32 Scale Diagrams

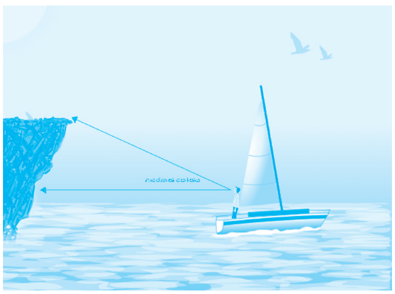
By studying this lesson, you will be able to

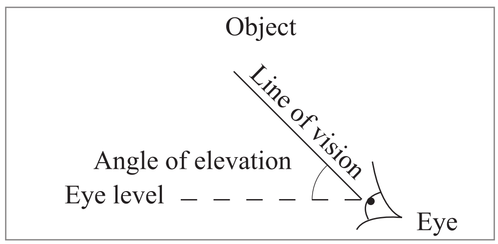
solve problems related to scale diagrams in a vertical plane.

32.1 Scale Diagrams

In the earlier grades, you used bearings and distances to indicate the position of a point on a horizontal plane.
In this lesson, you will learn how to find the position of a point on a vertical plane by drawing a scale diagram using angles of elevation and angles of depression.

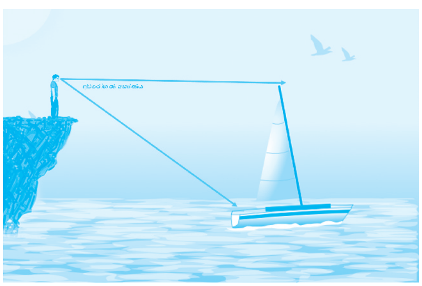
Angle of Elevation

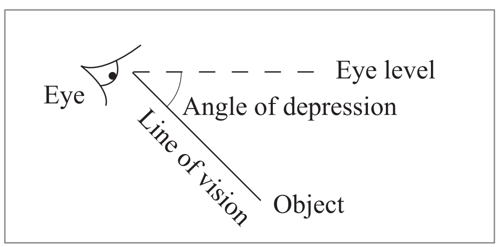




An angle of elevation is defined as the angle formed between the line of vision and the eye level (horizontal level) of an observer when the observer is looking at an object above the eye level.

Angle of Depression





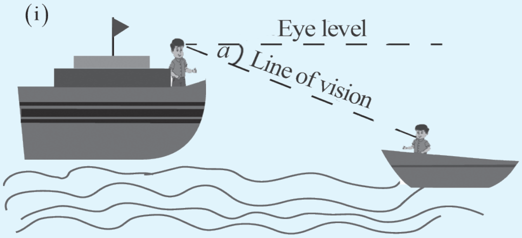
An angle of depression is defined as the angle formed between the line of vision and the eye level (horizontal level) of an observer when the observer is looking at an object below the eye level.

Note: Angles of elevation and angles of depression are always made with the horizontal level.

Exercise 32.1

1. Write down whether the angle marked with a letter in each picture is an angle of elevation or an angle of depression.

(i)



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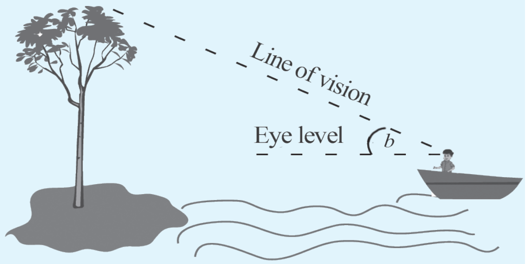

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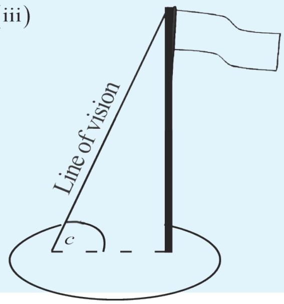

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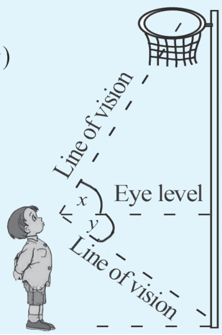
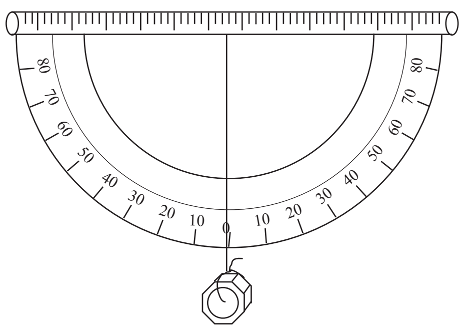
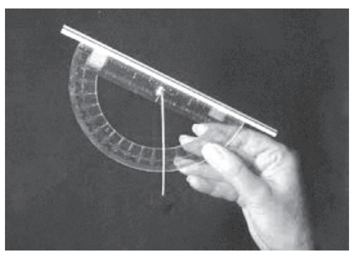


32.2 Clinometer

When stating the position of an object in a vertical plane, we need to state the magnitude of the angle of elevation or depression. We may use a clinometer for the purpose of measuring these angles.





A simple clinometer can be made in the classroom as follows:

- Cut out a semicircle with a radius of about 10 cm from a piece of cardboard.
- Mark both ends of the curved edge as 90° and the middle of the curved edge as 0°.
- Calibrate the curved edge in both directions from 0°, 10° by 10° as shown in the figure.
- Fix a straw along the straight edge of the semicircle.
- Attach a small weight to one end of a piece of string longer than 10 cm and fix the other end to the center of the semicircle.

When the straw is horizontal, the string goes through 0°. When the straw is inclined at an angle of 45°, for example, the string goes through 45°, in which case the straw shows a 45° vertical inclination.

Angles of elevation and depression can be measured in this manner using a clinometer.

Exercise 32.2

1. From a suitable position find the angle of elevation of each of the points given below using the clinometer you made.

(i) The top of the flagpole at your school.

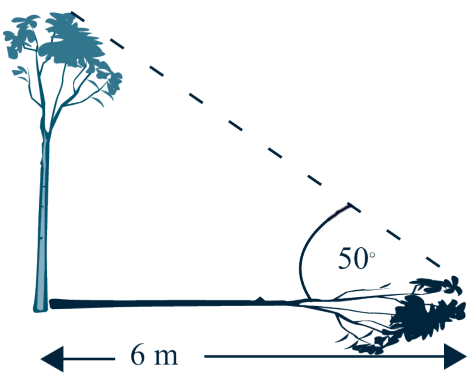
(ii) The top of a building.

(iii) The top of a tree in your school.

32.3 Scale diagrams in a vertical plane

Now let us consider several instances when scale diagrams are used to represent information in a vertical plane.

The figure below shows a tree and its shadow on the ground. Let us draw a scale diagram using the given information and hence find the height of the tree.

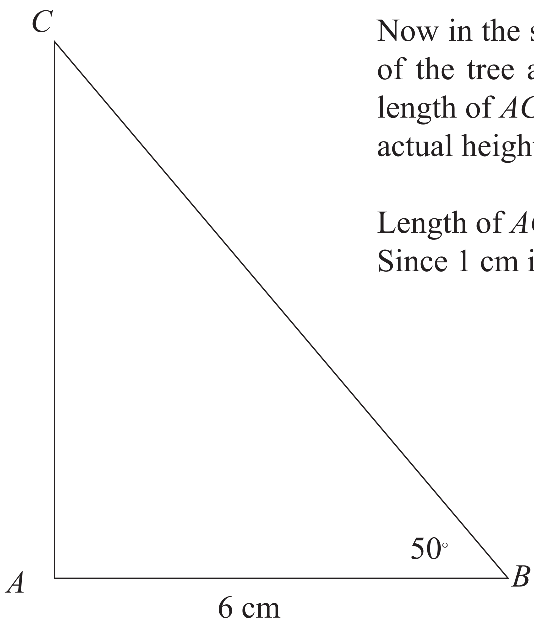


First we choose a suitable scale. Let 1 cm in the scale diagram represent an actual distance of 1 m.

In other words, let 1 cm represent 100 cm.

Therefore, the scale is 1:100.

Accordingly, we need to draw a line segment of 6 cm to represent 6 m. Let us represent this by a horizontal line segment AB, (see the figure below). Now let us draw an angle of 50° at B and complete the triangle ABC such that $B\hat{A}C = 90^\circ$ as shown in the figure.



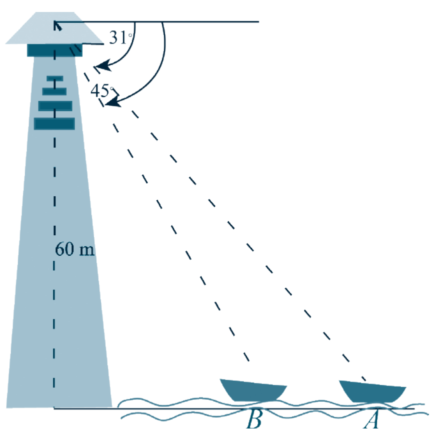
Now in the scale diagram, AC represents the height of the tree and you can measure and find that the length of AC is 7.2 cm. Hence, we can calculate the actual height of the tree as follows:

Length of AC = 7.2 cm.
Since 1 cm in the scale diagram represents 100 cm,

$$\begin{aligned} \text{height of the tree} &= 7.2 \text{ cm} \times 100 \\ &= 720 \text{ cm} \\ &= \underline{\underline{7.2 \text{ m}}} \end{aligned}$$

Example 1

From a sixty meter tall lighthouse a boat A is observed at sea with an angle of depression of 31° and another boat B with an angle of depression of 45° (see the figure). The two boats and the lighthouse are in the same vertical plane. Draw a scale diagram depicting the above information and find the distance between the boats A and B.



First let us draw a sketch diagram using the given information. Next let us choose a suitable scale. Let us represent 10 m by 1 cm.
Since 1 m = 100 cm, In the chosen scale, 1 cm represents 1000 cm.

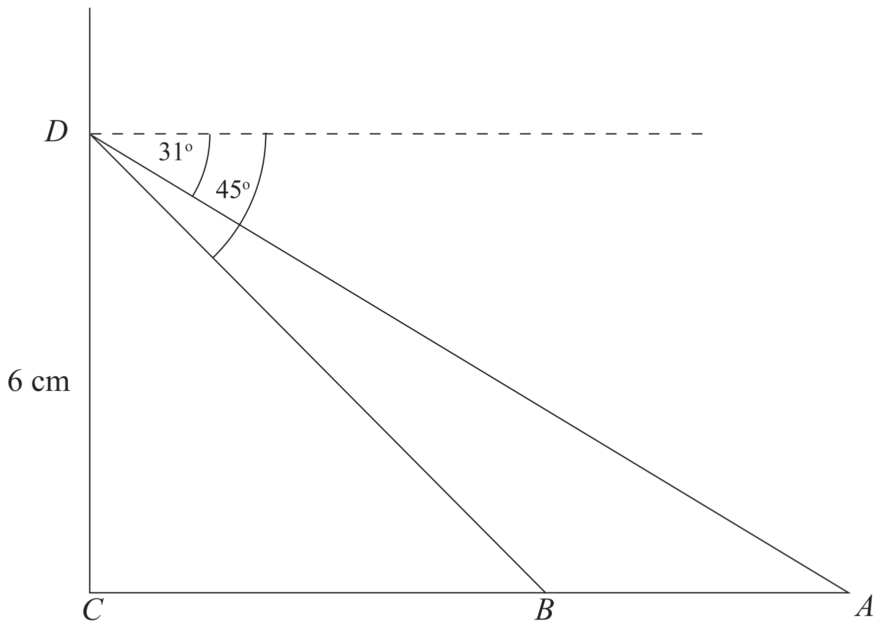
$\therefore$ the scale is 1: 1000.

Note: When drawing scale diagrams involving objects at great distances, a man's height being comparatively small can be ignored.

According to the scale, a 6 cm line must be drawn to represent the lighthouse's height. Denote this line by *CD*.

Now, let us draw the scale diagram.

- First draw a vertical line segment of length 6 cm and label it *CD*.
- Draw two lines perpendicular to *CD* at *C* and *D*.
- Draw an angle of depression of 31° at *D*. Extend this line to the point where it meets the line perpendicular to *CD* at *C* and label this point as *A*.
- Draw another angle of depression of 45° at *D*. Extend this line to the point where it meets the line perpendicular to *CD* at *C* and label this point as *B*.
- Now measure the length of *AB*. It should be 4 cm.



The distance between the two boats $= 4 \times 1000$ cm
$= 4000$ cm
$= \underline{\underline{40 \text{ m}}}$.

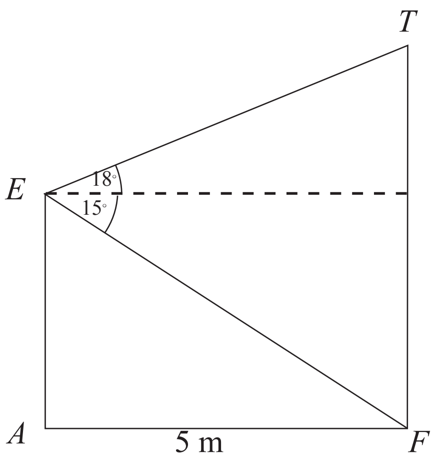
Example 2

In a horizontal playground, Dilini is standing at the location A, 5 m away from a netball goal post.

She can see the top of the goal post T, with an angle of elevation of 18° from her eye level E. She can see the base of the goal post F, from the same position with an angle of depression of 15°. Draw a scale diagram and find Dilini's height and the height of the goal post.

When a diagram is not given, it is best to draw a sketch diagram prior to drawing the scale diagram.

Sketch diagram:



Now, we have to choose a suitable scale for the scale diagram.
Let us take 2 cm to represents 1 m.
Then, 2 cm represent 100 cm.
$\therefore$ 1 cm represents 50 cm.
$\therefore$ the scale is 1: 50.
If 1 m is represented by 2 cm, 5 m is represented by 10 cm.

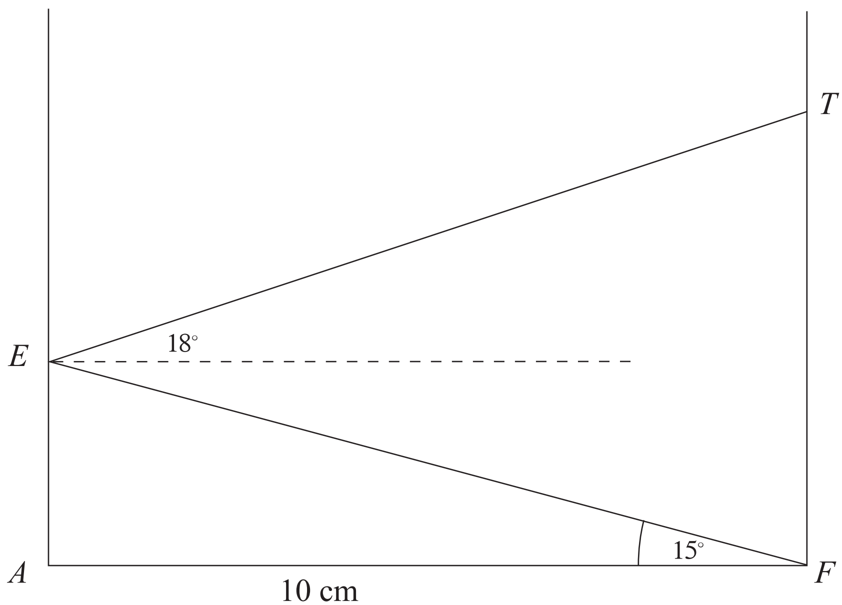
Note: Because the distance between the human and the pole is not large, we get a more accurate answer by taking the man's height into consideration when drawing the scale diagram.

Now let us draw the scale diagram.

- Because the distance between A and F is 5 m, draw a line segment of length 10 cm and label the two ends as A and F.
- Then, draw two lines at A and F, each perpendicular to AF.

- Because the point E is not yet determined, we cannot draw the angle of elevation or depression at E at once. Instead, we first find E as follows. Observe that $A\hat{F}E = 15^\circ$ because $A\hat{F}E$ and the angle of depression at E are alternate angles. Now draw an angle $A\hat{F}E$ of magnitude 15° at F such that E is on the line drawn perpendicular to AF at A.
- Now that we know point E, draw a line at E perpendicular AE.
- Draw the angle of elevation of 18° with the line drawn in the previous step. Label the point T which is the intersection point of the line of sight of this angle of elevation and the line drawn perpendicular to AF at F, as shown in the figure.
- Dilini's height is represented by AE and the height of the goal post is represented by TF.

Scale Drawing:



According to the scale diagram,

$AE = 2.6$ cm

$\therefore$ Dilini's height $= 2.6 \times 50$ cm

$= 130.0$ cm

$= \underline{\underline{1.3 \text{ m.}}}$

Also, from the scale diagram,

$TF = 6$ cm

$\therefore$ the height of the goal post $= 6 \times 50$ cm

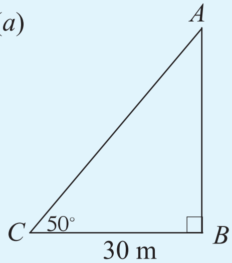
$= 300$ cm

$= \underline{\underline{3 \text{ m}}}.$

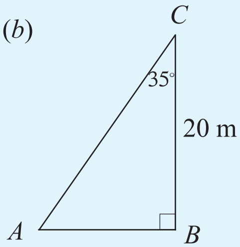
Exercise 32.3

1. Draw scale diagrams according to the given information and find the length *AB*.

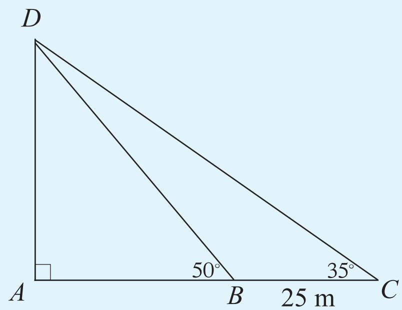
(*a*)



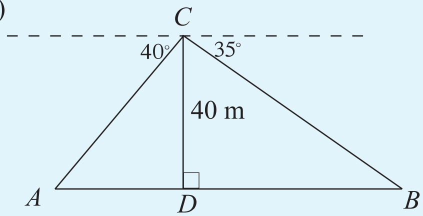
(*b*)



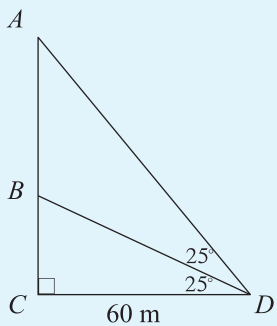
(*c*)



(*d*)



(*e*)



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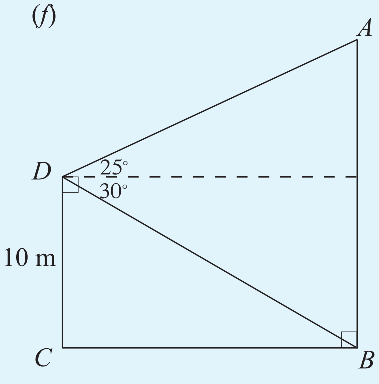

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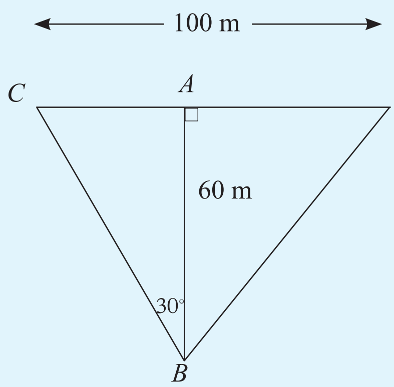
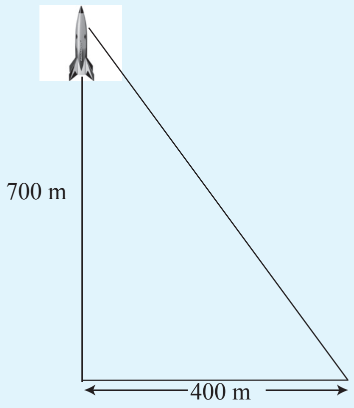


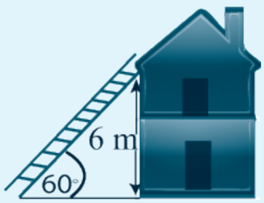
2. A person observes a rocket from a point 400 m horizontally away from the launching pad when the rocket has travelled 700 m vertically up from the launching pad. Using a scale diagram, find the angle of elevation of the rocket.



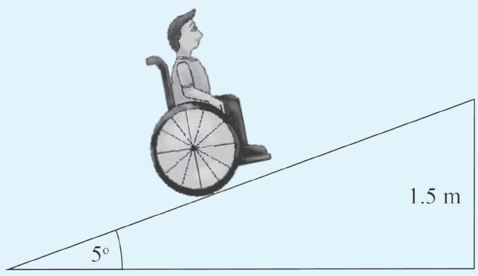
3. A ladder leaning against a wall is shown in the figure. Draw a scale diagram using the given information and find

(i) the length of the ladder and

(ii) the distance from the foot of the ladder to the wall.



4. A ramp for the use of wheelchairs to access a building is shown in the figure. Draw a suitable diagram using the given information and find the length of the ramp.

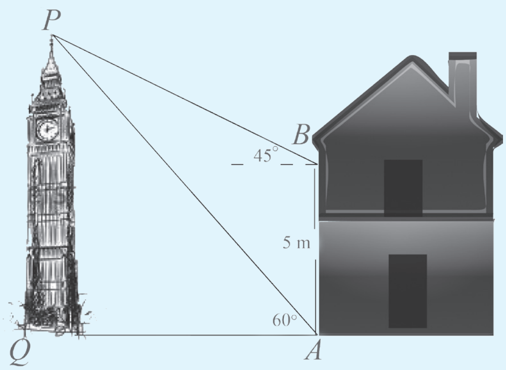


5. The mathematics teacher Mr. Weerasekera asked Chanuka to find the height of an inaccessible mango tree. Chanuka finds, using his clinometer, that the angle of elevation of the highest point P of the tree from a point A on the horizontal ground is 30° and that the angle of elevation of the same point P from a point B which is 10 m closer to the tree than A is 40°. The points A, B and the mango tree are in the same vertical plane. Find, the height of the mango tree by drawing a suitable scale diagram. (Ignore Chanuka's height).

6. Mr. Pieris observes that the angle of elevation to the top of a coconut tree from the upper floor balcony of his house is 40°. The distance from his house to the coconut tree is 6 m. Find the minimum length of a picking pole he could pluck coconuts with from the upper floor balcony.

7. The prefect Sithira was assigned the task of making arrangements for hoisting the national flag at the Independence Day celebration. Sithira needed to find the height of the flag pole. Form the upper floor of a building 10 m from the flagpole, using his clinometer, he observed the top of the flagpole with an angle of elevation of 20° and the bottom of the flagpole with an angle of depression of 50°. Draw a scale diagram using this information and find the height of the flag pole to the nearest metre.

8. The top, P, of a clock tower situated on a horizontal ground has an angle of elevation of 60° from a point A at the brink of a building. The angle of elevation of P from a point B in the building which is 5m directly above the point A, is 45° (see figure on the right). Using a suitable scale diagram, find the height of the clock tower and the distance from A to the foot Q of the clock tower.



9. An observer who stands on a horizontal ground 3m away from a bell tower, observes the top of a bell tower with an angle of elevation of 60° and the bottom of the clock tower with an angle of depression of 25°. Using a suitable scale diagram, find the height of the clock tower and the height of the observer.