10 E I

Second Term Test - Grade 13 - 2018

Index No : ....

Combined Mathematics I

Three hours only

Instructions:

- *This question paper consists of two parts.*
  Part A (Question 1 - 10) and **Part B** (Question 11 - 17)
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  Answer all questions. Write your answers to each question in the space provided. you may use additional sheets if more space is needed.
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For Examiner's Use only

(10) Combined Mathematics I		
Part	Question No	Marks Awarded
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
	Total	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	
Paper 1 total		
Percentage		

Paper I	
Paper II	
Total	
Final Marks	

Final Marks

In Numbers	
In Words	

Marking Examiner	
Marks Checked by 1	
2	
Supervised by	

Combined Mathematics (Part A)

01) Using the principle of Mathematical induction prove that $2^{n+2}+3^{2n+1}$ is a integer multiple of 7 for all $n \in Z^{+}$

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02) Find all real value of x satisfying the inequality $2x-x^{2} \geq |x-1|-1$

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03) If Prove that, $Ur = f(r+1) - f(r)$

Write down general term Ur of the series 1.1! + 2.2 ! + 3.3! + ... Find $f(r)$ such that $Ur = f(r+1) - f(r)$. Using above result evaluate. $\sum_{r=1}^{n} Ur$.

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04) Find the number of ways of standing n number of students in a raw such that none of two particular persons are in either side.

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05) Show that, $\lim_{x \to 0} \left\{ \frac{\cos(2x^3)-1}{Sin^6(2x)} \right\} = -\frac{1}{32}$

06) Find the area enclosed by the curve $y = x^2 + 2$ and the lines $y = x$, $x = 0$ and $x = 3$

07) Find the points on the curve $y = x^2 + 3x + 4$ such that the tangents drawn to the above curve are passing through the origin.

08) $A\,(-4,1)$, $C\,(8,4)$ are verticals of a parallelogram ABCD. Diagonal BD is parallel to the line $3X + 4Y = 0$ and its length is 5 units. D is on the 1st quadrant.

(i) Find the coordinates of B and D

(ii) Show that ABD is an isosceles triangle.

09) Equation of the tangent drawn to a circle at point $(2,-3)$ is $2x - 3y - 13 = 0$. Also if the circle passes through the point $(2,3)$, find the equation of the circle.

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10) Prove that, $\dfrac{1}{\cos 290^0} + \dfrac{1}{\sqrt{3}\,\sin 250^0} = \dfrac{4}{\sqrt{3}}$

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Combined Mathematics - 13 (Part B)

11) (a) Let $f(x) = ax^2 + bx + c = 0$ and $g(x) = (a + c - b)x^2 - 2(a + c)x + (a + b + c) = 0$ where, $a, b, c \in R$ and $a \neq 0, a + c \neq b$

i. Prove that roots of the equation $g(x) = 0$ are rational.

ii. If α and β are the roots of $f(x) = 0$ show that product of the roots of the equation $g(x) = 0$ is $\frac{(1-\alpha)(1-\beta)}{(1+\alpha)(1+\beta)}$

(b) α, β are roots of the equation $m^2(x^2 - x) + 2mx + 3 = 0$. $(m \neq 0)$ α, β exists such that satisfying the relation $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{4}{3}$. If m_1, m_2 be the two values that can take for m find the value of $\frac{m_1^2}{m_2} + \frac{m_2^2}{m_1}$.

(c) Remainders when the polynomial $f(x)$ is divided by $(x - 1)$ and $(x - 2)$ are 2 and 3 respectively. Find the remainders when the polynomial $f(x)$ is divided by $(x - 1)(x - 2)$

It is given that $f(x)$ is a third order polynomial with coefficient of x^3 is 1. If -1 is a root of $f(x) = 0$, find the polynomial $f(x)$ and show that there are no any other real root for $f(x)$

12) (a) How many permutations that can be prepared taking 4 digits out of 7 digits 1, 2, 3, 4, 5, 6, 7.Out of these permutations.

(i) How many permutations contain the digit 3.

(ii) How many permutations contain the digits 2 and 4.

(iii) How many permutations do not contain the digits 1 and 5.

(b) If the sum of the infinite series.... $1 + (1 + b)r + (1 + b + b^2)r^2 + (1 + b + b^2 + b^3)r^3 + \cdots$ is S, show that $s = \frac{1}{(1-r)(1-br)}$, Where $|b| < 1, |r| < 1$

(c) Resolve $\frac{4x^2 + 1}{(2x-1)(2x+1)}$ into partial fractions.

Write down general term Ur of the series $\frac{5}{1.3} + \frac{17}{3.5} + \frac{28}{5.7} + \cdots$

Using the above result find $f(r)$ such that $U_r - 1 = f(r) - f(r+1)$

Hence, show that $\sum_{r=1}^{n} Ur = \frac{2n^2 + 3n}{2n+1}$

Is the series $\sum_{r=1}^{\infty} Ur$ convergent? Justify your answer.

13) (a) Sketch the graph of $y = |3x-1| + x - 2$ and $y = 4 - 2x$ in the same diagram. Hence find all the real value of x satisfying the inequality, $|3x-1| + 3x \geq 6$

(b) Show that $\log_a x + \log_{a^2} x^2 + \log_{a^3} x^3 + \dots\dots\dots + \log_{a^{2018}} x^{2018} = \log_a x^{2018}$

(c) A committee of 6 members is to be selected out of 7 teachers including the sectional head and 4 students. Find the different ways in which this committee can be selected.

i. If 2 students and the sectional head is included.
ii. Including 3 students without sectional head.
iii. at least 4 teachers including the sectional head.
iv. Including both teachers and students such that the number of teachers are greater than the number of students.

14) a.) Let $f(x) = \dfrac{3x^2 - 1}{x^3 - x}$ for $x \neq 0, \pm 1$

Show that $f'(x)$ the derivative of $f(x)$ is given by $f^1(x) = \dfrac{-(3x^4+1)}{(x^3-x)^2}$ Hence sketch the graph of $y = f(x)$

Using the graph, find the number of roots of the equation $f(x) - x = 0$

b) Find the equation of the normal drawn to the curve $y = x^2$ at point $P \equiv (t, t^2), t \neq 0$

If this normal meets the curve again at point Q, Show that $PQ^2 = \left(\dfrac{4t^2+1}{2t}\right)^3$

Find the equation of the normal such that the length of PQ is minimum.

15) a) Using integration by parts find $\int_0^1 x.e^{2x+1} dx$

b) Evaluate using partial fractions. $\int \dfrac{1}{(1-x^2)(1+x)} dx$

Using a suitable substitution evaluate $\int \dfrac{1}{\sin\theta(1+\cos\theta)} \, d\theta$

Using above result evaluate $\int \dfrac{1}{\sin\theta + \tan\theta} \, d\theta$

c) Evaluate $\int_0^{\pi} \frac{1}{1+\sin x} dx$

Let $I = \int_0^a \phi(x)dx$ and $J = \int_0^a \phi(a-x)dx$. Where a is a positive constant and $\phi(x)$ is differentiable function of x. Prove that $I = J$. Using the above result or otherwise find $\int_0^{\pi} \frac{x\sin x}{1+\sin x} dx$

16) a) Show that the any straight line passing through the point of intersection of the two straight lines $l_1 = ax + by + c = 0$ and $l_2 = a^1x + b^1y + c^1 = 0$ can be represented as,
$ax + by + c + \lambda (a^1x + b^1y + c^1) = 0$ where λ is a parameter.
Variable line $l_3 = lx + my + n$ intersects the lines l_1 and l_2 at A and B respectively. If OA, is perpendicular to OB. Show that
$(aa^1 + bb^1)n^2 - (ac^1 + ca)ln - (bc^1 + cb^1)mn + (l^2 + m^2) \quad cc^1 = 0$
Where O is the origin c and c^1 are non zero constants.

b) A circle which its center is on the first quadrant touches both cordinate axes. Also it touches the line $4y - 3x - 6 = 0$
i. Find the equation of the circle.
ii. Find the coordinates of the touching point the circle and the straight line.

(17) a) Show that $\sin\alpha + \sin\beta + \sin\gamma - \sin(\alpha+\beta+\gamma) = 4\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\beta+\gamma}{2}\right)\sin\left(\frac{\gamma+\alpha}{2}\right)$

b) State the cosine rule.
In a triangle ABC, $BC = 5$, $CA = 4$, $AB = 3$ and point D and E are on the side BC such that $BD = DE = EC$. Prove that $\tan C\hat{A}E = \frac{3}{8}$

c) Solve following equations.
i. $1 - \sin 2x = cos\ x - \sin x$
ii. $Sin^{-1} 6x + Sin^{-1} 6\sqrt{3}x = -\frac{\pi}{2}$

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Provincial Department of Education - NWP

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Second Term Test - Grade 13 - 2018

Index No : ....

Combined Mathematics II

Three hours only

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	16	
	17	
	Total	
Paper 1 total		
Percentage		

Paper I	
Paper II	
Total	
Final Marks	

Final Marks

In Numbers	
In Words	

Marking Examiner	
Marks Checked by 1	
2	
Supervised by	

(Part B)

01) A glass ball which is placed on the smooth horizontal floor at the mid point in between two vertical walls distance 'a' apart and moves on the floor with velocity $'u'$ such that the ball hits a wall perpendicularly. If the coefficient of elasticity between the walls and the ball is e , $(0 < e < 1)$ draw the velocity time graph for the first three collisions. Hence show that the time taken for it is $\frac{a}{2e^2u}\,(e^2 + 2e + 2)$.

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02) When the time of flight of a particle projected at an angle θ to the horizontal is t, Show that the horizontal range S gives by $gt^2 = 2S\tan\theta$. Here g is the acceleration due to gravity.

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03) In a motor bike race held on a straight track, the motor bike A drives with a velocity of $30ms^{-1}$ with an uniform acceleration of $6\,ms^{-2}$ at a distance of $1.5km$ behind the final point. At the same instant the motor bike B drives with a velocity of $40\,ms^{-1}$ with an uniform acceleration of $10ms^{-2}$ at a distance of $600m$ behind A. By considering the motion of B relative to A or otherwise, Show that B passes A at a distance $375m$ before the final point.

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04) A vehicle of mass Mkg, with its engine working at power $H\,kw$ is moving with constant velocity $v\,ms^{-1}$ against a constant resistance. Then the engine is disconnected and breaks are applied to stop the vehicle in distance $s\,m$. If the constant resistance is same as before show that the retardative force of breaks is $\dfrac{MV^3 - 2000Hs}{2Vs}N$.

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05) $\underline{a}$ and $\underline{b}$ are two non-zero non parallel vectors. $|\underline{a}|=2$, $|\underline{b}|=1$, The angle between $\underline{a}$ and $\underline{b}$ is $\frac{2\pi}{3}$. Find the angle between $\underline{a}+\underline{b}$ and $\underline{a}-\underline{b}$

06) An uniform sphere of radius a weight W is in equilibrium rests in contact with a smooth vertical wall and is supported by a string of length a joining a point on the sphere to a point on the wall. Show that the tension in the string is $\frac{2W}{\sqrt{3}}$. Also find the reaction on the sphere by the wall.

07) A body is kept on a rough plane which is inclined at an angle θ to the horizontal. The minimum force which should be applied to move the body up the plane, is thrice the force which should be applied parallel to the plane to prevent the body sliding down the plane. Show that the coefficient to friction between the body and the plane is $\frac{\tan\theta}{2}$.

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08) P and Q are two fixed points on a vertical line distance $2a$ apart such that Q below P. One end of a light inextensible string of length $3a$ is attached to the point P and the other end is attached to the point Q and a particle R of mass m is attached to the mid point of the string. R moves in a horizontal circle with an angular velocity of $\frac{\sqrt{5g}}{a}$ about the vertical line. Show that the ratio between the tensions of the two parts of the string is 3:2.

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09) Let A and B are two events of a sample space Ω. If $P(A' \mid B) = \frac{3}{5}$, $P(A^1 \cap B^1) = \frac{1}{10}$, and $P(B) = \frac{1}{3}$, find $P(A \cap B)$ and $P(A)$.

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10) A box X contains two red balls and three blue balls with equal size. Another box Y contains two red balls and four blue balls with equal size. Two balls are taken out taking one at a time randomly without a replacement.

$i.$ Find the probability of obtaining two red balls.

$ii.$ If both selected balls are red, find the probability that it was selected from the box X.

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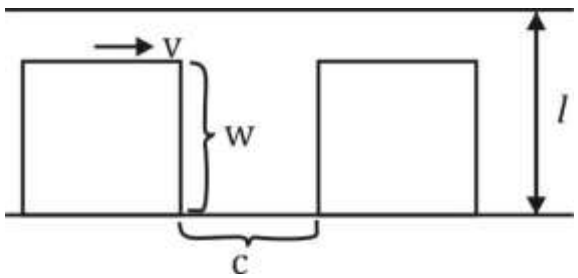
Combined Mathematics 13 - II (Part B)

11) (a) Two buses of $'w'$ width moving with uniform velocity $'v'$ one behind the other along a straight line almost touching to the edge of the road. A man who is in the same side of the road where buses travel, crosses the road, soon after one bus passes him. The width of the road is l and the constant distance between two busses is C. Find the magnitude and direction of the velocity required to cross the road safety for the man. Find the time taken for the man to cross the road.



(b) A rocket which starts from the rest from a point $'A'$ on the ground travels vertically upwards with uniform acceleration $\frac{g}{6}\ mS^{-2}$ for a time $2T$ and subsequently it travels vertically upwards with uniform acceleration $\frac{g}{3}\ mS^{-2}$ for another time T. After that it moves under gravity and comes to the initial point. Draw the velocity time graph for the vertical motion of the rocket. Hence show that,

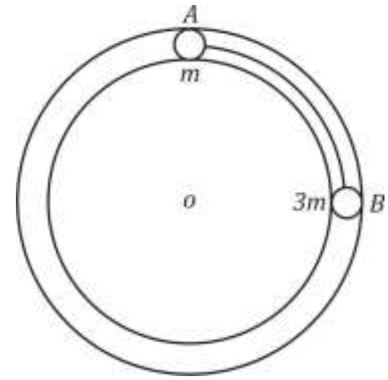
i. The time taken for the rocket to reach the maximum height is $11\frac{T}{3}\ S$

ii. The time taken to reach the ground from the maximum height is $\frac{\sqrt{19}}{3}T$

12) (a) A particle of mass mKg is kept on a fixed smooth plane of inclination 30^0 to the horizontal, on the floor. One end of an inextensible string is attached to it and passes around a fixed pulley at A, Which is on the top of he plane and the other end is attached to a particle of mass MKg which hangs vertically.

Initially the distance to mKg from A is $d_1\ m$ and the distance to the floor from Mkg is $d_2\ m$. When the system is released from rest with $d_1 > d_2$ and if $2M > m,$ show that the particle m reaches A with velocity $\sqrt{\frac{(2M-m)}{(M+m)}gd_1}\ \ m\,s^{-1}$ and the time taken to come to the point A is $t = 2\sqrt{\frac{d_1(M+m)}{(2M-m)g}}\ \ S$. Also show that the tension in the string is $\frac{3Mmg}{2(M+m)}$.

(b) A smooth thin circular tube with centre O and radius a, fixed in a vertical plane is shown in the figure. Two particles A and B with masses m and $3m$ respectively are connected by a light inextensible string with length $\frac{\pi a}{2}$ and are placed inside the tube. A is at the highest point of the tube and B is kept in the level of O and the system is released from rest. When OA makes an angle θ with the vertical, Such that the string is taut, show that $2a\dot{\theta}^2 = g\,(1 - \cos\theta + 3\sin\theta)$. Find the reaction between the tube and the particle A and the tension in the string in that position. Also show that the string slackens when $= \frac{\pi}{4}$.



13) (a) A smooth sphere A of mass $2m$ moves with constant velocity u and collides directly with another smooth sphere B of mass 3m, which is at rest.

i. Show that after the collision, ratio between the velocities of the spheres B and A is 2:1.

ii. Show that the impulse due to the collision is $\frac{3mu}{2}$

iii. Also show that the loss of kinetic energy due to collision is $\frac{9mu^2}{16}$

(b) The power of an engine of a lorry with mas Mkg is H kw and when it travels along a horizontal road its maximum velocity is $u\,mS^{-1}$. When the lorry moves upwards along a road of inclination α to the horizontal with the same power and against the same resistive force the maximum velocity is $V\,mS^{-1}$

Show that $H = \frac{MguvSin\,\alpha}{1000(u-V)}$ Here g is the acceleration due to gravity. When the lorry moves downwards along the same inclined plane with maximum velocity $2VmS^{-1}$ with the same power show that $V = \frac{3u}{4}$

14) (a) The position vectors of the points B and C relative to a point A are $\underline{b}$ and $\underline{c}$ respectively. The mid points of the sided AB, BC and CA are D, E and F respectively.

i. Find the position vectors of the points D, E and F.

ii. AE and BF intersect at G. Show that $\overrightarrow{AG} = \frac{1}{2}\left[\underline{c} + \lambda\,(2\underline{b} - \underline{c})\right]$

iii. Build up another expression for $\overrightarrow{AG}$ in terms of $\underline{c}$ and $\underline{b}$. Hence show that $AG: GE = 2:1$ and $BG: GF = 2:1$

iv. Also deduce that $CG: GD = 2:1$

v. Let, H is a point inside a triangle and its position vector is $\underline{h}$

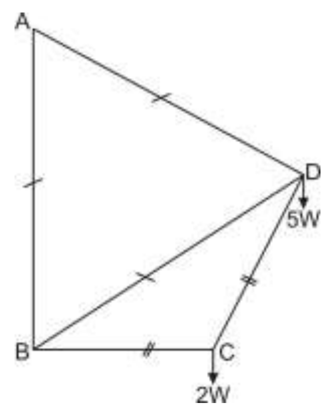
1. If HD is perpendicular to AB, show that $|\underline{b}|^2 = 2.\underline{b}\,.\,\underline{h}$
2. If HE is perpendicular to BC build up another similar expression between $\underline{h}$ and $\underline{b}$
3. Deduce that HF is perpendicular to AC.

(b) $PQRS$ is a square of length a. Forces of magnitudes $6, m, 3, 6, \sqrt{2}$ and $\sqrt{2}\,P$ Newton's act along the directions $\overrightarrow{PQ}, \overrightarrow{QR}, \overrightarrow{RS}, \overrightarrow{SP}, \overrightarrow{PR}$ and $\overrightarrow{QS}$ in the order of the letters respectively. Here m and P are positive integers.

i. Show that the system is not in equilibrium.

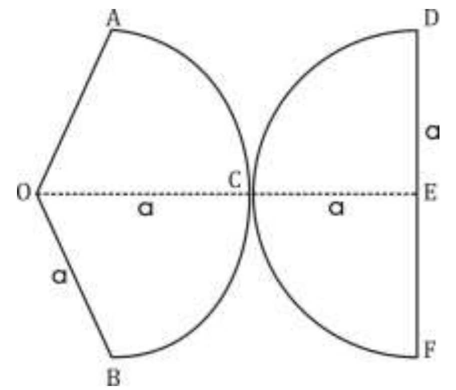
ii. If the system of forces reduces to a couple, shows that $m = 1$ and $p = 4$

15) (a) A Framework consisting of five light rods is shown in the diagram. Here ABD is an equilateral triangle. $A\hat{B}C = 90^0$, BC and CD are equal in length. The framework is hinged freely and smoothly at A and B. A weight of 2W is hung at C and a weight of $5w$ is hung at D such that AB is vertical. By drawing a stress a diagram, find the magnitudes of the stresses along the rods AD, CD, BD and BC classifying them in to tensions or thrusts.



(b) Four rods each of weight w with lengths $AB = DC = 2l$ and $BC = AD = l$ are jointed freely at the points A, B, C, D and a light inextensible string with length $\sqrt{3}l$ is, attached to the points D and B. AB and CD are kept horizontally by means of two vertical strings connected at A and B and to the two points in the same level. Find the tension of the string DB and the reaction at the joint C.

16) Show that the centre of mass of an uniform sector of radius a, subtending an angle 2α at the center lies a distance $\frac{2\,a\,\sin\alpha}{3\alpha}$ from the centre on the symmetrical axis of the sector.



The figure shows an uniform composite plane lamina consists of a sector $AOBCA$ of radius a and $A\hat{O}B = \frac{2\pi}{3}$ and a semicircular sector $DCFD$ of radius a both made of the same material are rigidly joined at C. Show that the centre of mass of the lamia is at a distance $2\left(\frac{3\pi+\sqrt{3}-2}{5\pi}\right)a$ from O on the symmetrical axis.

The lamina is hung freely in a vertical plane by means of a light inextensible string attached to the point A and a point a ceiling. If OE makes an acute angle θ with the upward vertical, show that $\tan\theta = \frac{5\sqrt{3}\,\pi}{7\pi+4\sqrt{3}-8}$

17) (a) A and B are any two events of a sample space Ω

Show that $P\,(A) = P\,(A\,\cap\,B') + P\,(A \cap B\,)$ Here B' is the complement event of B.

Also that $P\,(A \cup B\,) = P(A) + P(B) - P\,(A \cap B\,)$

Define the conditional probability of B given A, $P\,(B \mid A)$

Show that $P(B' \mid A) = 1 - P(B \mid A)$

If, $P(A) = \frac{3}{8}$, $P(A \cap B') = \frac{5}{24}$, $P(A' \cap B') = \frac{1}{3}$

Find,

i. $P(A \cap B)$ ii. P (B) iii. $P(A' \mid B)$

(b) Three tyre factories A, B and C supply tyres to a shop. Their contribution for supplying tyres to the shop is 60%, 30%, 10% respectively. The probabilities of having defective tyres from B and C are 0.04 and 0.02 respectively. If a tyre is selected at random from the shop the probability that the selected tyre is defective one is 0.044.

Find the probability that the tyres supplied by A is defective.

It is given that the selected tyre is a defective one, find the probability that it is supplied by C.