24   Arithmetic Progressions

By studying this lesson, you will be able to
recognize arithmetic progressions and solve problems related to arithmetic
progressions.

In earlier grades, you have learned various number patterns. A number pattern
when indicated as a list is called a sequence of numbers or simply a sequence. Let
us consider the following sequence.

      3, 8, 13, 18, ...

In this sequence, the first term is 3, the second term is 8, the third term is 13 etc.
A feature of this sequence is that, considering any two consecutive terms in the
sequence, when the first term is subtracted from the second term, a constant value
is obtained. In this case, the constant value is 5.
A similar sequence is shown below.

      8, 5, 2, –1, –4, ...

In this sequence too, when the first term is subtracted from the second term, for any
pair of consecutive terms, a constant value is obtained. In this case, the constant
value is –3.

Sequences having this feature are called arithmetic progressions. The constant
value obtained by subtracting any term from the term right after that term is called
the **common difference**, and is usually denoted by d.

> An arithmetic progression is a sequence of numbers such that
> a constant value is obtained when any term is subtracted from
> the term right after that term.

The common difference d, of an arithmetic progression can be found as follows:

common difference (d) = (any term other than the first term) – (the preceding term)

1. Determine whether each of the following sequences is an arithmetic progression.

(i) 9, 11, 13, 16, ...

(ii) − 8, − 5, − 1, 2, ...

(iii) 2.5, 2.55, 2.555, 2.5555, ...

(iv) $5\frac{1}{2}$, $5\frac{3}{4}$, 6, $6\frac{1}{2}$, ...

(v) 1, −1, 1, −1, ...

2. Find the common difference of each of the following arithmetic progressions.

(i) 12, 17, 22, ...

(ii) 10, 6, 2, ...

(iii) −5, −1, 3, ...

(iv) −2, − 8, − 14, ...

(v) 2.5, 4, 5.5, ...

24.1 n^{th} term of an Arithmetic Progression

The following notation is used to denote the terms of an arithmetic progression.

$T_1 = 1^{st}$ term
$T_2 = 2^{nd}$ term
$T_3 = 3^{rd}$ term etc.
For example, for the arithmetic progression 6, 8, 10, 12, 14,...

we may write $T_1 = 6$, $T_2 = 8$, $T_3 = 10$, $T_4 = 12$, $T_5 = 14$ etc .

What is the 25th term of this progression? In other words, what is the value of T_{25}? It is clear that if you continue writing the terms according to the above pattern, the 25th term appears when you write 25 terms. If you do this, you will get 54 as the 25th term. That is, $T_{25} = 54$.

Now, if you require to find the 500th term of this progression, how would you find it? For this you would have to write down 500 terms following the pattern, which is quite a tedious task. Let us see how we can derive a formula that can be used to

find any term of an arithmetic progression rather easily.

Let us illustrate this derivation using the above arithmetic progression 6, 8, 10, 12, ... For this progression, the first term is 6 and the common difference is 2. Observe carefully how the terms of this progression have been written in terms of the first term and the common difference in the following table.

Term	Value of the term	Value of the term in terms of the first term and the common difference	
T_1	6	6	$= 6 + (1 - 1) \times 2$
T_2	8	$6 + 2$	$= 6 + (2 - 1) \times 2$
T_3	10	$6 + 2 + 2$	$= 6 + (3 - 1) \times 2$
T_4	12	$6 + 2 + 2 + 2$	$= 6 + (4 - 1) \times 2$
...	...	...	...

Now, according to the pattern in the table,

$$\begin{aligned} T_{500} &= 6 + (500 - 1) \times 2 \\ &= 6 + 499 \times 2 \\ &= 6 + 998 \\ &= 1004 \end{aligned}$$

Hence, the 500th term is 1004.

Can you generalize the above pattern further? In other words, can you find a formula for the nth term T_n in terms of the first term a and the common difference d? For this, look at the expression $T_{500} = 6 + (500 - 1) \times 2$ again, where 6 is the 1st term and 2 is the common difference.

You can see that if you follow the above pattern to obtain the nth term T_n of the arithmetic progression with first term a and common difference d, you obtain $T_n = a + (n - 1) d$. In this formula, according to our notation, T_n denotes the nth term. Hence, the nth term T_n of the arithmetic progression with first term a and common difference d is given by

$$\boxed{T_n = a + (n - 1)\, d}$$

The importance of this formula is that it gives the relationship between the four unknowns a, d, n and T_n. This formula can be used to find the value of any one of the four unknowns when the values of the other 3 unknowns in an arithmetic progression are known.

Now let us consider how problems on arithmetic progressions are solved using this formula.

Example 1 (Finding T_n when a, d and n are known)

Find the 15th term of the arithmetic progression 3, 7, 11, 15....

Here, $a = 3$, $d = 7 - 3 = 4$, $n = 15$. Substitute these values in $T_n = a + (n - 1)d$ to get

$$T_{15} = 3 + (15 - 1) \times 4$$
$$= 3 + 56$$
$$= 59$$

Therefore the 15th term is 59.

Example 2 (Finding a when d, n and T_n are known)

Find the first term of the arithmetic progression with common difference 4 and 26th term equal to 105.

Here, $d = 4$ and $T_n = 105$ when $n = 26$.

Substituting these in $T_n = a + (n - 1)\, d$ we get,

$$T_{26} = a + (26 - 1) \times 4$$
$$105 = a + (26 - 1) \times 4$$
$$\therefore 105 - 100 = a$$
$$\therefore \ a = 5$$

Therefore the first term is 5.

Example 3 (Finding d when a, n and T_n are known)

Find the common difference of the arithmetic progression with 1st term – 32 and 12th term 1.

Here, $a = -32$ and $T_n = 1$ when $n = 12$

Substituting these in $T_n = a + (n - 1)\, d$, we get

$$1 \ = -32 + (12 - 1) \times d$$
$$\therefore \ \ 33 \ = 11 \times d$$
$$\therefore \quad \frac{33}{11} = d$$

$$\therefore \ \ d \ = 3$$

Therefore the common difference is 3.

Example 4 (Finding n when a, d and T_n are known)

Find which term is – 65 in the arithmetic progression 30, 25, 10, ...

Here, $a = 30$, $d = -5$, $T_n = -65$

Substituting these in $T_n = a + (n - 1)\, d$,

$$-65 = 30 \times (n - 1) \times (-5)$$
$$-65 = 30 - 5n + 5$$
$$-65 - 35 = -5n$$

$$\frac{-100}{-5} = n$$

$$\therefore n = 20. \qquad \therefore -65 \text{ is the } 20^{th} \text{ term}$$

For an arithmetic progression, when the values of 2 unknowns out of a, d, n and T_n are not given, the values of these two unknowns can be found by solving two simultaneous equations, when sufficient information has been provided.

Example 5

The 7th and 12th terms of an arithmetic progression are 38 and 63 respectively.
   Find, (i) the first term and the common difference
            (ii) the 20th term.
(i) Since $T_n = 38$ when $n = 7$ and $T_n = 63$ when $n = 12$, we get, by substituting in
   $T_n = a + (n - 1) d,$

$$T_7 = a + (7 - 1) \times d \qquad\qquad T_{12} = a + (12 - 1) \times d$$
$$38 = a + 6d \;\underline{\qquad}\; ① \qquad\qquad 63 = a + 11d \;\underline{\qquad}\; ②$$

Now, let us solve ① and ②
② − ① gives
   $63 - 38 = a + 11d - (a + 6d)$
      $25 = a + 11d - a - 6d$
   $\therefore \; 25 = 5d$
      $\therefore \; 5 = d$
         Substitute $d = 5$, in ① to get
      $38 = a + 6 \times 5$
$\therefore \; 38 - 30 = a$
      $\therefore \quad a = 8$
$\therefore$ the first term is 8 and the common difference is 5.
   (ii) Now that we know the first term and the common difference of the
      progression, let us substitute $a = 8$, $d = 5$ and $n = 20$ in $T_n = a + (n - 1) d$.
      $T_{20} \; = 8 + (20 - 1) \times 5$
            $= 8 + 19 \times 5$
            $= 8 + 95$
            $= 103$
$\therefore$ the 20th term is 103.

Example 6

In a certain sequence, the nth term T_n is given by $T_n = 3n + 4$.
   (i) Write down the first 4 terms of this sequence.
   (ii) Write down an expression for the $(n - 1)^{th}$ term of this sequence and hence
      show that this is an arithmetic progression.
   (iii) Which term is 169 in this arithmetic progression?

(iv) Show that no term in this progression is equal to 95.

(i) We have $T_n = 3n + 4$.

When $n = 1$, $T_1 = 3 \times 1 + 4 = 7$
When $n = 2$, $T_2 = 3 \times 2 + 4 = 10$
When $n = 3$, $T_3 = 3 \times 3 + 4 = 13$
When $n = 4$, $T_4 = 3 \times 4 + 4 = 16$

∴ the first four terms are 7, 10, 13 and 16, respectively.

(ii) Replacing n by $n - 1$ in $T_n = 3n + 4$, we get

$$T_{n-1} = 3(n - 1) + 4$$
$$= 3n - 3 + 4$$
$$= 3n + 1$$
$$T_n - T_{n-1} = (3n + 4) - (3n + 1)$$
$$= 3$$
$$= \text{constant}$$

∴ the sequence is an arithmetic progression.

(iii) It is given that $T_n = 169$.

Substituting in $T_n = 3n + 4$, we get

$$169 = 3n + 4$$
$$169 - 4 = 3n$$
$$\frac{165}{3} = n$$
$$55 = n$$

∴ 169 is the 55th term.

(iv) If there is a term with value 95, then there should be a positive integer n such that $T_n = 95$.

Then, $95 = 3n + 4$
$$3n = 95 - 4$$
$$= 91$$
$$\therefore n = \frac{91}{3}$$

n is not an integer

∴ no term is equal to 95.

Exercise 24.1

1. Find the first 5 terms of the arithmetic progression for each of the following situations.

(a) $a = 5$; $d = 2$

(b) $a = -3$; $d = 4$

(c) $a = 4.5$; $d = 2.5$

(d) $a = 10\frac{1}{4}$; $d = -\frac{1}{2}$

(e) $a = 2x$; $d = x + 3$

2. Find the indicated term of each of the following arithmetic progressions.

(a) 13, 15, 17, ... (10th term)

(b) 40, 38, 36, ... (21st term)

(c) –2, –7, –12, ... (15th term)

(d) –3, 2, 7, ... (20th term)

(e) 6.5, 8, 9.5, ... (12th term)

(f) $3\frac{1}{4}$, $3\frac{1}{2}$, $3\frac{3}{4}$, ... (11th term)

(g) $12\frac{1}{2}$, 12, $11\frac{1}{2}$, ... (18th term)

3. (*a*) Using the given information, find the first term of the relevant arithmetic progression for each of the following situations.

(i) $d = 5$; $T_{21} = 101$

(ii) $a = -3$; $T_{35} = -113$

(iii) $a = 2\frac{1}{2}$; $T_{37} = 93$

(*b*) Using the given information find the common difference of the relevant arithmetic progression for each of the following situations.

(i) $a = 60$; $T_{15} = 102$

(ii) $a = -30$; $T_{35} = -25$

(iii) $a = 4\frac{1}{4}$; $T_{37} = -7\frac{3}{4}$

(*c*) For each of the following situations, find the number of terms (*n*) of the relevant arithmetic progression.

(i) $a = 9$; $d = 4$; $T_n = 69$

(ii) $a = -20$; $d = \frac{1}{2}$; $T_n = 35$

(iii) $a = 7$; $d = \frac{1}{2}$; $T_n = 27$

4. For each of the following arithmetic progressions find the nth term in the simplest form.

(i) –15, –12, –9, – 6, ...

(ii) 7, 12, 17, 22, ...

(iii) $3\frac{1}{4}$, 4, $4\frac{3}{4}$, ...

(iv) 67, 64, 61, ...

5. Find the
(i) first three terms
(ii) common difference
(iii) 15th term
for each of the arithmetic progressions with n^{th} term given by
(i) $2n + 1$ (ii) $5n - 1$ (iii) $8 - n$ (iv) $20 - 5n$

6. Between 1 and 150, how many multiples of
(i) 2
(ii) 3
(iii) 5
are there?

7. (i) In an arithmetic progression, the third term is 7 and the sixth term is 13. Find the first term of this progression.
(ii) In an arithmetic progression, the fifth term is 34 and the fifteenth term is 9. Which term in this progression is -6?
(iii) In an arithmetic progression, the fifth term is 22 and the tenth term is 47. Show that the fifteenth term is six times the third term.
(iv) In an arithmetic progression, the sum of the third and sixth terms is 42 and the sum of the second and tenth terms is 54. Which term in this progression is 63? Show further that no term in this progression is 30.
(v) In an arithmetic progression, the second term is 10 and the value of the twelvth term is 12 more than that of the tenth term. Find the first term, the common difference and the 21st term of this progression.
(vi) Which term is 52 more than the 7th term of the arithmetic progression 3, 7, 11, ...

24.2 Sum of the first *n* terms of an arithmetic progression

Consider the arithmetic progression 3, 5, 7, 9, 11, 13, 15, 17, ... in which the first 8 terms are written down. The sum of these 8 terms is
$$3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 = 80.$$
We denote by S_n the sum of the first n terms of an arithmetic progression. In this notation, the sum of the first 8 terms of the above arithmetic progression can be written as
$$S_8 = 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17$$
$$S_8 = \underline{80}$$

When we have to find the sum of a large number of terms, this lengthy way of

adding all the terms one by one is a tedious task. In order to overcome this problem, let us derive a formula that can be used to find the sum of the initial terms of an arithmetic progression.

Considering the above example, we rewrite

$$S_8 = 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 \quad\text{———①}$$

and again by reversing the terms we write

$$S_8 = 17 + 15 + 13 + 11 + 9 + 7 + 5 + 3 \quad\text{———②}$$

Now from ① and ② we may write, rearranging the terms,

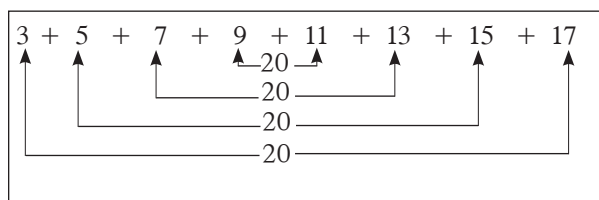$$2\,S_8 = (3 + 17) + (5 + 15) + (7 + 13) + (9 + 11) + (11 + 9) + (13 + 7) + (15 + 5) + (17 + 3)$$

$$2\,S_8 = 20 + 20 + 20 + 20 + 20 + 20 + 20 + 20$$

$$\therefore 2\,S_8 = 8 \times 20 \quad \text{(there are 8 terms of 20)}$$

$$\therefore \quad S_8 = \frac{8}{2} \times 20$$

$$= 80$$

We may illustrate the above method as follows.

There are 8 terms in the progression.

The first term and the last term add to 20.

The second and the penultimate term add to 20.

Continuing this, we see that the sum of the first 8 terms of the progression is a sum of 4 pairs.

The number of such pairs is one half the total number of terms to be added.

Then the sum of all the terms is the product,

$$\frac{\text{(number of terms)}}{2} \, (\text{first term} + \text{last term})$$

$$\therefore \quad S_8 = \frac{8}{2} [3 + 17]$$

Now, let us derive a formula using the above process, for the sum of the first n terms, S_n, of an arithmetic progression with first term a, common difference d and last (n^{th}) term l.

We may write,

$S_n = a + (a + d) + (a + 2d) + (a + 3d) + ... (l - 2d) + (l - d) + l$ ─── ①

and then reversing the terms,

$S_n = l + (l - d) + (l - 2d) + (l - 3d) + ... (a + 2d) + (a + d) + a$ ─── ②

Now ① and ② gives

$$2 S_n = (a + l) + (a + l) + (a + l) + ... (a + l) + (a + l) + (a + l)$$
$$2 S_n = n (a + l) \quad \text{[there are } n \text{ number of } a + l \text{ terms]}$$

$$\therefore \boxed{S_n = \frac{n}{2}(a+l)} .$$

As an example, let us find using the above formula, the sum of all integers from 1 to 100. The relevant progression is 1, 2, 3, 4, ...98, 99, 100, where $a = 1, l = 100$ and $n = 100$.

$$\therefore \quad S_{100} = \frac{100}{2}(1+100)$$

$$S_{100} = 50\,(101)$$
$$\therefore S_{100} = \underline{5050}.$$

$\therefore$ the sum of the integers from 1 to 100 is 5050.

The above formula can be used to find the sum of an arithmetic progression given the first term (a), the number of terms (n) and the last term (l).

Now we derive a formula for the sum, given the first term (a), the number of terms (n) and the common difference (d).

For this purpose, in the formula

$$S_n = \frac{n}{2}(a+l) ,$$

we substitute for l, which is actually T_n, the nth term given by $T_n = a + (n - 1) d$. we then get,

$$S_n = \frac{n}{2}\{a+a+(n-1)d\},$$

and this simplifies to

$$S_n = \frac{n}{2}\{2a+(n-1)d\}.$$

Therefore, to find the sum S_n of the first n terms of an arithmetic progression with first term a and common difference d, use the formula

$$\boxed{S_n = \frac{n}{2}\{2a+(n-1)d\}}$$

As an example, let us find the sum of the first 30 terms of the arithmetic progression 2, 4, 6, 8, ... In this progression, $a = 2$ $d = 2$ and $n = 30$.

Substituting in $S_n = \dfrac{n}{2}\{2a + (n-1)d\}$

we get

$$S_{30} = \dfrac{30}{2}\{2 \times 2 + (30-1) \times 2\}$$

$$= \dfrac{30}{2}\{4 + 29 \times 2\}$$

$$= \dfrac{30}{2}\{62\}$$

$$= 15 \times 62$$

$$\therefore S_{30} = 930$$

$\therefore$ the sum of the first 30 terms is 930.

In summary, we could use

★ $S_n = \dfrac{n}{2}(a + l)$ when the first term, the number of terms and the last term are known

★ $S_n = \dfrac{n}{2}\{2a + (n-1)d\}$ when the first term, the number of terms and the common difference are known

Let us look at some examples now.

Example 1   Find the sum of the first 12 terms of the arithmetic progression 5, 10, 15, 20, ...

Here $a = 5$, $d = 5$ and $n = 12$. Substituting these in $S_n = \dfrac{n}{2}\{2a + (n-1)d\}$,we get,

$$S_{12} = \dfrac{12}{2}\{2 \times 5 + (12-1) \times 5\}$$

$$= \dfrac{12}{2}\{10 + 11 \times 5\}$$

$$= 6\ \{10 + 55\}$$

$$= 6 \times 65$$

$$= 390$$

$\therefore$ the sum of the first 12 terms is 390.

Example 2   An arithmetic progression has 16 terms. The first term and the last term are 75 and 0 respectively. Find its sum if the common difference is – 5.

Here $n = 16$, $a = 75$, $d = -5$, and $l = 0$.

Substituting in $S_n = \dfrac{n}{2}(a+l)$,

$$S_{16} = \dfrac{16}{2}(75+0)$$

$$= \dfrac{16}{2} \times 75$$

$$= 8 \times 75$$

$$= 600$$

∴ the sum of the arithmetic progression is 600.

Example 3   Find the sum of the arithmetic progression 70, 66, 62, 58, ..., 2

Here $a = 70$, $l = 2$, and $d = -4$.
First we have to find the number of terms in the progression.
Using the formula $T_n = a + (n-1)d$, we get,

$$2 = 70 + (n-1) \times (-4)$$

$$2 = 70 - 4n + 4$$

$$2 - 74 = -4n$$

$$\dfrac{-72}{-4} = n$$

$$18 \quad = n$$

The progression has 18 terms and its sum can be found by using the formula

$$S_n = \dfrac{n}{2}(a+l).$$

$$S_{18} = \dfrac{18}{2}(70+2)$$

$$= \dfrac{18}{2} \times 72$$

$$= 9 \times 72$$

$$= 648$$

∴ the sum of the terms of the series is 648.

The formula $S_n = \dfrac{n}{2}(a+l)$ is in 4 unknowns, namely S_n, n, a and l. Whenever three of these unknowns are given, the remaining unknown can be found by using the formula. The case for the formula $S_n = \dfrac{n}{2}\{2a+(n-1)d\}$ is similar.

Let us look at some examples to illustrate such situations.

Example 4

The first term, the last term and the sum of all the terms of an arithmetic progression are 12, 99 and 1665 respectively. Find the number of terms, the common difference and the sum of the first 15 terms.

Here $a = 12$, $l = 99$, $S_n = 1665$.

We use $S_n = \dfrac{n}{2}(a+l)$ to find n. Substitution gives

$$1665 = \dfrac{n}{2}(12+99)$$
$$3330 = n \times 111$$
$$\dfrac{3330}{111} = n$$
$$30 = n$$

$\therefore$ there are 30 terms in the progression.

Substituting,

$T_n = l = 99$, $a = 12$ and $n = 30$ in the formula $T_n = a+(n-1)d$,

$99 = 12+(30-1)d$

$99-12 = 29d$

$$d = \dfrac{87}{29}$$
$$= 3$$

$\therefore$ the common difference is 3.

Finally, in order to find the sum of the first 15 terms, substitute $n = 15$, $a = 12$, $d = 3$ in $S_n = \dfrac{n}{2}\{2a+(n-1)d\}$ to get

$$S_{15} = \frac{15}{2}\{2\times12+(15-1)\times3\}$$

$$= \frac{15}{2}\{24+14\times3\}$$

$$= \frac{15}{2}\{24+42\}$$

$$= \frac{15}{2}\{66\}$$

$$= 15\times33$$

$$S_{15} = \underline{495}$$

$\therefore$ the sum of the first 15 terms is 495.

Example 5

How many terms (starting from the 1st term) of the arithmetic progression 13, 11, 9, ..., add to 40?

Substitute $a = 13$, $d = -2$, $S_n = 40$.

$$S_n = \frac{n}{2}\{2a+(n-1)d\}$$

$$40 = \frac{n}{2}\{2\times13+(n-1)\times(-2)\}$$

$$80 = n\{26-2n+2\}$$

$$80 = 28n-2n^2$$

$$2n^2-28n+80 = 0$$

$$n^2-14n+40 = 0$$

$$(n-10)(n-4) = 0$$

$$n-10=0 \text{ or } n-4=0$$

$$n=10 \text{ or } n=4.$$

Note:

Here there are two acceptable solutions for n.

Sum of the first four terms when n is 4 $= 13 + 11 + 9 + 7 = 40$

Sum of the first ten terms when n is 10 $= 13+11+9+7+5+3+1+-1+-3+-5$

$$= \underline{40}$$

Here, both values are acceptable. Therefore 4 or 10 terms can be taken for the sum to be 40.

Exercise 24.2

1. For each of the following cases, find the sum of the relevant arithmetic progression using the given data.
  (i) $a = 2$, $l = 62$ and $n = 31$
  (ii) $a = 95$, $l = 10$ and $n = 12$
  (iii) $a = 7\frac{1}{2}$, $d = \frac{1}{2}$ and $n = 15$
  (iv) $a = 3.25$, $d = 1.7$ and $n = 21$

2. Find the sum of the indicated number of terms in each of the following progressions
  (i) 3, 7, 9, ... first 11 terms
  (ii) -10, -9.7, -9.4, ... first 20 terms
  (iii) 1, $1\frac{3}{4}$, 2.5, ...   first 17 terms
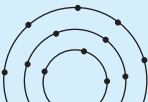 (iv) 67, 65, 63, ...   first 12 terms

3. (i) Find the number of odd numbers between 2 and 180 and then find their sum.
  (ii) Find the number of positive terms divisible by 5 below 200 and then find their sum.
  (iii) Find the number of terms between 3 and 200, whose remainder upon division by 4 is 1, and then find their sum.
  (iv) Find the sum of the terms between 5 and 170 which are not divisible by 3.

4. The sum of the first 4 terms of an arithmetic progression is 36 and the 11th term is 43. Find the first term and the common difference of this progression and also find the sum of the first 15 terms.

5.   The figure shows the way some small light bulbs have been connected in a circular pattern in the first three rings of a decoration. The last ring in this decoration contains 35 bulbs.
  (i) How many rings are there in the decoration?
  (ii) How many bulbs have been used in total?
 (iii) If one bulb costs 50 rupees, how much money was spent on the bulbs in total?

6. The installments per month and the number of months interest needs to be paid for a loan of Rs. 50 000 taken from two financial institutions P and Q are as given below:
 P: 11 000, 10 000, 9 000, … for 11 months
  Q: 14 000, 15 000, 16 000, … for 8 months
  To which institution does one have to pay less interest? Give reasons.

7. A father opens a bank account for his daughter by depositing Rs. 500 on her tenth birthday. Every month, he deposits in that account an amount of money equal to the amount he deposited in the previous month plus a constant amount. Find the constant amount of money the father should deposit so that the total amount in the account, without interest, at her 18th birthday is Rs.504 000.

8. The nth terms of an arithmetic progression is given by $T_n = 63 - 2n$.
    (i) Write down the first four terms.
    (ii) Find the sum of the first 21 terms.
    (iii) Find the 21st term.
    (iv) Find the number of terms that should be added to get a sum of 336.

9. Find the number of terms of an arithmetic progression needed to get the indicated sum in each of the following cases:
    (i) $a = 7$, $l = 10$, $S_n = 34$
    (ii) $a = 63$, $d = 3$, $S_n = 345$

Miscellaneous Exercises

1. In a shop, bars of soap are stacked on top of each other on a rack in such a way that the bottom row has 24 bars, the row above that has 21 bars, and the row above that has 18 bars and so on.

    (i) Find the number of bars in the 8th row from the bottom.
    (ii) If the top row has 3 bars of soap, find the total number of rows and the total number of bars of soap.
    (iii) If the bars of soap are 5 cm wide, find the minimum height the rack should be to enable all the rows of soap to be placed on it.

2.
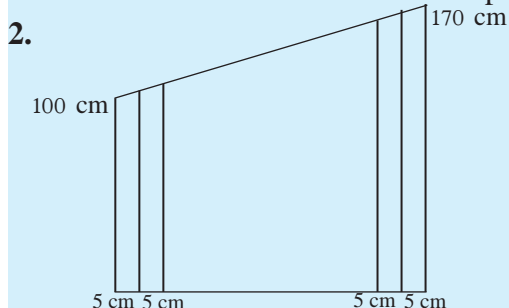
The figure shows a sketch of one part of a gate having two equal parts, which is made by fixing planks of wood together. Each plank is 5 cm wide. The shortest plank is 100 cm long and each plank fixed after the shortest is 5 cm longer than the previous plank. The longest plank is 170 cm long.

    (i) Find the number of planks used for one part of the gate.
    (ii) Find the minimum width of the gate.

(iii) Find the total length of all the planks used for the gate.

(iv) If one 30 cm long plank costs Rs. 50, then find the total cost of planks required for the gate.

3. The sum of the first n terms of a series is given by $S_n = n^2 - 8n$.

(i) Write the first term.

(ii) Find the sum of the first two terms.

(iii) Find the common difference.

(iv) How many terms, starting from the first term, add up to 180?

4. The pages numbered 3, 5, 7,...of a magazine are printed in a special pink colour. Thushan reads 5 pages of the magazine on the first day and thereafter, on each day, he reads 3 more pages than he read on the previous day.

(i) Find the number of pages he completes reading at the end of the fifth day.

(ii) Find the number of pages he completes reading at the end of the seventh day.

(iii) If Thushan finishes reading the whole magazine in 10 days, find the number of pages there are in the magazine.

(iv) What is the maximum number of pink pages the magazine can have?

(v) Thushan claims that the last page he read is pink. Determine the truth of his statement.