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| கணிதம் | I |
| Mathematics | I |

## 09．12．2019／0830－1030

ษ๘ จद్రమి
இரண்டு மணித்தியாலம்
Two hours

| Index Number：．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．． |
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| Signature of Invigilator |

## Important：

＊This question paper consists of $\mathbf{8}$ pages．
＊Write your Index Number correctly in the appropriate places on this page and on page three．
＊Answer all questions on this question paper itself．
＊Use the space provided under each question for working and writing the answer．
＊Indicate the relevant steps and the correct units when answering the questions．
＊Marks are awarded as follows：
In Part A
2 marks for each question
In Part B
10 marks for each question
＊Blank papers can be obtained for scratch work．

| For Marking Examiners＇Use Only |  |  |  |
| :---: | :---: | :---: | :---: |
| Part | Question Numbers |  | Marks |
| A | 1－25 |  |  |
| B | 1 |  |  |
|  | 2 |  |  |
|  | 3 |  |  |
|  | 4 |  |  |
|  | 5 |  |  |
| Total |  |  |  |
| First Examiner |  | Code Number |  |
| Second | Examiner | Code Number |  |
| Arithmetic Checker |  | Code Number |  |
| Chief Examiner |  | Code | umber |

## Part A

Answer all questions on this question paper itself.

- Area of the curved surface of a right circular cylinder of radius $r$ and height $h$ is $2 \pi r h$.

1. Customs duty of $9 \%$ is charged for a certain item that is imported. If the value of this item is 6000 rupees, find the amount that has to be paid as customs duty.
2. Find the factors: $x^{2}+3 x-10$
3. Find the value of $x$ according to the information given in the figure.

4. If it is given that $\log _{2} a=5$, write the value of $a$ as a power of 2 .
5. Find the time it takes to fill a tank of capacity 420 litres using a pipe through which water flows at a rate of 60 litres per minute.
6. The points $A, B, C$ and $D$ lie on the circle shown in the figure.
$A \hat{B} C=100^{\circ}$ and $\hat{C A} B=30^{\circ}$. Find the value of $x$.

7. The base radius of a solid right circular cylinder is 7 cm . Its height is 10 cm . Find the area of the curved surface of the cylinder. (Use $\frac{22}{7}$ for the value of $\pi$.)
8. Find the gradient of the straight line represented by $A B$ in the figure.

9. Simplify : $\frac{a x}{2} \div \frac{3 a}{4 x}$
10. In the given figure, the side $C B$ of the right angled triangle $A B C$ is produced to $F$. The midpoints of $A B$ and $C B$ are $E$ and $D$ respectively. If $D \hat{E} B=50^{\circ}$, find the magnitude of $E \hat{B} F$.

11. Solve: $2 x^{2}-8=0$
12. A man takes a loan of 5000 rupees for two years at an annual interest of $8 \%$ with the interest compounded annually. How much is the interest for the second year for this loan amount?
13. A cyclic quadrilateral $A B C D$ is shown in the figure. The side $A B$ is produced to $E$. Moreover, $B \hat{C} D=120^{\circ}$ and $C \hat{B} E=70^{\circ}$. Find the values of $x$ and $y$ according to the given information.

14. The students in a certain school who study the subjects Art, Dancing and Music are represented by the pie chart. If the number of students who study Music is 45 , how many students study Dancing?

15. Express the sixth term in the geometric progression $9,27,81, \ldots$ as a power of 3 .
16. A parallelogram $A B C D$ is given in the figure.

For each statement in the table, if it is correct mark a ' $\checkmark$ ' in front of it and if it is incorrect mark a ' $x$ ' in front of it.


| (1) | Area of the triangle $A B D=\frac{1}{2} \times$ area of the parallelogram $A B C D$ |  |
| :--- | :--- | :--- |
| (2) | The diagonal $D B$ bisects $A \hat{D} C$. |  |

17. Find the least common multiple of the following three algebraic terms:
$3 x^{2}, 6 x y, 2 y$
18. The centre of the circle in the given figure is $O$. Find the value of $x$ according to the information indicated in it.

19. Two vertical pillars $A B$ and $C D$ located on a level ground are shown in the figure. When observed from $D$, the angle of elevation of $B$ is $42^{\circ}$ and the angle of depression of $A$ is $58^{\circ}$. Represent this information in the figure.

20. A bag contains 35 identical marbles. A certain number of them are white while the rest are black. If the probability of a marble picked randomly from the bag being black is $\frac{5}{7}$, how many black marbles are there in the bag?
21. Fill in the blanks in the following statement using suitable geometric terms.
"The straight line joining the centre of a circle to the of a chord of the circle is $\qquad$ to the chord".
22. Find the value of $x$ if

$$
\left(\begin{array}{rr}
3 & 1 \\
-1 & 2
\end{array}\right)\left(\begin{array}{rr}
2 & -1 \\
-1 & 1
\end{array}\right)=\left(\begin{array}{rr}
5 & x \\
-4 & 3
\end{array}\right) \text {. }
$$

23. From the group of students who are represented in the given Venn diagram, the set of students who like Mathematics is represented by $A$ and the set of students who like Science is represented by $B$. In the Venn diagram, shade the regions that represent the students who like only one of these two subjects.

24. A distance-time graph that represents the motion of a motor vehicle is shown in the figure. Which part of the graph represents the motor vehicle travelling with the greater speed? What is this speed?

25. A portion of the locus of a point that moves at a constant distance of 5 m from the given point $P$ is indicated by the arc in this sketch. The straight line $R S$ is at a distance of 7 m from $P$. Indicate by a sketch on this figure, how the points on the arc which are at a distance of 5 m from the straight line $R S$ also, are found.

$\qquad$

## Part B

Answer all questions on this question paper itself.

1. $\frac{7}{15}$ of the total length of a drain was dug on the first day and $\frac{1}{4}$ of the remaining length was dug on the second day.
(i) At the end of the first day, what fraction of the total length of the drain remained to be dug?
(ii) What fraction of the total length of the drain was dug on the second day?
(iii) At the end of the first two days, a further length of 600 metres of the total length of the drain remained to be dug. Find the total length of the drain.
(iv) It has been estimated that 4 men will require 3 days to dig the remaining 600 metres of the drain. How many more men need to be engaged to dig this length in two days?
2. A sheet consists of a portion $A B C D$ in the shape of a trapezium and a semi-circular portion with diameter $D C$, as shown in the figure. (Take the value of $\pi$ as $\frac{22}{7}$.)
(i) It has been decided to attach small buttons along the edge of the semi-circular portion, starting from $D$ and ending at $C$, such that the distance between every two adjacent buttons is 2 cm . How many buttons are required for this?

(ii) Calculate the total area of the sheet.
(iii) If a rectangular sheet is made with its area equal to the area of the semi-circular portion and its length equal to the length of $A D$, then find its breadth.
3. Kumara runs a business within an urban council limits.
(a) The assessed annual value of his business place is 40000 rupees. The urban council charges annual rates of $22 \%$.
(i) Find the amount that has to be paid annually as rates.
(ii) How much does he have to pay as rates for a quarter?
(b)

| Annual income (rupees) | Income tax percentage |  |
| :--- | :--- | :---: |
| Initial | 500000 | tax free |
| Next | 500000 | $4 \%$ |
| Next | 500000 | $8 \%$ |

Kumara pays 12000 rupees as income tax for a year, based on the above table. What is his annual income?
4. The following incomplete frequency distribution and corresponding incomplete histogram have been prepared using the marks obtained in a test by 40 students in a class. Here 10-20 denotes the marks interval "greater than 10 and less than or equal to 20 ", and the other intervals denote similarly.

| Marks | Number of students |
| :---: | :---: |
| $10-20$ | 6 |
| $20-40$ | $\ldots$ |
| $40-50$ | $\ldots$ |
| $50-80$ | 15 |
| $80-90$ | $\ldots$ |
| Total | 40 |


(i) Complete the above frequency table and histogram.
(ii) Express the number of students who obtained more than 40 marks as a percentage of the total number of students.

5．（a）It is given that the first two symbols of a certain password are two different digits from the digits $1,2,3,4,5$ ．To choose the first digit of the password，a student randomly picks a card from five identical cards on each of which one of these digits is written．Then，without replacing it，he randomly picks another card to choose the second digit．
（i）Using the symbol＇$x$＇，mark the sample space of the two digits on the cards that the student randomly picks，on the given grid．

（ii）It was later discovered that the first digit which was obtained for the password was odd and that this digit was less than the second digit that was obtained．Indicate the event that satisfies these conditions on the sample space and obtain its probability．
（b）During a period when a certain disease was spreading， 20 males and 16 females who displayed the symptoms of this disease arrived to obtain medication from a physician．Although all the females actually had the disease，the probability of a person picked at random from among the males actually having the disease was 0.6 ．
A portion of a tree diagram drawn to indicate the probabilities of a person picked at random from all these people，being a male or a female and being a person having the disease or not having it is shown below．

（i）Complete the tree diagram by indicating all the relevant probabilities．
（ii）Find the probability of a person picked at random being a person having the disease．


கல்விப் பொதுத் தராதரப் பத்திர（சாதாரண தர）ப் பரீட்ணை， 2019 டிசெம்ப்ர்
General Certificate of Education（Ord．Level）Examination，December 2019

| のூூற II <br> கணிதம் II <br> Mathematics II | 09．12．2019／1300－1610 |
| :---: | :---: |
|  セுணன்று மணேித்திய｜Tナம் Three hours |  |

Use additional reading time to go through the question paper，select the questions and decide on the questions that you give priority in answering．

## Important：

＊Answer ten questions selecting five questions from Part A and five questions from Part B．
＊Write the relevant steps and the correct units in answering the questions．
＊Each question carries $\mathbf{1 0}$ marks．
＊The volume of a sphere of radius $r$ is $\frac{4}{3} \pi r^{3}$ ．

## Part A

Answer five questions only．
1．The following notices have been published by Bank $\mathbf{A}$ and Company $\mathbf{B}$

| Bank A |
| :---: |
| An interest of $9 \%$ per annum is <br> paid for fixed deposits． |


| Company B |
| :---: |
| Price of a share is 25 rupees |
| and dividends of 1.50 rupees |
| per share is paid annually． |

（i）Kamal deposited exactly half of the amount of 100000 rupees he had in a fixed deposit in Bank $\mathbf{A}$ and spent the remaining amount in buying shares in Company B．At the end of a year he obtains the dividends from Company B and sells all the shares at 26 rupees per share．Show giving reasons，from which investment he receives a greater income at the end of a year．
（ii）Express the total income he receives at the end of a year as a percentage of the total amount invested．

2．An incomplete table showing the $y$－values corresponding to several $x$－values of the quadratic function $y=x^{2}-2 x$ ，within the interval $-2 \leq x \leq 4$ ，is given below．

| $x$ | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 8 | 3 | 0 | -1 | 0 | $\ldots$ | 8 |

（i）Find the value of $y$ when $x=3$ ．
（ii）Using the standard system of axes and a suitable scale，draw the graph of the given quadratic function on a graph paper，according to the above table of values．
（iii）Write the coordinates of the turning point of the graph．
（iv）Express the given quadratic function in the form $y=(x-a)^{2}+b$ ．Here，$a$ and $b$ are two numbers．
（v）Write the interval of values of $x$ on which the function increases within the interval $-1<y \leq 3$ ．
3. (a) There are 5 boys and 4 girls in a group of dancers. In a certain performance, all the boys wore costumes of the same price and all the girls too wore costumes of the same price. The price of two boys' costumes was 1000 rupees more than the price of three girls' costumes. The cost of the costumes of all the children in the group was 14000 rupees.
(i) Take the price of a boy's costume as $x$ rupees and the price of a girl's costume as $y$ rupees and construct a pair of simultaneous equations that represents the above information.
(ii) By solving these equations, find separately the price of a boy's costume and the price of a girl's costume.
(b) Simplify: $\frac{5 x}{x^{2}-1}-\frac{4}{x+1}$
4. A lamina consisting of a semi-circle $A$ of radius $x$ units and a sector $B$ with angle at the centre $60^{\circ}$, which is concentric with the semi-circle, is shown in the figure. If the area of $A$ and the area of $B$ are equal, show that $x$ satisfies the quadratic equation $x^{2}-4 x-8=0$ and show with reasons that $x$ can take exactly one value.
By using 1.73 for the value of $\sqrt{3}$, find an approximate value for the radius of the sector $B$, to the first decimal place.

5. A man standing at point $A$ of a level ground observes a mango tree at point $C$, a distance of 100 metres away from him on a bearing of $127^{\circ}$. He also observes a coconut tree at point $B$ which is to the south of point $A$ and to the west of point $C$.
A rough sketch of the locations of the points $A, B$ and $C$ is given in the figure.
(i) Copy the given figure onto your answer script and indicate the above information in it.
(ii) Using the trigonometric tables, find the distance $A B$ between
 the man and the coconut tree, to the nearest metre.
(iii) On the copied figure, mark the point $D$ which is exactly halfway between the man and the coconut tree, and the point $E$ which is 118 metres to the west of the coconut tree at point $B$. Using the approximate value obtained in part (ii) above for the distance $A B$, and the trigonometric tables, find the magnitude of $B \hat{D} E$.
6. Information collected on the masses of a sample of 40 bags of items that were brought to be loaded into a lorry is given in the following frequency table. Here, $0-10$ denotes the mass interval "greater than 0 and less than or equal to 10 " and the other intervals denote similarly.

| Mass of a bag of items (kg) | $0-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ | $50-60$ | $60-70$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of bags (frequency) | 2 | 5 | 7 | 9 | 8 | 6 | 3 |

(i) Find the mean mass of a bag of items according to the given information.
(ii) If the above sample has been obtained from a stock of 200 bags of items, estimate the total mass of the stock of bags of items.
(iii) If it is given that the maximum mass that can be loaded into the lorry is 1500 kg , give reasons to show that there can be instances when the above 40 bags of items cannot be loaded into the lorry.

## Part B

Answer five questions only.
7. Sunitha who hopes to participate in a sports event trains by jogging daily. She spends 105 minutes in the first week and 119 minutes in the second week for this. The time she spends training each week when taken consecutively, lie in an arithmetic progression.
(i) Find the common difference of this arithmetic progression.
(ii) Find in minutes, the time she spends training in the 7th week.
(iii) In which week does the time she spends training during a week first exceed 221 minutes?
(iv) (a) Find the total time she spends jogging during the first 10 weeks of training.
(b) If the average speed at which she jogs is $6 \mathrm{~km} \mathrm{~h}^{-1}$, find the total distance she jogs during that time.
8. Use only a straight edge with a $\mathrm{cm} / \mathrm{mm}$ scale and a pair of compasses for the following constructions.

The construction lines should be drawn clearly.
(i) Construct a straight line segment $A C$ of length 6 cm and construct the line $A B$ such that $C \hat{A} B=60^{\circ}$.
(ii) Construct the angle bisector of $C \hat{A} B$.
(iii) Construct the circle that has its centre $O$ on the above constructed angle bisector and touches $A C$ at $C$. Produce the line $A O$ such that it meets the circle at $D$.
(iv) Construct the tangent to the circle at $D$ and mark the point of intersection of this tangent and $A C$ produced as $P$.
(v) Give reasons why $D \hat{P} C=A \hat{O} C$.
9. In the given figure, $A B C$ is a triangle. $X$ and $Y$ are two points on $A B$ such that $A X=B Y$. Moreover, $P$ is a point on $A C$ such that $X P / / B C$ and $Q$ is a point on $B C$ such that $Y Q / / A C$. Copy the given figure onto your answer script and indicate the above information in it.
(i) Show that $\triangle A X P \equiv \triangle B Y Q$.
(ii) Draw the straight line $P Q$ and show that $P Q \| A B$.
(iii) The lines $P X$ and $Q Y$ produced meet at $D$. If $D X=X P$, show that $X Y=\frac{1}{2} P Q$.
10. In the cyclic quadrilateral $A B C D$ shown in the figure, $\hat{D A B}=A \hat{D} C=45^{\circ}$. The straight line drawn from $B$ perpendicular to $A D$ meets the circle at $E$. The line $P A Q$ is the tangent drawn to the circle at $A$.
Prove that $C E$ is a diameter of the circle and that it is parallel to the tangent PAQ.


11．A survey was conducted on 115 families that own pets．Information on the families that own dogs， cats and fish as pets from these families and an incomplete Venn diagram drawn corresponding to it are given below．

－ 4 families own all the above three types of pets．
－The number of families that own only dogs is 19.
－ 24 families own both dogs and cats while 21 families own both dogs and fish．
－ 11 families do not own any of the above three types of pets．
（i）Copy the given Venn diagram onto your answer script and include the above information in it．
（ii）The number of families that own dogs is twice the number of families that own fish．Find the number of families that own fish but do not own dogs．
（iii）How many families own only cats？
（iv）The number of families that own only fish is twice the number of families that own cats and fish but not dogs．Find the probability of a family selected at random from those surveyed being a family that owns only fish．

12．A hemispherical container of radius $r$ is completely filled with water． This water is poured into a glass container in the shape of a prism， having a triangular cross section with the measurements shown in the figure，such that no water spills out．Then the water fills this glass container to a height of 10 cm ．Show that the radius $r$ of the hemispherical container is obtained by $r=\sqrt[3]{\frac{180}{\pi}} \mathrm{~cm}$ ，and taking the value of $\pi$ as

3.14 ，find the value of $r$ in centimetres to the first decimal place．
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