Provincial Department of Education - NWP

10 | E | I

Second Term Test - Grade 12 - 2018

Index No : .....

Combined Mathematics I

Three hours only

Instructions:

* *This question paper consists of two parts.*
  Part A (Question 1 - 10) and **Part B** (Question 11 - 17)
* **Part A**
  Answer all questions. Write your answers to each question in the space provided. you may use additional sheets if more space is needed.
* **Part B**
  Answer five questions only. Write your answers on the sheets provided.
* *At the end of the time allocated, tie the answers of the two parts together so that* **Part A** *is on top of* **part B** *before handing them over to the supervisor.*
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For Examiner's Use only

(10) Combined Mathematics I		
Part	Question No	Marks Awarded
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
	Total	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	
Paper 1 total		
Percentage		

Paper I	
Paper II	
Total	
Final Marks	

Final Marks

In Numbers	
In Words	

Marking Examiner	
Marks Checked by 1	
2	
Supervised by	

Combined mathematics 12 - I (Part A)

01) When polynomial $x^4 - px^2 + q, x \in R$ is divided by $(x+1)^2$ remainder is $5x - 2$. Find the constants p and q.

02) Find all values of x satisfying the inequality $\frac{3}{x} < |x - 2|$

03) Find the values of k for having common root for the equations $x^2+3x+k=0, x^2+2kx+2k=0$. Here $k \neq 0$.

04) Resolve $\frac{4x^3-x+2}{x(x+1)^2}$ in to partial fractions.

05) Evaluate, $x \xrightarrow{\lim} 0 \quad \dfrac{(1+\sin x)^{1/3} - (1-\sin x)^{1/3}}{x}$

06) If, $a^2 + b^2 = 23ab$ Prove that $\log a + \log b = 2\log\left(\dfrac{a+b}{5}\right)$

07) If $A(0,-1), B(2,1), C(0,3)$ and $D(-2,1)$ Prove that $ABCD$ is a square.

08) Let, $f(x) = x^2 - 1$, $g(x) = \sqrt{x^2+1}, h(x) = \begin{cases} 0; x = 0 \\ x; x \neq 0 \end{cases}$ Find the compound function $(hofog)x$

09) If $k = \sin\frac{\pi}{18} \quad \sin\frac{5\pi}{18} \quad \sin\frac{7\pi}{18}$ Find the value of k numerically.

10) Solve for x; $\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \cot^{-1}\left(\frac{1-x^2}{2x}\right) = \frac{\pi}{3}; x > 0$

Combined Mathematics 12 - I (Part - B)

Answer Five questions only.

11) a) α and β are real and distinct roots of the equation $(x+1) = kx(1-kx)$

Write down the equation $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = a - 2$ interms of k.

If λ and μ the two values of k of the above equations,

Prove that $\frac{\lambda^2}{\mu^2} + \frac{\mu^2}{\lambda^2} + \frac{2}{(a-1)^2} = 4\left(\frac{a+1}{(a-1)^2}\right)^2$

(b) Roots of the equation $ax^2 + bx + c = 0$ are such that difference of the roots is equal to exactly half of the sum of its reciprocals. Prove that,
$b^2(4c^2 - a^2) = 16ac^3$

(c) $(x^2 + 4)$ is a factor of fourth order polynomial of $f(x)$. Remainder when $f(x)$ is divided by $(x+1)^2$ is -15 and the coefficient of x^4 is 1. Find $f(x)$

12) a) Sketch the graph of $y = 2|x+1| - 3$ and $y = x + 2|x-1|$ on a same coordinate plane. Hence find the set of solutions of the inequality $x + 2|x-1| > 2|x+1| - 3$

b) Evaluate

i. $\lim_{x \to 1^-} \frac{x^2-1}{|x+1|}$ ii. $\lim_{x \to 5^+} \frac{x-5}{|x-5|}$

c) $(3,1), (5,6)$ and $(-3,2)$ are the mid point coordinates of the sides of a triangle. Find the coordinates of the vertices of the triangle. Hence find the coordinates of the centroid of the triangle.

13) a) Evaluate following limits.

(i) $\lim_{x\to 2} \dfrac{\sqrt{1+\sqrt{2+x}}-\sqrt{3}}{x-2}$

(ii) $\lim_{x\to \frac{\pi}{4}} \dfrac{\sin x-\cos x}{x-\pi/4}$

b) Let $f(x)=\begin{cases} 3ax+b\ ; & x>1 \\ 11\ \ \ ; & x=1 \\ 5ax-2b; & x<1 \end{cases}$

If the function $f(x)$ is continuous at $x=1,$. Find the value of a and b.

c) Find K and $f(x)$ such that,

$\dfrac{1}{(x-2)(x-1)^3}=\dfrac{k}{(x-2)}+\dfrac{f(x)}{(x-1)^3}$. Represent $f(x)$ as polynomial of $(x-1)$

Hence, resolve $\dfrac{1}{(x-2)(x-1)^3}$ in to partial fractions.

d) Let $f(x)=\dfrac{3x+2}{5x-3}$ Show that $f^{-1}(x)$ is exists and prove that $f^{-1}(x)=f(x)$.

14) a) Sketch the graph of $y=x^2-5x+6$. Hence discuss the nature of roots of the following equations.

i. $x^2-5x-3=0$

ii. $2x^2+4x-7=0$

iii. $x^2+6x+9=0$

b) Prove that the roots of the equation $a(b-c)x^2+b(c-a)x+c(a-b)=0$ are rational where $a,b,c \in R$, $a\neq 0$ and $b-c\neq 0$

c) Let $f(x)=2x^2+6x+1+k(x^2+2)$ Find the range of values of k such that $f(x)>0$ for all the values of x

d) Roots of the equation $x^2+ax+b=0$ are α,β and if $S_n=\alpha^n+\beta^n$, $(n\in N)$

Prove that $S_{2018}=-\left[aS_{2017}+bS_{2016}\right]$

15) a) i. Represent $\dfrac{2.814814..... \times 2.2525.....}{1.8585.....}$ as a rational number.

ii. Rationalize the denominator; $\dfrac{12}{3+\sqrt{5}+2\sqrt{2}}$

b) Prove that $\log_a^b = \dfrac{\log_c^b}{\log_c^a}$ Where $a, b, c \in R$ and $a \neq 0, \quad c \neq 0$

Hence, If, $a = \log_{12} 18, b = \log_{24} 54$ Prove that the value of $ab + 5(a-b)$ is 1.

c) Sketch the graph of the function $f(x) = \dfrac{2|x-1|}{x-1}$

Hence show that the function is not defined at $x = 1$.

Further is $\lim\limits_{x \to 1} f(x)$ exists? Explain your answer.

16) a) If $5\theta = 90^0$, Prove that $4\sin^3\theta - 2\sin^2\theta - 3\sin\theta + 1 = 0$

Hence, show that $\sin 18^0 = \dfrac{\sqrt{5}-1}{4}$. Deduce that $\cos 36^0 = \dfrac{\sqrt{5}+1}{4}$

Using the above results show that $\tan 6^0 \tan 42^0 \tan 66^0 \tan 78^0 = 1$

b) In the usual rotation, state and prove the cosine rule for any triangle.
Hence Prove that,

$$4\left(bc\cos^2\frac{A}{2} + ca\cos^2\frac{B}{2} + ab\cos^2\frac{C}{2}\right) = (a+b+c)^2$$

c) If, $\operatorname{cosec}\theta - \sin\theta = m$ and $\sec\theta - \cos\theta = n$ Show that, $m = \dfrac{\cos^2\theta}{\sin\theta}$ and $n = \dfrac{\sin^2\theta}{\cos\theta}$.

Hence prove that, $(m^2 n)^{2/3} + (nm^2)^{2/3} = 1$

17) a) If, $\sin x + \sin^2 x + \sin^3 x = 1$ Prove that $\cos^6 x - 4\cos^4 x + 8\cos^2 x = 4$

b) In the usual notation state the sine rule and the cosine rule for any triangle.

i. For a triangle ABC it is given that $a\cos A = b\cos B$. What can you say about triangle ABC.

ii. If for a triangle ABC $\dfrac{Sin\,(A-B)}{Sin\,(A+B)} = \dfrac{a^2-b^2}{a^2+b^2}$

Prove that the triangle is either isosceles or right angled triangle.

c) Solve the equation $\cos\theta\cos 2\theta\cos 3\theta = \dfrac{1}{4}$

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Second Term Test - Grade 12 - 2018

Index No : .....

Combined Mathematics II

Three hours only

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(10) Combined Mathematics II		
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	2	
	3	
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Percentage		

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Paper II	
Total	
Final Marks	

Final Marks

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Marking Examiner	
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Supervised by	

(Part A)

1) In the usual notation, relative to a fixed origin O the position vectors of the two points A and B are $3\underline{i} - 2\underline{j}$ and $\underline{i} - 5\underline{j}$

a) Find the angle $A\hat{O}B$

b) The position vector of C is $\lambda\underline{i} - 2\underline{j}$. If $\overrightarrow{OC}$, is perpendicular to $\overrightarrow{AB}$ find the value of λ.

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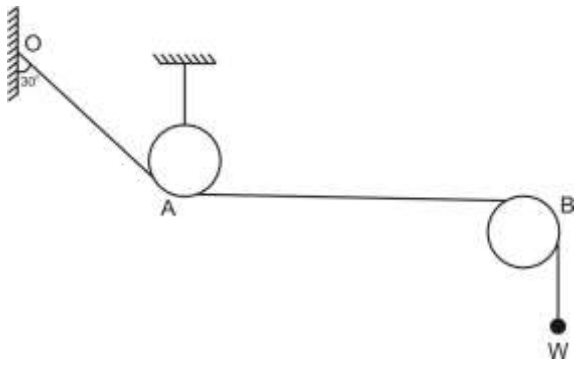
2) One end of a light inextensible string is attached to a fixed point O and passes below a fixed smooth pulley A and above a fixed smooth pulley B and a weight W is hung at the other end of it. The part AB of the string is horizontal and OA makes an angle 30^0 with the vertical. Find the magnitude and direction of the reactions on the pulleys A and B by the string in terms of W.



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3) The sum of two inclined forces is $18N$. The resultant of these two forces is a force of $12N$, which acts perpendicular to the small force. Find the magnitudes of the two forces.

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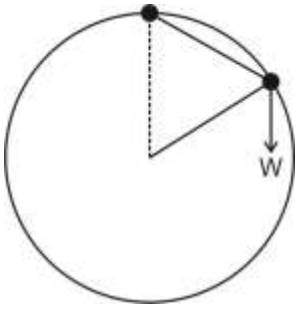
4) State the Lami's theorem for the equilibrium of three coplanar forces.
A smooth circular wire is fixed in a vertical plane and one end of a light inextensible string of length equal to the radius is attached to the highest point of the wire and the other end is attached to a light ring which is movable along the wire. A particle of weight W is hung at the ring. The system is in equilibrium like in the given figure. Show that the tension in the string and the reaction on the ring by the wire is W each.



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5) A, B and C are three points which lie in order on a straight line. The distance between A and B is $15m$. A particle P starts from rest from the point B and moves along the straight line with uniform acceleration $4\,ms^{-2}$. At the same instant, from the point A, a particle Q travels forward with a velocity $2\,ms^{-1}$ and with an acceleration $6\,ms^{-2}$. Q passes P at point C. Using a velocity time graph or using equations of motion, find the time taken for Q to pass P and the distance between A and C.

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6) A ball is dropped from rest form a point O which is vertically above a horizontal floor. After the balls hits the ground, it bounces with a velocity of $\frac{2}{5}$ of the velocity which it hits the ground and reaches a vertical height of $2.4\,m$. Find the vertical height to the point O from the horizontal floor. (let $g = 10\,ms^{-2}$)

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7) The system of forces $3\underline{i} + \underline{j}$, $2\underline{i} + 4\underline{j}$, $\underline{i} + 5\underline{j}$ and $p\underline{i} + Q\underline{j}$ act at the points $(-6,0), (-1,-4), (1,2)$ and $(2,3)$ respectively. If the system of forces reduces only to a couple of moment G, Find the values of P, Q and G.

8) Find the two unlike parallel forces which act at a distance $24\ m$ each other and the resultant of these forces equals to a force of $60\ N$. The small force among the two forces is at a distance $30m$ from the resultant.

9) ABC is an equilateral triangle with length of a side is $2a$ metres. D, E, F are mid points of the sides AB, BC and AC respectively. The forces of Newton $\sqrt{3}$, P, Q, $3\sqrt{3}, 2\sqrt{3}$ and $8\sqrt{3}$ act along the sides AB, BC, AC, BF, AE and CD in the order of the letters respectively. If the system of forces is in equilibrium, find values of P and Q.

10) The position vectors of two points A and B are $\underline{a} = 7\underline{i} + \underline{j}$ and $\underline{b} = \lambda\underline{i} - \underline{j}$. If the angle between $\underline{a}$ and $\underline{b}$ is $\cos^{-1}\left(\frac{3}{5}\right)$, find the value of λ ($\lambda > 0$). If the pints C lies on AB, such that $2AC = CB$, Find the unit vector along OC.

Combined Mathematics 12 - II (Part - B)

Answer five questions only.

11) A lift with open stage, travels vertically upwards with uniform velocity u releases an object from the lift when it is at a height h from the ground floor and the object starts to moves under gravity. By considering the upward motion, draw the velocity – time graph for the motion of the object. Hence,

i. Show that the time taken for the object to reach the maximum height after it releases from the lift is $\frac{u}{g}$

ii. Then show that the height from the floor to the object is $h + \frac{1}{2}\frac{u^2}{g}$.

iii. Show that the velocity which the object hits the ground is $(u^2 + 2gh)^{\frac{1}{2}}$.

iv. Find the total time which the object travels.

v. Show that the height from the floor to the lift, within that time interval is $\frac{u}{g}\left[\frac{gh}{u} + u + \sqrt{u^2 + 2gh}\right]$

12) a) Define the scalar product of two vectors.

The position vectors of the points A, B and C relative to the origin O are $\underline{a}$, $\underline{b}$, and $\underline{c}$ respectively. D lies on the line BC such that $DC : BC = 1:10$. Show that the position vector of D is, $\overrightarrow{OD} = \frac{1}{10}(9\underline{c} + \underline{b})$.

It is given that AD is perpendicular to BC. using the scalar product, show that

$(9\underline{c} + \underline{b}).(\underline{c} - \underline{b}) = 10\,\underline{a}.(\underline{c} - \underline{b})$

b) $\overrightarrow{OP} = \underline{p} + 2\underline{q}$, $\overrightarrow{OQ} = 3\underline{p} - \underline{q}$ and $OP \perp OQ$

Show that $\underline{p}\,.\,\underline{q} = \frac{2}{5}\,|\,\underline{q}|^2 - \frac{3}{5}\,|\,\underline{P}|^2$. If it is given that $\left|\underline{p}\right| = \left|\underline{q}\right| = 1$, find the angle between $\underline{p}$ and $\underline{q}$.

13) A system of coplanar forces consisting of 5 forces in the OXY plane with origin O is given below.

point of action	Force
$A \quad (4\underline{i})$	$5\underline{i} + \underline{j}$
$B \quad (6\underline{i})$	$3\underline{i} + 2\underline{j}$
$C \quad (3\underline{i} + 3\underline{j})$	$2\underline{i} + 3\underline{j}$
$D \quad (5\underline{i} + 3\underline{j})$	$5\underline{i} + 4\underline{j}$
$E \quad (-\underline{i} + 2\underline{j})$	$-3\underline{i} + 6\underline{j}$

Here $\underline{i}$ and $\underline{j}$ are unit vectors along the axes OX and OY respectively.

i. Express the resultant $\underline{R}$ of the system of forces in the from $\underline{R} = X\underline{i} + Y\underline{j}$. Here X and Y are to be determined. Hence find the magnitude and the direction of the resultant of the system of forces.

ii. Find the moment about the point $(2,2)$ and about the origin O. Find the direction of them also.

iii. Find the point which the lies of action of the resultant meet the X axis and hence find the equation of the line of action of the resultant.

iv. If the system of forces reduces to a couple with a single force $|\underline{R}|$ at the point $\left(-\frac{5}{2}, 0\right)$, find the moment of the couple.

14) a) The four points $A, B, C,$ and D lie on a plane such that $\overrightarrow{AB} = \underline{a}$, $\overrightarrow{BC} = \underline{b}$ and $\overrightarrow{DC} = \frac{a}{3}$, Here $\underline{a}$ and $\underline{b}$ are two non zero non parallel vectors. The point E lies on AD such that $AE:ED = 2:1$ and F lies on BC such that $BF:FC = 3:1$. Lines BE and AF intersect at G. When α and β are two scalars $\overrightarrow{AG} = \alpha\,\overrightarrow{AF}$ and $\overrightarrow{BG} = \beta\,\overrightarrow{BE}$. Find the values of α and β .

Show that $\overrightarrow{AG} = \frac{8}{13}\underline{a} + \frac{6}{13}\underline{b}$

Also find $\overrightarrow{BG}$ in terms of $\underline{a}$ and $\underline{b}$.

b) In the trapezium $ABCD$, AB and DC are parallel, $D\hat{A}B = \frac{\pi}{2}, AB = 7a\,m, DC = 4a\,m$ ç, $AD = 3a\,m$ The foot of the perpendicular drawn from C to AB is N. The forces with magnitudes $3F, 3\sqrt{2}F, 2F, 4\sqrt{3}\,F$ and $5F$ Newton's act along the sides $\overrightarrow{AB}$, $\overrightarrow{CB}$, $\overrightarrow{DC}$, $\overrightarrow{AD}$ and $\overrightarrow{ND}$ in the directions of the order of the letter respectively.

i. Find the magnitude and direction of the resultant of the system of forces and the distance from A, which the line of action of the resultant meet AB.

ii. If a couple of forces which act in the same plane is added to that system of forces such that line of action of the resultant passes through the point A, find the magnitude of that moment of the couple.

15) a) The position vectors of two points A and B relative to the origin O are $\underline{a}$ and $\underline{b}$. C is a point on AB such that $AC:CB = 1:3$. The line drawn parallel to OB through A meet the produced line OC to D.

i. Show that $\overrightarrow{OD} = \mu\,(\underline{b} + 3\underline{a})$. μ is a scalar to be determined.

ii. Obtain another linear relationship for $\overrightarrow{OD}$ in terms of $\underline{a}$ and $\underline{b}$ and hence show that $\overrightarrow{OD} = \frac{1}{3}(\underline{b} + 3\underline{a})$

iii. If E lies on the produced line AD such that $\overrightarrow{AE} = \frac{4}{3}\underline{b}$ Using the vector methods show that $ODEB$ is a parallelogram.

iv. F is a point which lies on BA such that $BF:FA = 3:4$, Show that O, E, F are collinear.

b) In the above triangle, the perpendicular drawn from the point O to the side AB and the perpendicular drawn from the point B to the side OA, meet at H. If the position vector of H is $\underline{h}$. Show that $\underline{h}\,.\,(\underline{b} - \underline{a})$ Here. Show that the line AH is perpendicular to OB.

16) a) $ABCD$ is a square of side a meters. The forces with magnitudes $5, 2, 4, 6, 6\sqrt{2}$ and $3\sqrt{2}$ Newtons act along the sides AB, BC, DC, DA, AC and BD in the direction of the order of the letters respectively. Find the magnitude and direction of the resultant by resolving the system of forces along the directions. $\overrightarrow{AB}$ and $\overrightarrow{AD}$

Find the distance from A to the point which the line of action of the resultant cut AB .

Wit this resultant If the system of forces reduces to a couple of moment $\frac{13\,a}{2}\,Nm$ in the anticlokwie direction, find the single force which should be added to the system and the distance to it from A.

b) One end of a light inextensible string is attached to a fixed point A and a weight of $3W$ is hung at the other end C of the string. A horizontal force P is applied to that weight at C. A weight of $6W$ is hung at point B which is in between A and C. AB is inclined at an angle 30^0 to the vertical. B lies below A and C lies below B. If the system is in equilibrium, and ABC is in the same vertical plane show that the inclination of the string BC to the vertical is $\frac{\pi}{3}$ and the magnitude of the force P is $3\sqrt{3}\,W$.

17) a) A motor car P passes a point A at $t = 0$ with a velocity $4u\ ms^{-1}$ and travels with the same uniform velocity. After time t = T seconds the motor car Q starts to travel from rest from the point A and travels with uniform acceleration amS^{-2} and the obtain a maximum velocity of $5u\ ms^{-1}$ Subsequently the motor car Q travels with the same uniform velocity until Q passes the motor car P at the point B . Draw the velocity time graphs for the motion of the both motor cars in the same diagram. Hence

i. Find the distance which the motor car Q travels with uniform acceleration.

ii. Show that the time which the motor car Q travels with the uniform velocity is $4T + \frac{15u}{2a}$

iii. If $AB = d\ m$

Show that $d = \frac{10u}{a}\ (2aT + 5u)$

b) A balloon starts from rest at $t = 0$ and travels vertically upwards with uniform acceleration $\frac{g}{3}\ ms^{-2}$. After $t = T\ S$ an object is released gently from it. Draw the velocity time graph for the moton till the object hits the ground from the starting point and hence show that the total time taken for it is $2T$ seconds.