + q \Rightarrow p + q = -3 \rightsquigarrow$ (1)

$x = 2$

$f(2) = 2p + q \Rightarrow 2p + q = -3 \rightsquigarrow$ (2)

(2) - (1) $p = 0$ (5)

(1) $\Rightarrow$ $q = -3$ (5)

$f(x) = -3 \Rightarrow x^4 - 3x^3 + x^2 + 3x - 2 = 0.$

$(x-1)(x-2)(x-1)(x+1) = 0$

$\cancel{x=0}$ $x = 1$ or $x = 2$ or $x = -1$ (5)

$\boxed{25}$

12) a. $\dfrac{12!}{3!\,2!\,4!} = 1663200 \cdot$ (10)

i). $\dfrac{11!}{3!\,2!\,4!} = 138600$ (10)

ii) $\dfrac{8!}{3!\,2!} \times \dfrac{5!}{4!} = 16800$ (10)

iv) $\dfrac{10!}{3!\,2!\,4!} = 12600 \cdot$ (10)

b. i) If $x_1, x_2, \ldots, x_n$ are in arithmetic Progression.,

$$x_2 - x_1 = x_3 - x_2 = x_4 - x_3 = \cdots = x_n - x_{n-1} = d \quad (5)$$

Here, d is the common difference.

$$\frac{1}{\sqrt{x_1}+\sqrt{x_2}} + \frac{1}{\sqrt{x_2}+\sqrt{x_3}} + \cdots + \frac{1}{\sqrt{x_{n-1}}+\sqrt{x_n}}$$

$$= \frac{\sqrt{x_1}-\sqrt{x_2}}{x_1-x_2} + \frac{\sqrt{x_2}-\sqrt{x_3}}{x_2-x_3} + \cdots + \frac{\sqrt{x_{n-1}}-\sqrt{x_n}}{x_{n-1}-x_n} \quad (5)$$

$$= \frac{\sqrt{x_1}-\sqrt{x_2}}{-d} + \frac{\sqrt{x_2}-\sqrt{x_3}}{-d} + \cdots + \frac{\sqrt{x_{n-1}}-\sqrt{x_n}}{-d} \quad (5)$$

$$= -\frac{1}{d}\left\{\sqrt{x_1}-\sqrt{x_2}+\sqrt{x_2}-\sqrt{x_3}+\cdots+\sqrt{x_{n-1}}-\sqrt{x_n}\right\}$$

$$= \frac{-1}{d}\left\{\sqrt{x_1}-\sqrt{x_n}\right\} \quad (5)$$

$$= -\frac{1}{d}\left\{\frac{x_1 - x_n}{\sqrt{x_1}+\sqrt{x_n}}\right\}.$$

5

$$= -\frac{1}{d}\left\{\frac{x_1 - x_1 - d(n-1)}{\sqrt{x_1}+\sqrt{x_n}}\right\}$$

$$= \frac{n-1}{\sqrt{x_1}+\sqrt{x_n}} \qquad \boxed{25}$$

ii) $$\frac{1}{x_1x_2} + \frac{1}{x_2x_3} + \frac{1}{x_3x_4} + \cdots + \frac{1}{x_{n-1}x_n}.$$

5 $$\frac{1}{d}\left\{\frac{d}{x_1x_2} + \frac{d}{x_2x_3} + \frac{d}{x_3x_4} + \cdots + \frac{d}{x_{n-1}x_n}\right\}$$

5

$$\frac{1}{d}\left\{\frac{x_2-x_1}{x_1x_2} + \frac{x_3-x_2}{x_2x_3} + \frac{x_4-x_3}{x_3x_4} + \cdots + \frac{x_n-x_{n-1}}{x_nx_n.}\right\}$$

$$\frac{1}{d}\left\{\frac{1}{x_1} - \frac{1}{x_2} + \frac{1}{x_2} - \frac{1}{x_3} + \frac{1}{x_3} - \frac{1}{x_4} + \cdots + \frac{1}{x_{n-1}} - \frac{1}{x_n}\right\}$$

5

$$\frac{1}{d}\left\{\frac{1}{x_1} - \frac{1}{x_n}\right\}$$

5

$$\frac{1}{d}\left\{\frac{x_n - x_1}{x_1x_n}\right\}$$

5

$$\frac{1}{d}\left\{\frac{x_1 + d(n-1) - x_1}{x_1x_n}\right\}$$

$$\frac{n-1}{x_1x_n} \qquad \boxed{25}$$

c.) $u_r = \dfrac{2r+1}{r^2 (r+1)^2}$ 5 + 5

$(r+1)^2 - r^2 \equiv 2r+1$

5 $\dfrac{(r+1)^2}{r^2(r+1)^2} - \dfrac{r^2}{r^2(r+1)^2} \equiv \dfrac{2r+1}{r^2(r+1)^2}$

$\dfrac{1}{r^2} - \dfrac{1}{(r+1)^2} \equiv \dfrac{2r+1}{r^2(r+1)^2}$ 5

$f(r) - f(r+1) = u_r \; ; \; f(r) = \dfrac{1}{r^2}$ 5

$r=1 \; ; \; f(1) - f(2) = u_1$ 5

$r=2 \; ; \; f(2) - f(3) = u_2$

$r=3 \; ; \; f(3) - f(4) = u_3$

$\vdots \quad \vdots \quad \vdots \quad \vdots$

$r=n-1 \; ; \; f(n-1) - f(n) = u_{n-1}$ 5

$r=n \; ; \; f(n) - f(n+1) = u_n$

$f(1) - f(n+1) = \sum_{r=1}^{n} u_r$ 5

$\sum_{r=1}^{n} u_r = 1 - \dfrac{1}{(n+1)^2}$ 5

$= \dfrac{(n+1)^2 - 1}{(n+1)^2}$

$= \dfrac{n^2+2n}{(n+1)^2} = \underline{\underline{\dfrac{n(n+2)}{(n+1)^2}}}$ 5

$\boxed{50}$

13. a.

$$A = \begin{pmatrix} 3 & 1 \\ 7 & 5 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 3 & 1 \\ 7 & 5 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 7 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 16 & 8 \\ 56 & 32 \end{pmatrix} \quad 10$$

$$A^2 + xI = yA$$

$$\begin{pmatrix} 16 & 8 \\ 56 & 32 \end{pmatrix} + x \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = y \begin{pmatrix} 3 & 1 \\ 7 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 16+x & 8 \\ 56 & 32+x \end{pmatrix} = \begin{pmatrix} 3y & y \\ 7y & 5y \end{pmatrix} \quad 5$$

$\underline{\underline{y = 8}}$ 5

$$16 + x = 3y$$

$$x = 3 \times 8 - 16$$

$$= \underline{\underline{8}} \quad 5$$

$$A^2 + 8I = 8A$$

$$AAA^{-1} + 8IA^{-1} = 8AA^{-1} \quad 5$$

$$A + 8A^{-1} = 8I$$

$$8A^{-1} = 8I - A$$

$$5 \quad = 8 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 3 & 1 \\ 7 & 5 \end{pmatrix}$$

$$8A^{-1} = \begin{pmatrix} 5 & -1 \\ -7 & 3 \end{pmatrix} \Rightarrow A^{-1} = \begin{pmatrix} 5/8 & -1/8 \\ -7/8 & 3/8 \end{pmatrix} \quad 5$$

$\boxed{40}$

b. i) $z = \dfrac{(1+\sqrt{3}i)^2}{4i(1-\sqrt{3}i)}$

$= \dfrac{1+2\sqrt{3}i+3i^2}{4i-4\sqrt{3}i^2}$

$= \dfrac{1+2\sqrt{3}i-3}{4i+4\sqrt{3}}$

$= \dfrac{2(\sqrt{3}i-1)}{4(i+\sqrt{3})}$

$= \dfrac{1}{2} \cdot \dfrac{(\sqrt{3}i-1)(i-\sqrt{3})}{i^2-3.}$

$= -\dfrac{1}{8} \cdot (-\sqrt{3}-3i-i+\sqrt{3})$

$= -\dfrac{1}{8} \times (-4i) = \dfrac{1}{2} i$ (20)

$\therefore Arg(z) = \pi/2$ (5) $\boxed{25}$

ii) $\dfrac{z_1-z_3}{z_2-z_3} = \dfrac{1}{2} - \dfrac{\sqrt{3}}{2} i$

$\left| \dfrac{z_1-z_3}{z_2-z_3} \right| = \sqrt{\left(\dfrac{1}{2}\right)^2 + \left(-\dfrac{\sqrt{3}}{2}\right)^2}$ (5)

(5) $\dfrac{|z_1-z_3|}{|z_2-z_3|} = 1$

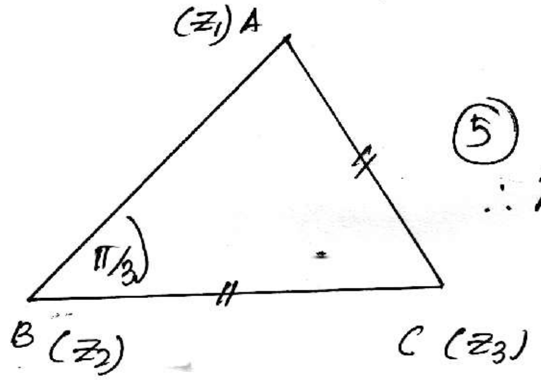
$\Rightarrow |z_1-z_3| = |z_2-z_3|.$ (5)

$\Rightarrow AC = BC$

$$\arg\left(\frac{z_1 - z_3}{z_2 - z_3}\right) = \frac{2\pi}{3} \quad 5$$

$$\arg(z_1 - z_3) - \arg(z_2 - z_3) = \frac{2\pi}{3} \quad 5$$

$$\Rightarrow A\hat{B}C = \pi/3 \quad 5$$



5

$\hat{B} = \hat{A}$ $(\because BC = AC)$

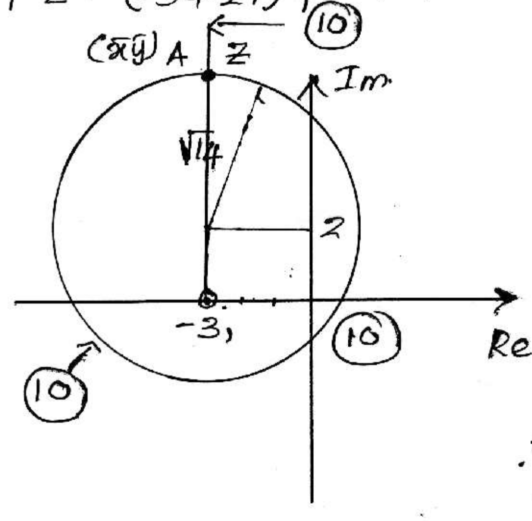
$\therefore \hat{A} = \hat{B} = \hat{C} = \pi/3$

$\therefore ABC$ is a equilateral triangle.

$\boxed{35}$

iii) $|z + 3 - 2i| = \sqrt{14}$

$|z - (-3 + 2i)| = \sqrt{14}$ 5 10



$\arg(z+3) = \pi/2$

$\arg[z - (-3 + 0i)] = \pi/2$ 5

$A(\bar{x}\,\bar{y}) \Rightarrow$

$\bar{x} = -3$ 5

$\bar{y} = 2 + \sqrt{14}$ 5

$\therefore z = -3 + (2 + \sqrt{14})\,i$.

$\boxed{50}$

14) i. $y = \dfrac{3(x+3)}{(x-1)(x+2)}$

$$\frac{dy}{dx} = \frac{(x-1)(x+2)(3) - 3(x+3)(x-1+x+2)}{(x-1)^2(x+2)^2}$$

$$= \frac{3\{x^2+x-2-(x+3)(2x+1)\}}{(x-1)^2(x+2)^2}$$

$$= \frac{3\{x^2+x-2-2x^2-7x-3\}}{(x-1)^2(x+2)^2}$$

$$= \frac{3\{-x^2-6x-5\}}{(x-1)^2(x+2)^2}$$

$$= \frac{-3\{x^2+6x+5\}}{(x-1)^2(x+2)^2}$$

$$= \frac{-3(x+5)(x+1)}{(x-1)^2(x+2)^2}$$ $\boxed{20}$

Horizontal Asymptote ; $\lim\limits_{x \to -\infty} f(x) = 0$

Hence it is $y = 0$. 5

Vertical Asymptote ; $x = 1$ and $x = -2$.
10

$\dfrac{dy}{dx} = 0 \Leftrightarrow x = -5$ and $x = -1$.
5 5

	$-\infty < x < -5$	$x = -5$	$-5 < x < -2$	$-2 < x < -1$	$x = -1$	$-1 < x < 1$	$1 < x < \infty$
$\frac{dy}{dx}$	$-$	0	$+$	$+$	0	$-$	$-$

(25)

When $x = -5$
$y = -\frac{1}{3}$

When
$x = -1$
$y = -3$.

There are two turning points;

$\left(-5, -\frac{1}{3}\right)$ - Local minimum. (5)

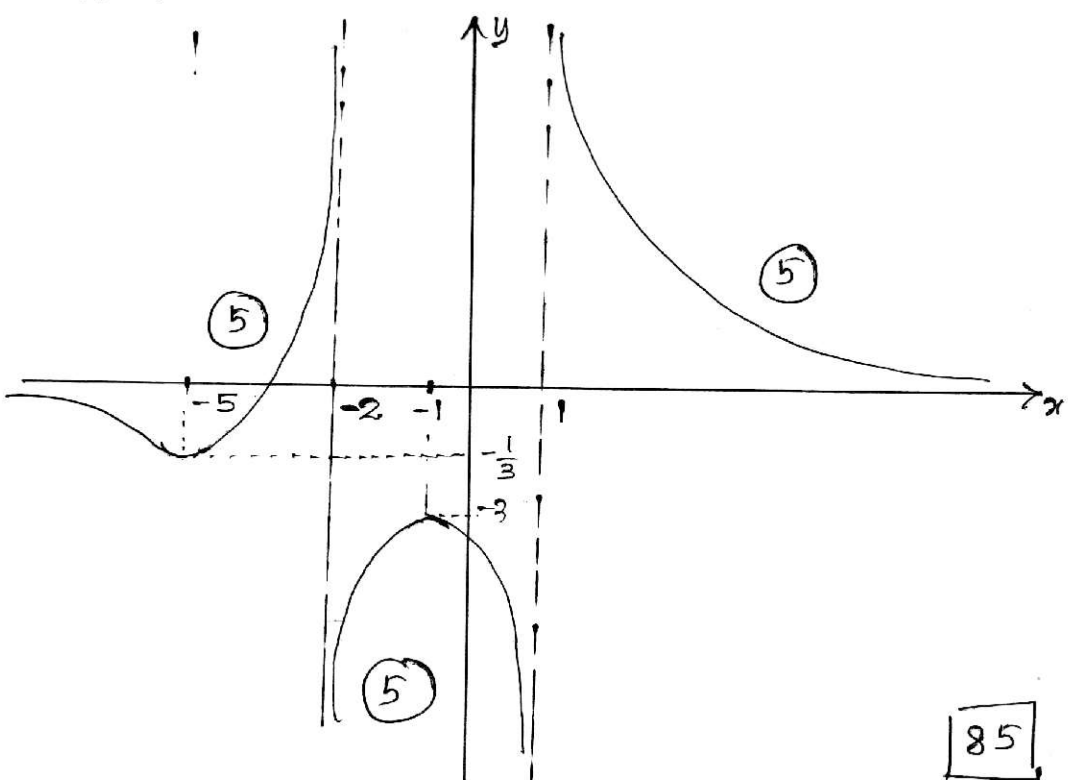
(5) $(-1, -3)$ - Local maximum.

$x \xrightarrow{\lim} -2^-$ $y \xrightarrow{\lim} +\infty$

$x \xrightarrow{\lim} -2^+$ $y \xrightarrow{\lim} -\infty$ (5)

$x \xrightarrow{\lim} 1^-$ $y \xrightarrow{\lim} -\infty$

$x \xrightarrow{\lim} 1^+$ $y \xrightarrow{\lim} +\infty$. (5)



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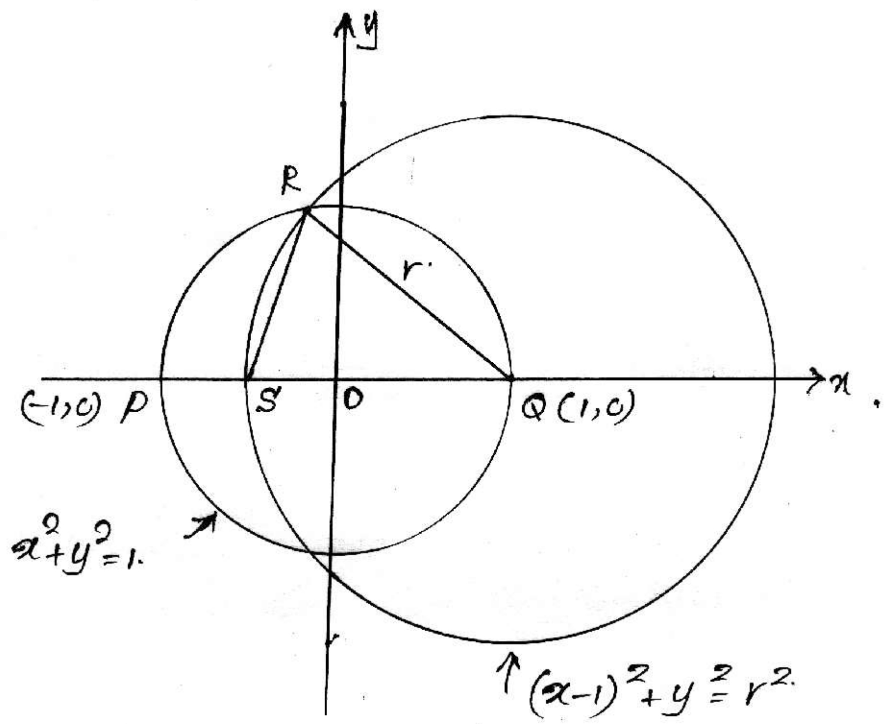
ii)



$x^2+y^2-2x+1-r^2=0$

$x^2+y^2=1 \rightsquigarrow ①$

$x^2+y^2-2x+1-r^2=0 \rightsquigarrow ②$

①, ② $\Rightarrow 1-2x+1-r^2=0$

$$2-2x=r^2$$

$$x=\left(\frac{2-r^2}{2}\right)$$

$$y^2+\frac{1}{4}(2-r^2)^2=1$$

$$y^2+1-r^2+\frac{1}{4}r^4=1$$

$$y^2=r^2-\frac{1}{4}r^4$$

(5) $r=\frac{r}{2}\sqrt{4-r^2}$

$\therefore$ Area of $\Delta RSQ = A = \frac{1}{2}\cdot r\cdot\frac{r}{2}\sqrt{4-r^2}$.

$A=\frac{r^2}{4}\sqrt{4-r^2}$. (5)

$$\frac{dA}{dr} = \frac{1}{4}\left\{ r^2 \cdot \frac{(-\cancel{2}r)}{\cancel{2}\sqrt{4-r^2}} + \sqrt{4-r^2}\cdot 2r \right\} \quad (10)$$

$$= \frac{1}{4}\left\{ \frac{-r^3}{\sqrt{4-r^2}} + 2r\sqrt{4-r^2} \right\}$$

$$= \frac{r}{4}\left\{ \frac{-r^2+2(4-r^2)}{\sqrt{4-r^2}} \right\}$$

$$= \frac{r}{4}\left\{ \frac{-r^2+8-2r^2}{\sqrt{4-r^2}} \right\}$$

$$= \frac{r}{4}\left\{ \frac{8-3r^2}{\sqrt{4-r^2}} \right\}$$

(5)

$$\frac{dA}{dr} = 0 \iff \frac{r}{4}\left\{ \frac{8-3r^2}{\sqrt{4-r^2}} \right\} = 0$$

$$r^2 = 8/3$$

$$(5) \quad r = \frac{2\sqrt{2}}{\sqrt{3}}$$

$\frac{dA}{dr} > 0$ for $0 < r < \frac{2\sqrt{2}}{\sqrt{3}}$ and.

$\frac{dA}{dr} > 0$ for $r > \frac{2\sqrt{2}}{\sqrt{3}}$ (10)

$\therefore$ A maximum when $r = \frac{2\sqrt{2}}{\sqrt{3}}$

$$\therefore A_{max} = \frac{1}{4}\cdot\frac{4\times 2}{3}\sqrt{4-\frac{4\times 2}{3}}$$

$$= \frac{2}{3}\times 2\sqrt{\frac{1}{3}} = \frac{4}{3\sqrt{3}} \quad (5) \qquad \boxed{45}$$

15) a. $\int x^2 \cos 2x \, dx = \int x^2 \frac{d}{dx}\left(\frac{\sin 2x}{2}\right) dx$

$$= \frac{x^2 \sin 2x}{2} - \int \frac{\sin 2x}{\cancel{2}} \cdot \cancel{2}x \, dx.$$

$$= \frac{x^2 \sin 2x}{2} - \int x \sin 2x \, dx$$

$$= \frac{x^2 \sin 2x}{2} - \int x \frac{d}{dx}\left(-\frac{\cos 2x}{2}\right) dx$$

$$= \frac{x^2 \sin 2x}{2} - \left\{ -\frac{x \cos 2x}{2} + \int \frac{\cos 2x}{2} dx \right\}$$

$$= \frac{x^2 \sin 2x}{2} + \frac{x \cos 2x}{2} - \frac{1}{2} \cdot \frac{\sin 2x}{2}$$

$$= \frac{1}{4} \left\{ 2x^2 \sin 2x + 2x \cos 2x - \sin 2x \right\} + c \quad \boxed{35}$$

where c is an arbitrary constant.

b. $\frac{x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$

$x \equiv A(x+2) + B(x+1)$

$x \sim A + B = 1$ — ①

Constant $\sim$ $2A + B = 0$ $\sim$ ②

② − ① $\underline{\underline{A = -1}}$

① ⇒ $\underline{\underline{B = 2}}$

$$\frac{x}{(x+1)(x+2)} = \frac{2}{x+2} - \frac{1}{x+1} \quad \textcircled{10}$$

$$\int \frac{x}{(x+1)(x+2)}\, dx = \int \frac{2}{x+2}\, dx - \int \frac{1}{x+1}\, dx \quad \textcircled{5}$$

$$= 2\ln|x+2| - \ln|x+1| + c' \qquad \textcircled{5}\ \textcircled{5}$$

where c' is an arbitrary constant.

$\boxed{25}$

$x = \cos\theta$

$$\frac{dx}{d\theta} = -\sin\theta \quad \textcircled{5}$$

$$\int \frac{\cos\theta\,(-\sin\theta\, d\theta)}{(\cos\theta+1)(\cos\theta+2)} = 2\ln|\cos\theta+2| - \ln|\cos\theta+1| + c \quad \textcircled{10}$$

$$-\int \frac{\sin\theta\cos\theta}{\cos^2\theta + 3\cos\theta + 2}\, dx = 2\ln|\cos\theta+2| - \ln|\cos\theta+1| + c$$

$$\int \frac{\sin\theta\cos\theta}{\cos^2\theta + 3\cos\theta + 2}\, d\theta = \ln|\cos\theta+1| - 2\ln|\cos\theta+2| + c \quad \textcircled{10}$$

where c is an arbitrary constant

$\boxed{25}$

c. $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$. (10)

$$I = \int_0^{\pi/2} \frac{(\sin x)^{2018}}{(\cos x)^{2018} + (\sin x)^{2018}}\,dx$$

$$= \int_0^{\pi/2} \frac{\left[\sin\left(\frac{\pi}{2}-x\right)\right]^{2018}}{\left[\cos\left(\frac{\pi}{2}-x\right)\right]^{2018} + \left[\sin\left(\frac{\pi}{2}-x\right)\right]^{2018}}\,dx \quad (10)$$

$$= \int_0^{\pi/2} \frac{(\cos x)^{2018}}{(\cos x)^{2018} + (\sin x)^{2018}}\,dx \quad (5)$$

$= J$

$\therefore\ I = J \rightsquigarrow (1)$

$$I + J = \int_0^{\pi/2} \frac{(\sin x)^{2018}}{(\cos x)^{2018} + (\sin x)^{2018}}\,dx + \int_0^{\pi/2} \frac{(\cos x)^{2018}}{(\cos x)^{2018} + (\sin x)^{2018}}\,dx$$

$$= \int_0^{\pi/2} \frac{(\sin x)^{2018} + (\cos x)^{2018}}{(\cos x)^{2018} + (\sin x)^{2018}}\,dx \quad (10)$$

$$= \int_0^{\pi/2} 1\,dx$$

$$= (x)_0^{\pi/2} \quad (5)$$

$I + J = \frac{\pi}{2} \rightsquigarrow (2) \quad (5)$

(1) and (2) $\Rightarrow$ $I = J = \frac{\pi}{4}$ // (5) $\boxed{35}$

$$\int_0^{\pi/2} \frac{7(\sin x)^{2018} - 3(\cos x)^{2018}}{(\cos x)^{2018} + (\sin x)^{2018}}\, dx$$

$$7\int_0^{\pi/2} \frac{(\sin x)^{2018}}{(\cos x)^{2018} + (\sin x)^{2018}}\, dx - 3\int_0^{\pi/2} \frac{(\cos x)^{2018}}{(\cos x)^{2018} + (\sin x)^{2018}}\, dx$$

10

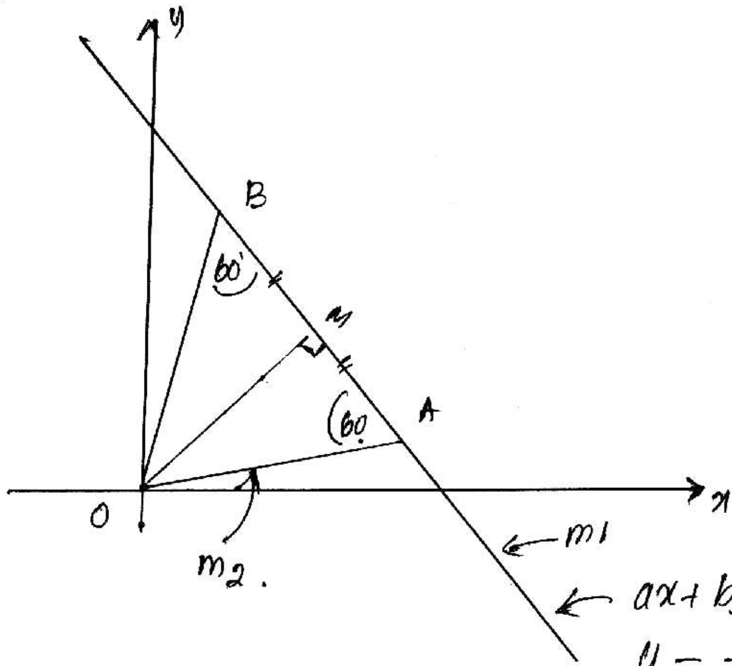
$$7I - 3J$$

$$4 \times \frac{\pi}{4}$$

$$\underline{\underline{\pi}}$$ 5

$\boxed{15}$

16) a.



$$ax+by+c=0$$

$$y=-\frac{a}{b}x-\frac{c}{b}$$

$$\tan 60 = \left|\frac{m_2-m_1}{1+m_1m_2}\right| \quad 5$$

$$m_1=\frac{-a}{b} \quad 5$$

$$\sqrt{3}=\left|\frac{m_2+\frac{a}{b}}{1-\frac{am_2}{b}}\right| \quad 5$$

$$\pm\sqrt{3}=\frac{bm_2+a}{b-am_2} \overset{\oplus}{\Longrightarrow} b\sqrt{3}-am_2\sqrt{3}=bm_2+a.$$

$$b\sqrt{3}-a=m_2(a\sqrt{3}+b)$$

$$m_2=\frac{b\sqrt{3}-a}{a\sqrt{3}+b}. \quad 5$$

$$\overset{\ominus}{\Longrightarrow} -\sqrt{3}b+am_2\sqrt{3}=bm_2+a.$$

$$m_2(a\sqrt{3}-b)=a+b\sqrt{3}$$

$$m_2=\left(\frac{a+b\sqrt{3}}{a\sqrt{3}-b}\right) //. \quad 5$$

$\therefore$ The eq^n of OA and OB are

$$y = \left(\frac{b\sqrt{3} - a}{a\sqrt{3} + b}\right) x \text{ // } (5) \text{ and}$$

$$y = \left(\frac{a + b\sqrt{3}}{a\sqrt{3} - b}\right) x \text{ // } (5)$$

$\boxed{35}$

$$\text{Area of } \Delta\, OAB = \tfrac{1}{2} \cdot OM \cdot AB.$$

$$= \frac{1}{2} \cdot OM \cdot \frac{2OM}{\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}} (OM)^2$$

$$= \frac{1}{\sqrt{3}} \left\{ \left| \frac{a \times 0 + b \times 0 + c}{\sqrt{a^2 + b^2}} \right| \right\}$$

$$\tan 60 = \frac{OM}{AM}.$$

$$\sqrt{3} = \frac{OM}{\frac{1}{2} AB}$$

$$AB = \frac{2OM}{\sqrt{3}}.$$

$$= \frac{c^2}{\sqrt{3}(a^2 + b^2)}$$

$\boxed{25}$

b). let the eq^n of the circle is.

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$A(0, 3) \sim. \quad 0 + 9 + 0 + 6f + c = 0$$

$$6f + c = -9 \sim (1) \quad (5)$$

$$B(\sqrt{3}, 0) \sim \quad 3 + 0 + 2\sqrt{3}g + c = 0.$$

$$2\sqrt{3}g + c = -3 \sim (2) \quad (5)$$

$C \equiv (-\sqrt{3}, 0) \Rightarrow \quad 3 + 0 + 2g(-\sqrt{3}) + 0 + c = 0$

$$-2\sqrt{3}g + c = -3 \quad \text{③} \quad (5)$$

$$\text{②} + \text{③} \qquad 2c = -6$$

$$\underline{\underline{c = -3}}.$$

$$\text{①} \Rightarrow \quad 6f - 3 = -9$$

$$6f = -6$$

$$\underline{\underline{f = -1}}$$

$$\text{③} \Rightarrow \quad -2\sqrt{3}g + c = -3$$

$$-2\sqrt{3}g - 3 = -3 \qquad (15)$$

$$\underline{\underline{g = 0}}$$

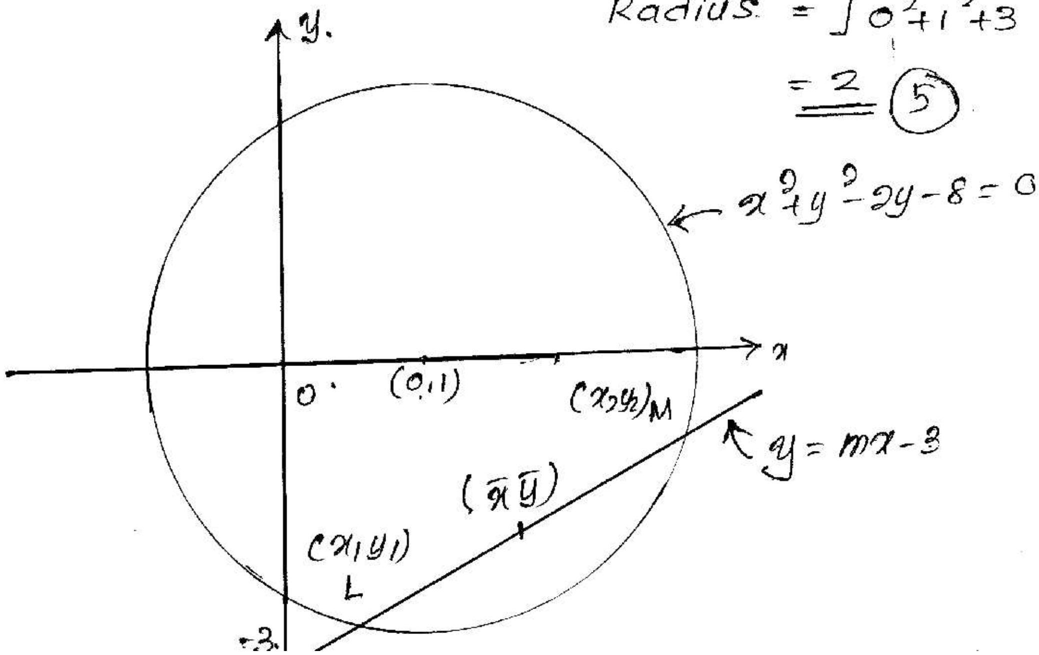
$\therefore$ The eq^{n} is.

$$x^2 + y^2 - 2y - 3 = 0 \;// \quad (10) \qquad \boxed{40}$$

$x^2 + y^2 - 2y - 3 = 0 \longrightarrow$ Center $\equiv (0, 1)$

Radius $= \sqrt{0^2 + 1^2 + 3}$

$\underline{\underline{= 2}}$ (5)



$x^2+y^2-2y-8=0 \sim ①$

$y=mx-3 \sim ②$

① and ② ; $x^2+(mx-3)^2-2(mx-3)-8=0$

$x^2+m^2x^2-6mx+9-2mx+6-8=0$

$(1+m^2)x^2-8mx+7=0$

$$x_1+x_2=\frac{8m}{1+m^2}$$

$$\frac{x_1+x_2}{2}=\frac{4m}{1+m^2} \Longrightarrow \bar{x}=\frac{4m}{1+m^2} \text{ — } Ⓐ \quad (15)$$

① and ②

$$\left(\frac{y+3}{m}\right)^2+y^2-2y-8=0$$

$y^2+6y+9+m^2y^2-2m^2y-8m^2=0$

$(1+m^2)y^2+(6-2m^2)y+9-8m^2=0$

$$y_1+y_2=\frac{-2(3-m^2)}{1+m^2}$$

$$\frac{y_1+y_2}{2}=\frac{-(3-m^2)}{1+m^2} \Longrightarrow \bar{y}=\frac{m^2-3}{m^2+1} \text{ — } Ⓑ \quad (15)$$

Ⓑ/Ⓐ $\frac{\cancel{y}}{\cancel{x}}=\frac{\cancel{m^2-3}}{4m}$

$m^2\bar{y}+\bar{y}=m^2-3$

$m^2(1-\bar{y})=\bar{y}+3$

$$m^2=\left(\frac{\bar{y}+3}{1-\bar{y}}\right)$$

$m^2 = \left(\frac{\bar{y}+3}{1-\bar{y}}\right)$ Substitute in (A)

$$\bar{x}^2\{1+m^2\}^2 = 16m^2$$

$$\bar{x}^2\left\{1+\frac{\bar{y}+3}{1-\bar{y}}\right\}^2 = 16\left(\frac{\bar{y}+3}{1-\bar{y}}\right)$$

$$\bar{x}^2\left\{\frac{1-\bar{y}+\bar{y}+3}{1-\bar{y}}\right\}^2 = 16\left(\frac{\bar{y}+3}{1-\bar{y}}\right)$$

$$\bar{x}^2\left\{\frac{4}{1-\bar{y}}\right\}^2 = 16\left(\frac{\bar{y}+3}{1-\bar{y}}\right)$$

$$16\bar{x}^2 = 16(\bar{y}+3)(1-\bar{y})$$

$$\bar{x}^2 = \bar{y}-\bar{y}^2+3-3\bar{y}$$

$$\bar{x}^2+\bar{y}^2+2\bar{y}-3=0$$

$\bar{y} \equiv y \quad \bar{x} \equiv x$

15

$x^2+y^2+2y-3=0$ //

$\boxed{50}$

17) a. For sin rule. $\boxed{20}$

let $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = k$

$\dfrac{a}{\sin A} = k \Rightarrow a = k\sin A$

$\dfrac{b}{\sin B} = k \Rightarrow b = k\sin B$ (5)

$\dfrac{c}{\sin C} = k \Rightarrow c = k\sin C.$

$$\frac{\cos^2\left(\frac{B-C}{2}\right)}{(b+c)^2} + \frac{\sin^2\left(\frac{B-C}{2}\right)}{(b-c)^2} = \frac{1}{a^2}.$$

$$LHS = \frac{\cos^2\left(\frac{B-C}{2}\right)}{(b+c)^2} + \frac{\sin^2\left(\frac{B-C}{2}\right)}{(b-c)^2}$$

$$= \frac{\cos^2\left(\frac{B-C}{2}\right)}{(k\sin B + k\sin C)^2} + \frac{\sin^2\left(\frac{B-C}{2}\right)}{(k\sin B - k\sin C)^2} \quad (10)$$

$$= \frac{1}{k^2}\left\{\frac{\cos^2\left(\frac{B-C}{2}\right)}{4\sin^2\left(\frac{B+C}{2}\right)\cos^2\left(\frac{B-C}{2}\right)} + \frac{\sin^2\left(\frac{B-C}{2}\right)}{4\cos^2\left(\frac{B+C}{2}\right)\sin^2\left(\frac{B-C}{2}\right)}\right\} \quad (10)$$

$$= \frac{1}{4k^2}\left\{\frac{1}{\sin^2\left(\frac{B+C}{2}\right)} + \frac{1}{\cos^2\left(\frac{B+C}{2}\right)}\right\} \quad (5)$$

$$= \frac{1}{k^2}\left\{\frac{1}{4\sin^2\left(\frac{B+C}{2}\right)\cos^2\left(\frac{B+C}{2}\right)}\right\} \quad (5)$$

$$= \frac{1}{k^2} \cdot \frac{1}{\sin^2(B+C)}$$

(5)

$$= \frac{1}{k^2} \cdot \frac{1}{\sin^2(\pi - A)}$$

(5)

$$= \frac{1}{k^2} \cdot \frac{1}{\sin^2 A}$$

$$= \frac{1}{k^2} \cdot \frac{1}{\frac{a^2}{k^2}}$$

(5)

$$= \frac{1}{a^2}.$$

$\boxed{50}$

$\therefore$ LHS = RHS.

b). $f(x) = \sin x \cos\left(x + \frac{\pi}{4}\right)$

$$= \frac{1}{2} \cdot \left\{2\sin x \cos\left(x + \frac{\pi}{4}\right)\right\}$$

$$= \frac{1}{2}\left\{\sin\left(2x + \frac{\pi}{4}\right) + \sin\left(-\frac{\pi}{4}\right)\right\}$$

$$= \frac{1}{2}\left\{\sin\left(2x + \frac{\pi}{4}\right) - \sin\frac{\pi}{4}\right\}$$

$$= \frac{1}{2}\sin\left(2x + \frac{\pi}{4}\right) - \frac{1}{2\sqrt{2}}$$

$$= \frac{1}{2}\sin\left[\frac{\pi}{2} + \left(2x - \frac{\pi}{4}\right)\right] - \frac{1}{2\sqrt{2}}$$

$$= \frac{1}{2}\cos\left(2x - \frac{\pi}{4}\right) - \frac{1}{2\sqrt{2}}.$$

Here $a = \frac{1}{2}$, $b = 2$, $\alpha = \frac{\pi}{4}$ and $c = -\frac{1}{2\sqrt{2}}$.

(20)

$$g(x) = 4f(x) + \sqrt{2}$$

$$= 4\left\{\frac{1}{2}\cos\left(2x - \frac{\pi}{4}\right) - \frac{1}{2\sqrt{2}}\right\} + \sqrt{2}$$

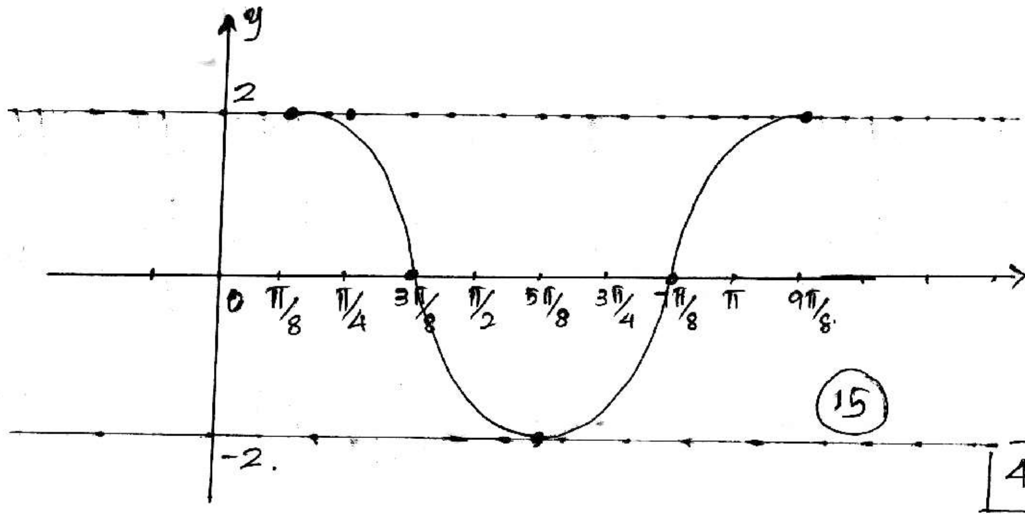
$$= 2\cos\left(2x - \frac{\pi}{4}\right) - \sqrt{2} + \sqrt{2}$$

$$= \underline{\underline{2\cos\left(2x - \frac{\pi}{4}\right)}} \quad (10)$$

$$y = g(x)$$

$$y = 2\cos\left(2x - \frac{\pi}{4}\right)$$

$y_{max} = 2$, $y_{min} = -2$ Period $= \pi$

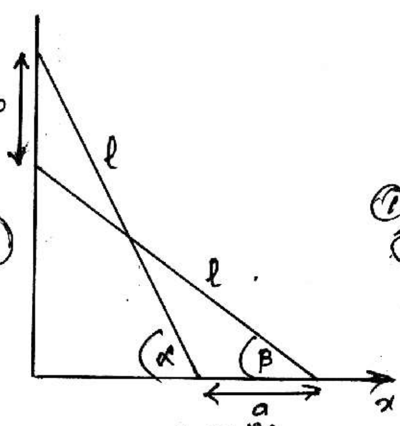


(15)

$\boxed{45}$

c.

(5)



$a = l\cos\beta - l\cos\alpha \rightsquigarrow (1)$ (5)

$b = l\sin\alpha - l\sin\beta \rightsquigarrow (2)$ (5)

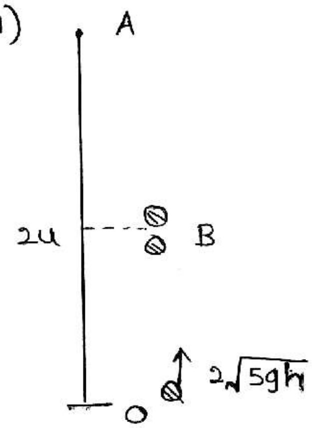
$$\frac{(1)}{(2)} \quad \frac{a}{b} \overset{(5)}{=} \frac{\cos\beta - \cos\alpha}{\sin\alpha - \sin\beta}$$

(5)

$$= \frac{-2\sin\left(\frac{\beta+\alpha}{2}\right)\sin\left(\frac{\beta-\alpha}{2}\right)}{2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}$$

(5) $a = b\tan\left(\frac{\alpha+\beta}{2}\right) \Longleftarrow \frac{a}{b} = \tan\left(\frac{\alpha+\beta}{2}\right)$ (5) $\boxed{35}$

01)



$O \rightarrow B$, $v^2 = u^2 + 2as$

$v_1^2 = 20gh - 2gh$

$v_1^2 = 18gh$

$v_1 = 3\sqrt{2gh}$ 5

$A \rightarrow B$, $\downarrow$ $v_2^2 = 0 + 2gh$

$v_2 = \sqrt{2gh}$ 5

using the principle on linear momentum,

$\uparrow$ $mv_1 - mv_2 = 2mv_3$ 10

$3\sqrt{2gh} - \sqrt{2gh} = 2v_3$

$\sqrt{2gh} = v_3$

For the combined particle,

$v^2 = u^2 + 2as$

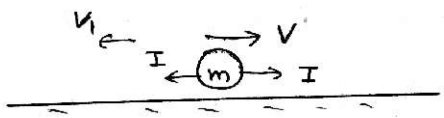
$v_4^2 = v_3^2 - 2gh$

$= 2gh - 2gh$

$v_4 = 0$ 5

25

(2)



$$I = \Delta mV$$

for the frog $\rightarrow$ $I = m(V - V_1)$ — (1) (5)

$\leftarrow$ $I = MV_1$ — (2) (5)

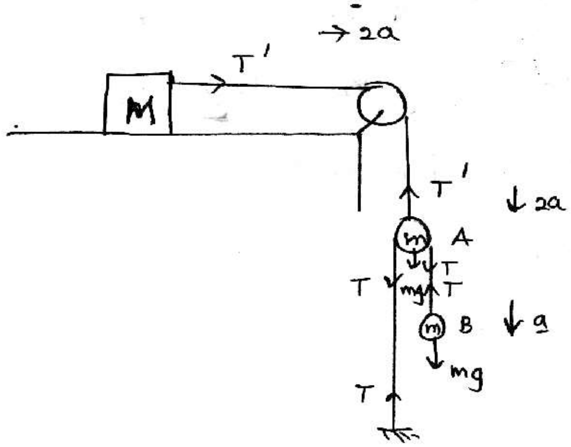
$$m(V - V_1) = MV_1 \quad (5)$$

$$V_1 = \frac{mV}{m+M} \quad (5)$$

$$I = \frac{Mmv}{M+m} \quad (5)$$

25

(3)



(5)

$F = ma$

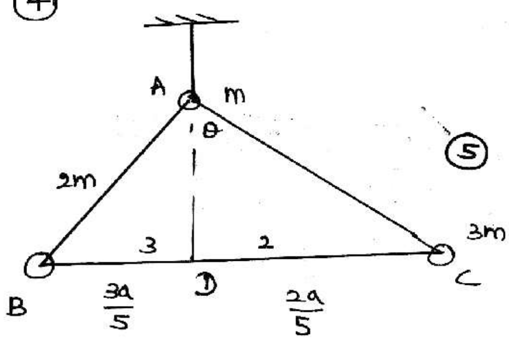
for (m) $T' = M \times 2a$ — (1) (5)

for (A) $\downarrow$ $mg + 2T - T' = m \times 2a$ — (2) (10)

$mg - T = ma$ — (3) (5)

25

(4)



Using the sine rule for the triangle ACD.

$$\frac{\frac{2a}{5}}{\sin\theta} = \frac{a}{\sin[180-(60+\theta)]} \quad (5)$$

$$2\sin(60+\theta) = 5\sin\theta \quad (5)$$

$$2[\sin 60\cos\theta + \cos 60\sin\theta] = 5\sin\theta \quad (5)$$

$$2\left[\frac{\sqrt{3}}{2} + \frac{1}{2}\tan\theta\right] = 5\tan\theta$$

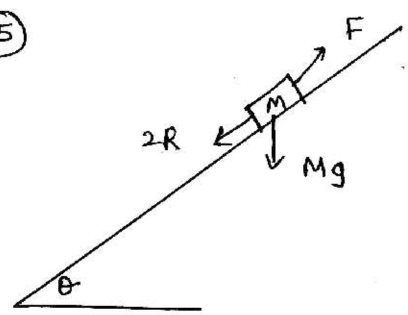
$$\sqrt{3} + \tan\theta = 5\tan\theta$$

$$\tan\theta = \frac{\sqrt{3}}{4} \quad (5)$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{4}\right)$$

25

(5)



$\nearrow$ $F = ma$

$F - 2R - Mg\sin\theta = Ma$ (10)

$F = 2R + Mg\sin\theta + Ma$

$H = FV$

$1000H = (2R + Mg\sin\theta + Ma)V$ (5)

when the vehicle travels with maximum velocity $a = 0$

$H = 7$, $R = 400$, $g = 9.8$, $M = 1200$

$$1000 \times 7 = \left(800 + 1200 \times 9.8 \times \frac{1}{19.6}\right)V \quad (5)$$

$$\frac{7000}{1400} = V$$

$$5\ ms^{-1} = V \quad (5)$$

25

6 $\quad P = 3\underline{i} + 5\underline{j}$ $\qquad |\underline{q}| = 1$

$$\sqrt{\alpha^2+\beta^2} = 1 \qquad 5$$

$$(3\underline{i} + 5\underline{j})(\alpha\underline{i} + \beta\underline{j}) = 0 \qquad 5$$

$$3\alpha + 5\beta = 0$$

$$\alpha^2 + \frac{9\alpha^2}{25} = 1 \qquad 5$$

$$\frac{34\alpha^2}{25} = 1$$

$$\alpha = \frac{-5}{\sqrt{34}} \qquad (\alpha < 0) \qquad 5$$

$$\beta = \frac{-3\alpha}{5}.$$

$$\beta = \frac{3}{\sqrt{34}} \qquad 5$$

25

7 $\quad P(A \cup B) = \frac{11}{12}$ $\qquad P(B) = \frac{2}{3}$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \qquad 5$$

$$\frac{11}{12} = P(A) + \frac{2}{3} - P(A)\,P(B)$$

$$\frac{11}{12} = P(A) + \frac{2}{3} - P(A)\,\frac{2}{3} \qquad 5$$

$$\frac{11}{12} - \frac{8}{12} = P(A)\left[1 - \frac{2}{3}\right]$$

$$\frac{1}{4} = P(A)\left(\frac{1}{3}\right)$$

$$P(A) = \frac{3}{4} \qquad 5$$

$$P(A|B') = \frac{P(A \cap B')}{P(B')} \quad 5$$

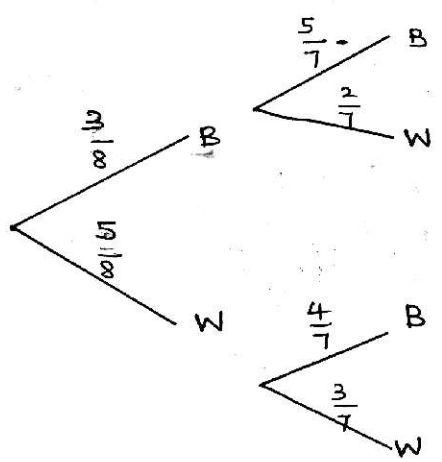
$$= \frac{P(A) - P(A \cap B)}{1 - P(B)}$$

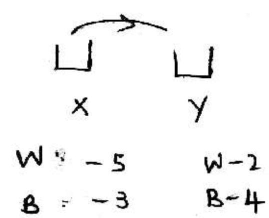
$$= \frac{\frac{3}{4} - \frac{3}{4} \times \frac{2}{3}}{\frac{1}{3}}$$

$$= \frac{\frac{3}{4} - \frac{2}{4}}{\frac{1}{3}} = \underline{\underline{\frac{3}{4}}} \quad 5$$

25

8





5

I) $\frac{3}{8} \times \frac{5}{7} + \frac{5}{8} \times \frac{4}{7} = \frac{15}{56} + \frac{20}{56} = \underline{\underline{\frac{5}{8}}}$ 10

II $\frac{\frac{5}{8} \times \frac{4}{7}}{\frac{5}{8}} = \underline{\underline{\frac{4}{7}}}$ 10

25

(9)

$3, 4, 5, a, b, c, d, 8$

$\frac{a+b}{2} = 6$

$a+b = 12$ (5)

since $a, b > 5$, $a = 6$; $b = 6$ (5)

$\frac{20 + a+b+c+d}{8} = 6$ (5)

$32 + c + d = 48$

$c + d = 16$ (5)

$c, d < 8$

$\underline{\underline{c = 8, \ d = 8}}$ (5)

25

(10)

$S^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$ (5)

$12 = \frac{104}{n} - \frac{64}{n^2}$ (5)

$12n^2 - 104n + 64 = 0$ (5)

$3n^2 - 26n + 16 = 0$

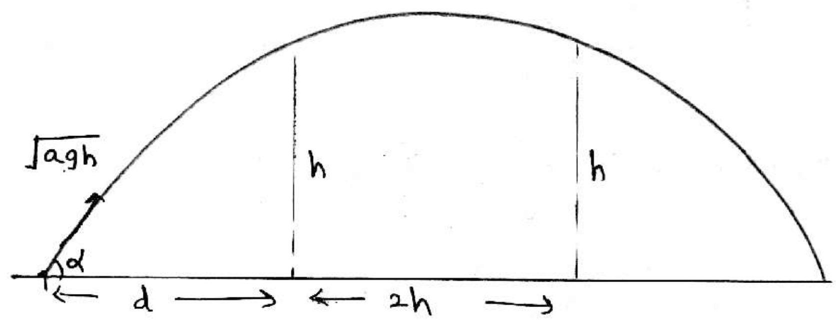
$(n-8)(3n-2) = 0$ (5)

$n = 8$ or $n = \frac{2}{3}$

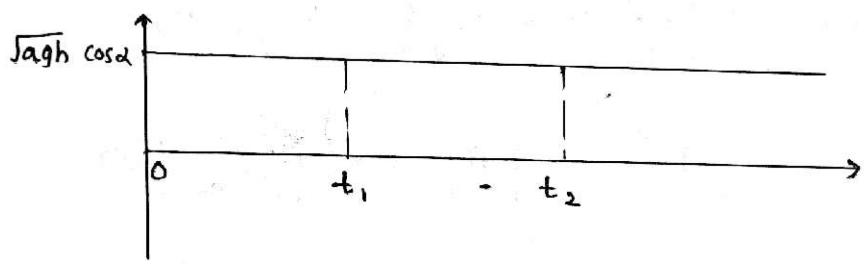
$\therefore \underline{\underline{n = 8}}$ (5)

25

11 a)



Horizontal component

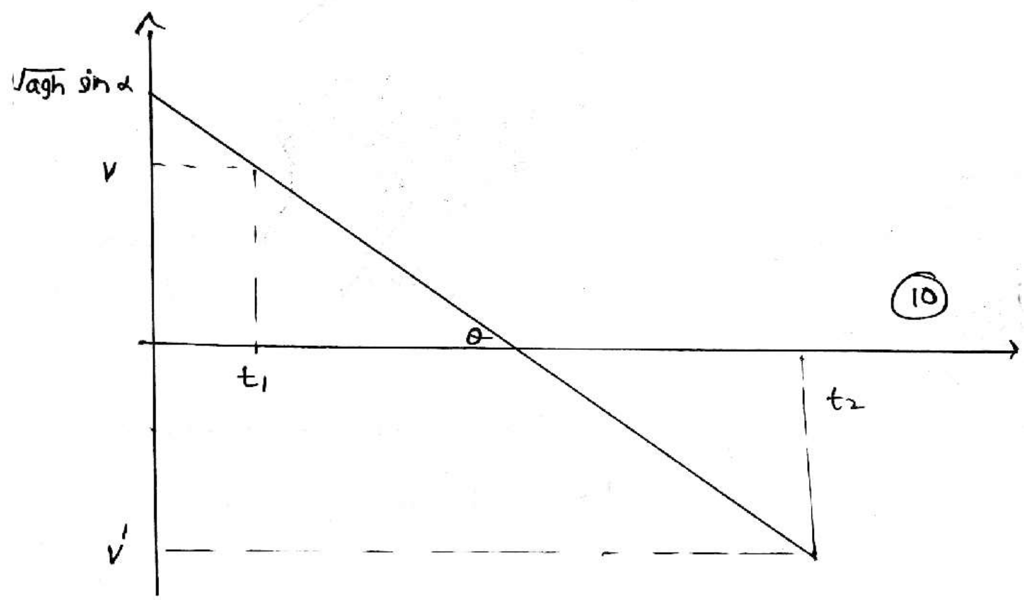


10

$$(t_2 - t_1)\sqrt{agh}\cos\alpha = 2h$$ 10

$$t_2 - t_1 = \frac{2h}{\sqrt{agh}\cos\alpha}$$ 5

25



10

$$\tan\theta = \frac{\sqrt{agh}\,\sin\alpha}{t_1 + \left(\frac{t_2 - t_1}{2}\right)} \quad 5 \quad 5$$

$$g = \frac{2\sqrt{agh}\,\sin\alpha}{t_1 + t_2} \quad 5$$

25

$$t_1 + t_2 = 2\sqrt{\frac{ha}{g}}\,\sin\alpha \quad \text{—}(1)$$

$$t_2 - t_1 = \frac{2h}{\sqrt{agh}\,\cos\alpha} \quad \text{—}(2)$$

(1) − (2) $\quad 2t_1 = 2\sqrt{\frac{ha}{g}}\,\sin\alpha - 2\sqrt{\frac{h}{ag}}\,\frac{1}{\cos\alpha}$ $\quad$ 10

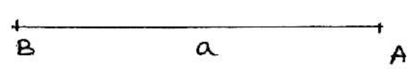
$$d = 2\sqrt{gha}\,\cos\alpha\left[\sqrt{\frac{ha}{g}}\,\sin\alpha - \frac{h}{ag}\cdot\frac{1}{\cos\alpha}\right] \quad 10$$

$$\underline{\underline{d = h\,[a\sin\alpha\cos\alpha - 1]}}$$

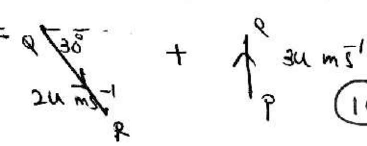
20

b) $\underline{V}_{A,E} = \uparrow 3u\ ms^{-1}$

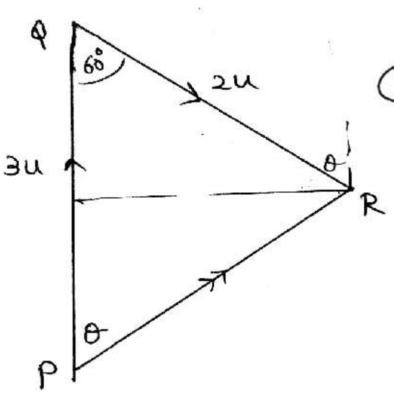
$V_{BA} =$ 



$\underline{V}_{B,E} = \underline{V}_{B,A} + \underline{V}_{A,E}$

$=$ 

10



15

$$(PR)^2 = 9u^2 + 4u^2 - 2\times 3u\times 2u\times\frac{1}{2}$$

$$PR = \sqrt{7}\,u \quad 10$$

35

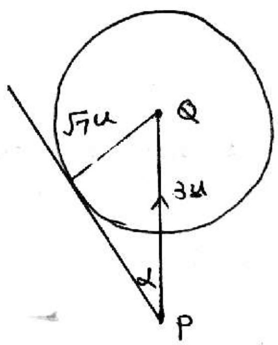
$$\sin\theta = \frac{2u\sin 60^\circ}{PR} \quad (10)$$

$$= \frac{7u \times \frac{\sqrt{3}}{2}}{\sqrt{7}u} = \sqrt{\frac{3}{7}} \quad (5)$$

15

$$\underline{V}_{A,B} = \underline{V}_{A,E} + V_{E,B}$$

$$= \overset{Q}{\underset{P}{\uparrow}} 3u + \sqrt{7}u_{\,QR} \quad (10)$$



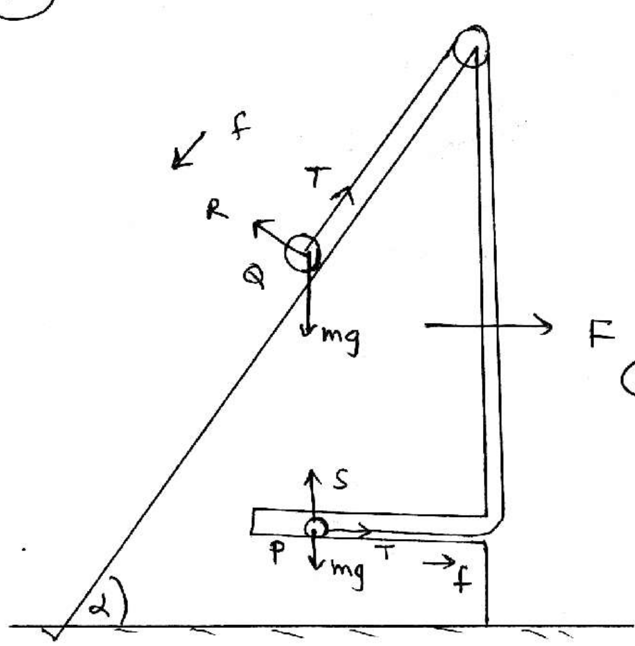
(15)

$$\sin\alpha = \frac{\sqrt{7}u}{3u} = \frac{\sqrt{7}}{3}$$

$$\alpha = \sin^{-1}\left(\frac{\sqrt{7}}{3}\right) \quad (5)$$

30

(12)



(5)

$$\underline{a}_{Q,E} = \overset{f}{\swarrow}_{\alpha} \rightarrow F \quad (5)$$

$$\underline{a}_{P,E} = \rightarrow f + F$$

$F = ma$

$$mg\sin\alpha - T = m[f - F\cos\alpha] \quad \text{—} (1) \quad (10)$$

$$T = m(f + F) \quad \text{—} (2) \quad (10)$$

for the system

$$0 = m(f+F) + m[F - f\cos\alpha]$$

(1) + (2) $\quad mg\sin\alpha = 2mf + mF(1-\cos\alpha) \quad (10)$

$$0 = m(f+F) + m[F - f\cos\alpha] \quad (15)$$

$$0 = F - f\cos\alpha + f \quad (5)$$

$$2F = f(\cos\alpha - 1)$$

$$mg\sin\alpha = 2m\left[\frac{2F}{\cos\alpha - 1}\right] + mF(1-\cos\alpha)$$

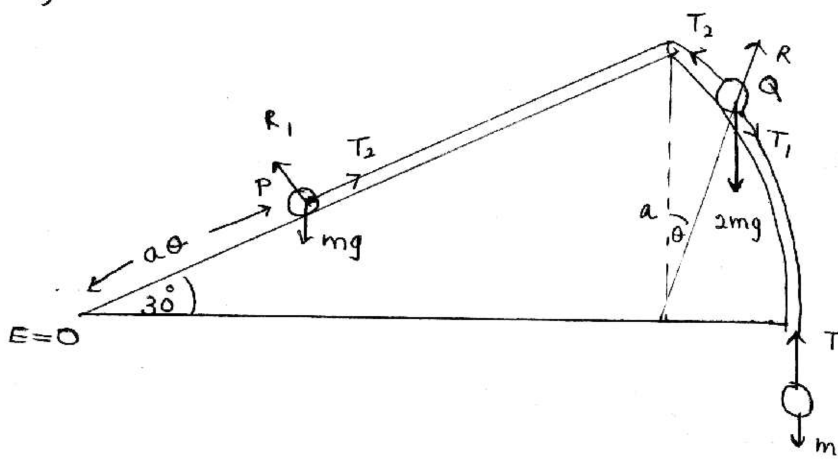
$$= \frac{4F - F(1-\cos\alpha)^2}{\cos\alpha - 1} \quad (10)$$

$$g\sin\alpha(\cos\alpha - 1) = F\left[4 - (1-\cos\alpha)^2\right] \quad (5)$$

$$F = \frac{g\sin\alpha(1-\cos\alpha)}{(1-\cos\alpha)^2 - 4}$$

75

b)



(5)

K.E -10
P.E -10
Eq^n - 10

Using the principle of Conservation of energy,

$$2mga - mgh = mga\theta \sin 30^\circ + 2mga\cos\theta - mg(h + a\theta) + \frac{1}{2}m(a\dot{\theta})^2 \times 2 + \frac{1}{2}\times 2m(a^2\dot{\theta}^2) \qquad (30)$$

$$2mga = mga\theta \times \frac{1}{2} + 2mga\cos\theta - mga\theta + ma^2\dot{\theta}^2 + ma^2\dot{\theta}^2$$

35

$$4g = g\theta + 4g\cos\theta - 2g\theta + 4a\dot{\theta}^2 \qquad (10)$$

$$4a\dot{\theta}^2 = 4g + g\theta - 4g\cos\theta \qquad (10)$$

$$4a\dot{\theta}^2 = g\left[\theta + 4(1-\cos\theta)\right]$$

F = ma

$$2mg\cos\theta - R = 2ma\dot{\theta}^2 \qquad (10)$$

$$R = 2mg\cos\theta - 2ma\dot{\theta}^2$$

$$= 2mg\cos\theta - 2m\frac{g}{4}\left[\theta + 4(1-\cos\theta)\right] \qquad (5)$$

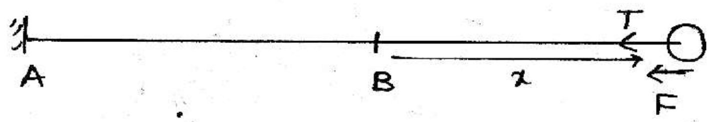
$$= 2mg\cos\theta - \frac{g}{2}\theta m g - 2(1-\cos\theta)mg$$

$$= \frac{mg}{2}\left[8\cos\theta - \theta - 4\right] \qquad (5)$$

$$= \frac{mg}{2}\left[4(2\cos\theta - 1) - \theta\right]$$

40

(13)



$T = mg\dfrac{x}{a}$ (5) $\qquad F = \mu R = \dfrac{1}{\sqrt{2}} mg$ (5)

for (m) $\underline{F} = m\underline{a}$

$\rightarrow \quad -T - F = m\ddot{x}$ (10)

$$-\left(\frac{mgx}{a} + \frac{1}{\sqrt{2}} mg\right) = m\ddot{x} \quad (5)$$

$$\ddot{x} = -\frac{g}{a}\left[x + \frac{a}{\sqrt{2}}\right]$$

$$\ddot{x} + \frac{g}{a}\left[x + \frac{a}{\sqrt{2}}\right] = 0 \quad (5)$$

$$x + \frac{a}{\sqrt{2}} = X \quad (5)$$

differentiating with respect to t twice,

$$\ddot{x} = \ddot{X} \quad (5)$$

$$\ddot{X} + \frac{g}{a} X = 0$$

$\therefore$ It is in the form $\ddot{X} = -\omega^2 X$. (5)

$\therefore$ The motion is simple harmonic.

45

At centre, $\ddot{X} = 0, \quad X = 0$

$$x = -\frac{a}{\sqrt{2}}$$

$\therefore$ The centre lies at a distance $a - \dfrac{a}{\sqrt{2}}$ from A. (5)

$$\dot{X}^2 = \omega^2\left[A^2 - X^2\right]$$

$x = 0, \quad \dot{X} = \dot{x} = 2\sqrt{ag}$ (5)

$X = \dfrac{a}{\sqrt{2}}, \quad 4ag = \dfrac{g}{a}\left[A^2 - \dfrac{a^2}{2}\right]$ (10)

$$4a^2 + \frac{a^2}{2} = A^2$$

$$\sqrt{\frac{9a^2}{2}} = A$$

$$\frac{3a}{\sqrt{2}} = A \quad 5$$

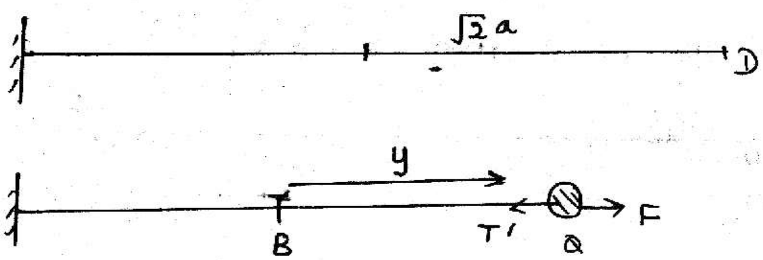
Amplitude of the motion $= \frac{3a}{\sqrt{2}}$

The maximum length of the string $\Big\} = \frac{3a}{\sqrt{2}} - \frac{a}{\sqrt{2}}$

$$= \underline{\underline{\sqrt{2}\,a}} \quad 5$$

30

<u>Backward motion</u>



Total energy at P $=$ Total energy at Q

K.E – 5
P.E – 5
Eqⁿ – 10

$$\frac{1}{2} mg \frac{(\sqrt{2}a)^2}{a} = \frac{1}{2} m\dot{y}^2 + \frac{1}{2} mg \frac{y^2}{a} + F(\sqrt{2}a - y)$$

$$\frac{1}{2} mg \cdot 2a = \frac{1}{2} m\dot{y}^2 + \frac{1}{2} mg \frac{y^2}{a} + \frac{2}{2} \frac{mg}{\sqrt{2}} [\sqrt{2}a - y] \quad 20$$

$$2ag = \dot{y}^2 + \frac{y^2 g}{a} + \sqrt{2} g [\sqrt{2} a - y]$$

$$\dot{y}^2 + \frac{g y^2}{a} - \sqrt{2} g y = 0 \quad 10$$

when $y = 0$, $\dot{y} = 0$

$\therefore$ mass m comes to rest at B. 10

40

differentiating w.r.t. t,

$$2\dot{y}\ddot{y} + \frac{2y\dot{y}g}{a} - \sqrt{2}\cdot g\dot{y} = 0 \qquad 5$$

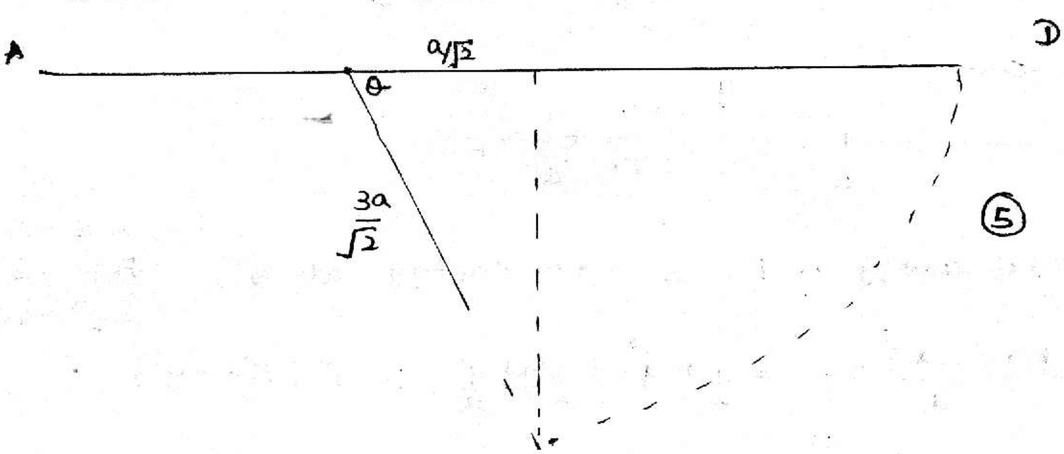
$$\ddot{y} + \frac{gy}{a} - \frac{g}{\sqrt{2}} = 0 \qquad 5$$

$$\ddot{y} = -\frac{g}{a}\left[y - \frac{a}{\sqrt{2}}\right] = 0$$

when $\dot{y} = 0$ centre $y = \frac{a}{\sqrt{2}}$

amplitude $= \sqrt{2}a - \frac{a}{\sqrt{2}} = \frac{a}{\sqrt{2}}$ 5

time taken to go to D from B $= t_1$

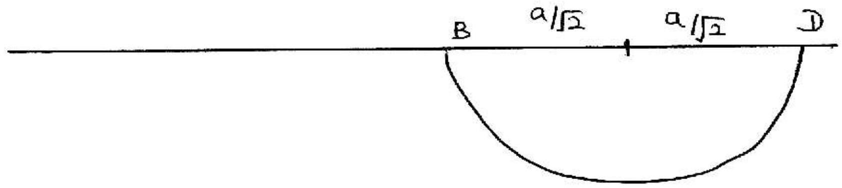


5

$$\cos\theta = \frac{a/\sqrt{2}}{3a/\sqrt{2}} = \frac{1}{3} \qquad \theta = \cos^{-1}\left(\frac{1}{3}\right)$$

$$t_1 = \frac{\theta}{\omega} = \sqrt{\frac{a}{g}}\left[\cos^{-1}\left(\frac{1}{3}\right)\right] \qquad 5$$

Time taken to go to B from D $= t_2$

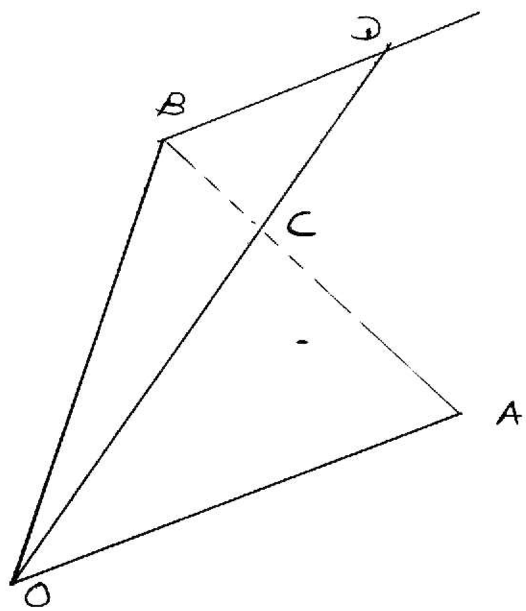


(5)

$t_2 = \frac{\pi}{\omega} = \sqrt{\frac{a}{g}}\,\pi$ (5)

Total time $= t_1 + t_2 = \sqrt{\frac{a}{g}}\left[\pi + \cos^{-1}\left(\frac{1}{3}\right)\right]$ 35

14)



(5)

$\overrightarrow{OA} = \underline{a}$, $\overrightarrow{OB} = \underline{b}$, $\overrightarrow{OC} = \frac{1}{3}\underline{a} + \frac{2}{3}\underline{b}$

$3\underline{c} = \underline{a} + 2\underline{b}$

$2(\underline{c} - \underline{b}) = \underline{a} - \underline{c}$

$2\overrightarrow{BC} = \overrightarrow{CA}$ (10)

$\frac{BC}{CA} = \frac{1}{2}$

$AC : CB = \underline{\underline{2:1}}$ (5)

$\therefore$ ABC is collinear.

20

$\overrightarrow{OD} = \lambda\ (\underline{a} + 2\underline{b})$

$\overrightarrow{BD} = \mu\, \underline{a}$ 5

$\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD}$

$= \underline{b} + \mu \underline{a}$

$\underline{b} + \mu \underline{a} = \lambda\ [\underline{a} + 2\underline{b}]$

$\lambda = \mu$

$\underline{a}\ [\lambda - \mu] + \underline{b}\ [2\lambda - 1] = 0 \Rightarrow \lambda = \frac{1}{2}$

$OD = \frac{1}{2}(\underline{a} + 2\underline{b})$ $\mu = \frac{1}{2}$ 10

$\overrightarrow{OD} = \frac{3}{2}\ \overrightarrow{OC}$

$\frac{|\overrightarrow{OD}|}{|\overrightarrow{OC}|} = \frac{3}{2}$ 5

$OC : CD = \underline{\underline{2 : 1}}$ 5

25

$\overrightarrow{OA} \cdot \overrightarrow{OD} = 0$

$\underline{a} \cdot \frac{(\underline{a} + 2\underline{b})}{2} = 0$ 5

$\underline{a} \cdot \underline{a} + 2\, \underline{a} \cdot \underline{b} = 0$

5 5

$|\underline{a}|^2 + 2\ |\underline{a}|\,|\underline{b}|\ \cos\theta = 0$

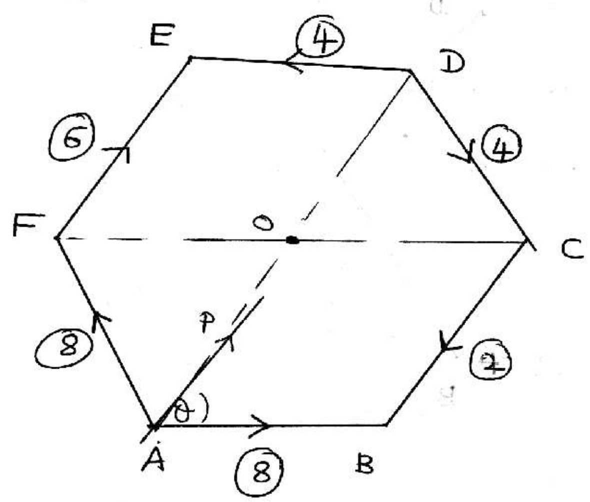
$|\underline{a}|^2 + 2\,|\underline{a}|^2 \cos\theta = 0$

$\cos\theta = -\frac{1}{2}$ 5

$\theta = \underline{\underline{\frac{2\pi}{3}}}$

20

b)



$\rightarrow X = 8 - 2\cos 60^\circ + 4\cos 60^\circ - 4 + 6\cos 60^\circ - 8\cos 60^\circ$ 5

$= 8 - 1 + 2 - 4 + 3 - 4$

$= \underline{\underline{4\ N}}$ 5

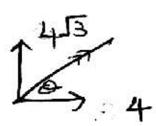
$\uparrow Y = 14\sin 60^\circ - 6\sin 60^\circ$ 5

$= 8 \times \frac{\sqrt{3}}{2} = \underline{\underline{4\sqrt{3}\ N}}$ 5

$R = \sqrt{X^2 + Y^2} = \sqrt{16 + 48} = \sqrt{64}$

8 $R = \underline{\underline{8\ N}}$ 5

30



$\tan\theta = \frac{4\sqrt{3}}{4} = \sqrt{3}$

$\theta = \underline{\underline{60^\circ}}$

A

$4\sqrt{3}\ d = -2\sin 60^\circ \times 2a - 4 \times 4a\cos 30^\circ + 4 \times 4a\cos 30^\circ - 6 \times 2a\sin 60^\circ$ 10

$4\sqrt{3}\ d = -4a \times \frac{\sqrt{3}}{2} - 12a \times \frac{\sqrt{3}}{2}$

2

$d = -2a$ 5

15

I) $\rightarrow$ $P \cos\theta + 4 = 0$ (5)

$\uparrow$ $P \sin\theta + 4\sqrt{3} = 0$

$\tan\theta = \frac{4\sqrt{3}}{4} = \sqrt{3}$

$\theta = \frac{\pi}{3}$ (5)

$P \times \frac{1}{2} = -4$

$P = -8N$ (5)

15

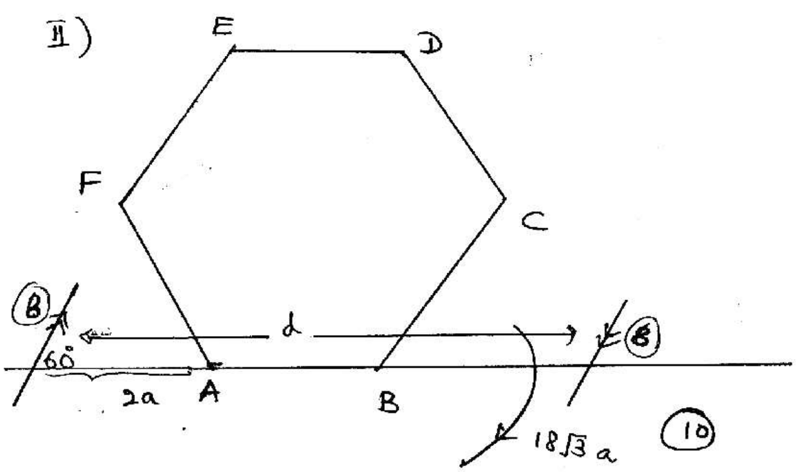
O) $G = d(4-6-8+8-2-4)$

$= (-8) \times 2a \sin 60$

$= -8\sqrt{3}\,a$.

$\therefore$ The system reduces to a couple of moment $8\sqrt{3}\,a$ Nm. (10)

II)



(10)

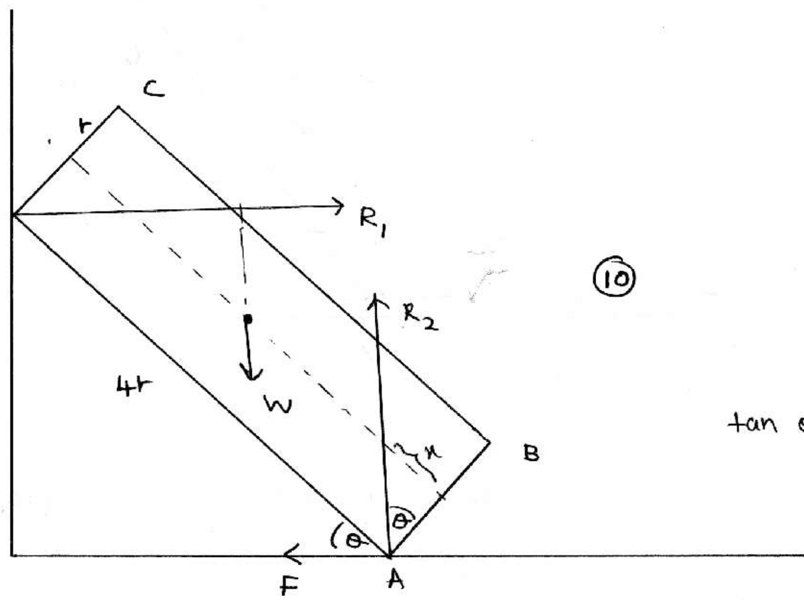
$8d \cos 30^\circ = 18\sqrt{3}\,a$ (10)

$8d \times \frac{\sqrt{3}}{2} = 18\sqrt{3}\,a$

$d = \frac{9}{2}a$ (5)

25

15) a)



(10)

$\tan\theta = \frac{x}{r}$

$\uparrow$ $R_2 - W = 0 \quad \Rightarrow \quad R_2 = W$ (5)

A $\curvearrowright$ $\quad R_1 \times 4r\sin\theta - W(2r - r\tan\theta)\cos\theta = 0$ (10)

$$4R_1 \sin\theta - W(2 - \tan\theta)\cos\theta = 0$$

$$R_1 = \frac{W(2-\tan\theta)\cos\theta}{4\sin\theta}$$

$$R_1 = \frac{W}{4}(2-\tan\theta)\cot\theta \quad (10)$$

$\rightarrow$ $\quad F = R_1$ (5)

$F \leq \mu R_2$ (10)

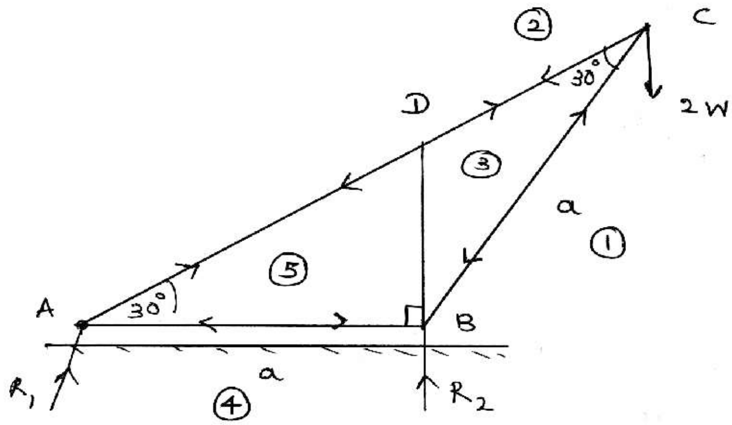
$\frac{W}{4}(2-\tan\theta)\cot\theta \leq \mu W$ $\qquad \tan\theta = \frac{4}{3}$

$\frac{\left(\frac{2\times3}{4} - 1\right)}{4} \leq \mu$ (15)

$\underline{\underline{\frac{1}{8} \leq \mu}}$

65

b)

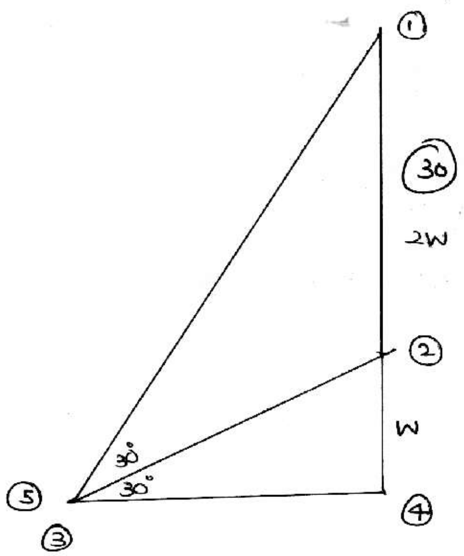


A ↻ $R_2 \times a - 2W\,(a + a\cos 60^\circ) = 0$ ⑩

$R_2 = 2W\left(\frac{3}{2}\right)$

$R_2 = \underline{\underline{3W}}$ ⑤

15



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rod	Stresses		Magnitude
	Tension	Thrust	
CD ②③	✓	–	2W
AD ⑤②	✓	–	2W
AB ⑤④	–	✓	$\sqrt{3}$W
DB	–	–	0
BC ③①	–	✓	$2\sqrt{3}$ W

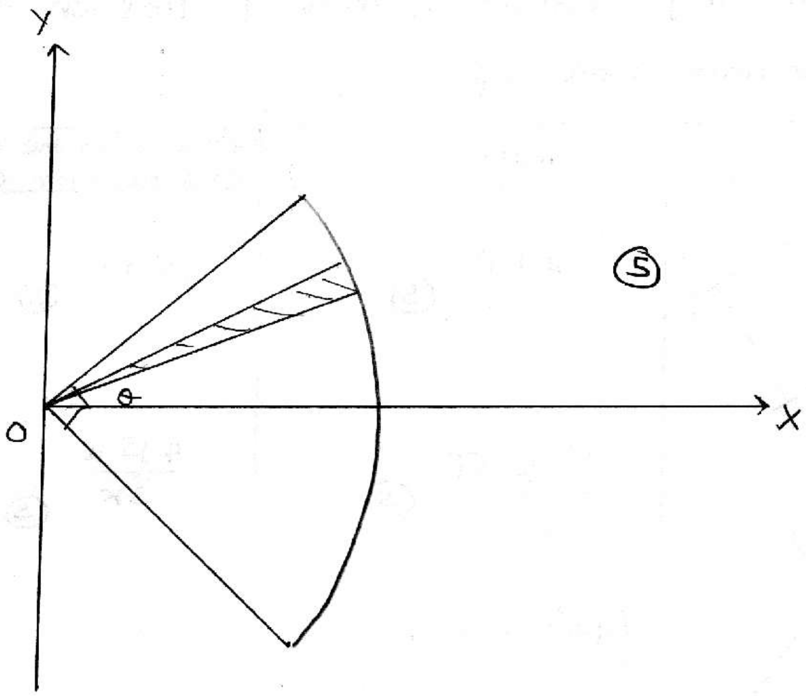
40

$\sin 30^\circ = \frac{W}{②③}$ $\tan 60^\circ = \frac{3W}{④⑤}$

$\sin 60^\circ = \frac{3W}{①③}$

70

(16)



By symmetry the centre of gravity lies on the X axis. (5)

mass per unit area $= \rho$

$dm = \frac{1}{2} a^2 \delta\theta \rho$

$x_i = \frac{2}{3} a \cos\theta$

$$\bar{x} = \frac{\int_{-\pi/4}^{\pi/4} \frac{1}{2} a^2 \frac{2}{3} a \cos\theta \, d\theta \, \rho}{\int_{-\pi/4}^{\pi/4} \frac{1}{2} a^2 \, d\theta \, \rho}$$ (5) (5)

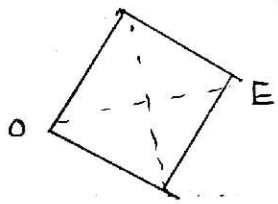
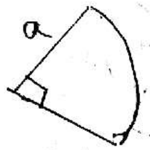
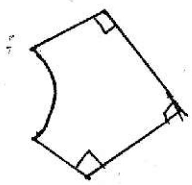
$$= \frac{\frac{2}{3} a [\sin\theta]_{-\pi/4}^{\pi/4}}{[\theta]_{-\pi/4}^{\pi/4}}$$ (5) (5)

$$= \frac{4\sqrt{2} a}{3\pi}$$

30

By symmetry centre of gravity lies on OE. 5

mass per unit area $= \rho$

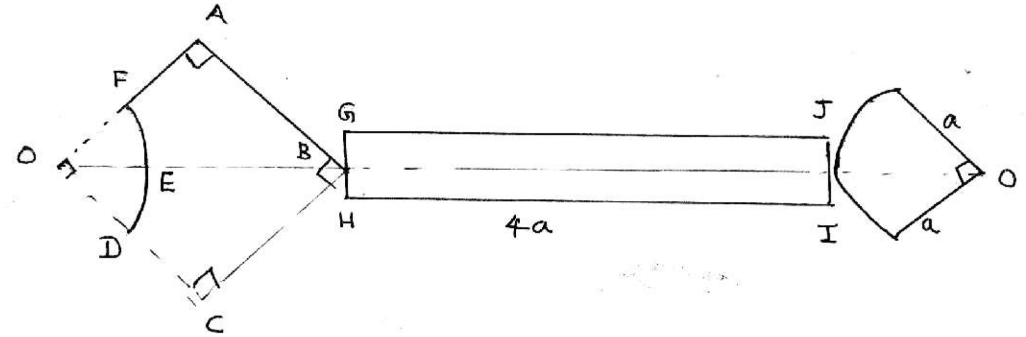
Object	mass	distance to the centre of gravity from O
	$4a^2\rho$ 5	$\sqrt{2}\,a$ 5
	$\frac{1}{2}\,\frac{\pi}{2}\,a^2\rho$ 5	$\frac{4\sqrt{2}\,a}{3\pi}$ 5
	$\left(4a^2 - \frac{\pi}{4}a^2\right)\rho$ 5	$\bar{x}$

$$a^2\left(4 - \frac{\pi}{4}\right)\rho\,\bar{x} = 4a^2\rho\,\sqrt{2}a - \frac{\pi}{4}a^2\,\frac{4a\sqrt{2}\rho}{3\pi} \quad 10$$

$$\bar{x} = \frac{\frac{11\sqrt{2}}{3}\,a}{\left(\frac{16-\pi}{4}\right)} \quad 5$$

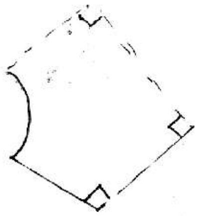
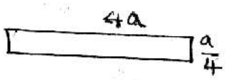
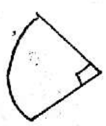
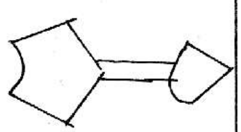
$$\bar{x} = \frac{44\sqrt{2}}{3(16-\pi)}\,a$$

45



By symmetry the centre of gravity lies on OO'.

mass per unit area $= \rho$ (5)

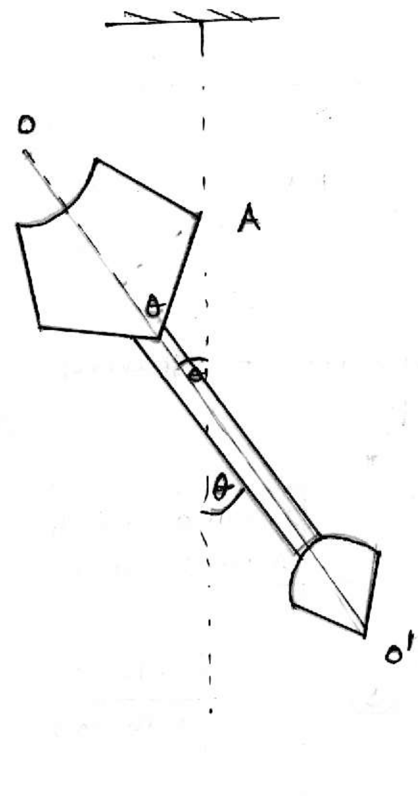
Object	mass	distance to the centre of gravity from O
	$\left(4-\frac{\pi}{4}\right)a^2\rho$ (5)	$\frac{44\sqrt{2}\,a}{3(16-\pi)}$ (5)
	$4a\times\frac{a}{4}\rho$ (5)	$(2+2\sqrt{2})a$ (5)
	$\frac{\pi a^2}{4}\rho$ (5)	$(4+2\sqrt{2})a+a-\frac{4\sqrt{2}a}{3\pi}$ (5)
	$\frac{(16-\pi)}{4}a^2\rho+a^2\rho+\frac{\pi a^2}{4}\rho$ (5)	$\bar{X}$

$$\bar{x}\,\frac{(16-\pi+4+\pi)}{4}\,a^2\rho = \left(4-\frac{\pi}{4}\right)a^2\rho\times\frac{44\sqrt{2}\,a}{3(16-\pi)} + a^2\rho\,(2+2\sqrt{2})a + \frac{\pi a^2\rho}{4}\cdot\left[4+2\sqrt{2}+1-\frac{4\sqrt{2}}{3\pi}\right]a$$ (10)

$$5\bar{x} = \left[\frac{11\sqrt{2}}{3}+2+2\sqrt{2}+\frac{(15\pi+6\sqrt{2}\pi-4\sqrt{2})}{12}\right]a$$ (5)

$$\bar{x} = \frac{a}{60}\left[64\sqrt{2}+24+3(5+2\sqrt{2})\pi\right]$$

55



(5)

$$\tan\theta = \frac{\sqrt{2}a}{\bar{x} - \sqrt{2}a}$$ (10)

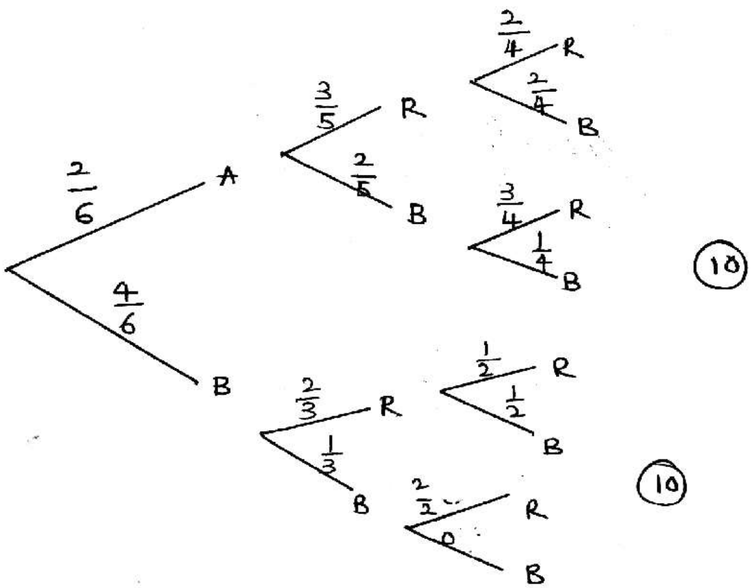
$$= \frac{\sqrt{2}\,a}{\frac{a}{60}\left[64\sqrt{2} + 24 + 3(5+2\sqrt{2})\pi\right] - \sqrt{2}\,a}$$ (5)

$$\theta = \tan^{-1}\left(\frac{60\sqrt{2}}{4\sqrt{2} + 24 + 3(5+2\sqrt{2})\pi}\right)$$

20

17 a)

A	B
B – 2	B – 1
R – 3	R – 2



20

I $\quad \frac{2}{6}\times\frac{3}{5} + \frac{4}{6}\times\frac{2}{3} = \frac{1}{5} + \frac{4}{9} = \underline{\underline{\frac{29}{45}}}$ (10) (5)

15

II $\quad \frac{2}{6}\times\frac{3}{5}\times\frac{2}{4} + \frac{4}{6}\times\frac{2}{3}\times\frac{1}{2} = \frac{1}{10} + \frac{2}{9} = \underline{\underline{\frac{29}{90}}}$ (10) (5)

15

III $\quad \dfrac{\frac{2}{6}\times\frac{3}{5}\times\frac{2}{4}}{29/90} = \dfrac{\frac{1}{10}}{29/90} = \underline{\underline{\frac{9}{29}}}$ (10) (5)

15

b) $\bar{x} = \dfrac{76 + 78 + 67 + 87 + 92}{5}$ 10

$= \underline{\underline{80}}$ 5

15

$s^2 = \dfrac{(76-80)^2 + (78-80)^2 + (67-80)^2 + (87-80)^2 + (92-80)^2}{5}$

$= \dfrac{16 + 4 + 169 + 49 + 144}{5}$ 10

$= \dfrac{382}{5} = 76 \cdot 4$ 5

$S_A = \sqrt{76 \cdot 4} = \underline{\underline{8 \cdot 8}}$ 5

20

$\bar{x}_B = \dfrac{375}{5} = \underline{\underline{75}}$ 10

$S_B^2 = \dfrac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n} = \dfrac{\Sigma x_i^2}{n} - \bar{x}^2$

$s^2 = \dfrac{28445}{5} - (75 \times 75)$ 10

$= 5689 - 5625$

$= 64$ 5

$S_B = \underline{\underline{8}}$ 5

30

Coefficient of Variation of A $= \dfrac{8 \cdot 8}{80} \times 100 = 11$ 5

Coefficient of Variation of B $= \dfrac{8}{75} \times 100 = \dfrac{32}{3} = 10 \cdot 67$ 5

$\therefore$ The Variation of marks of A is higher than the variation of marks of B 10

20