25 Algebraic Inequalities

By studying this lesson you will be able to

- solve inequalities and representing the solutions on a number line,
- represent inequalities on a coordinate plane.

Let us recall what has been learnt earlier about inequalities by considering the following examples.

Example 1

Solve the inequality $x + 20 > 50$ and

(i) write down the set of integral values that x can take.

(ii) represent the integral values that x can take on a number line.

$$x + 20 > 50$$
$$x > 50 - 20$$
$$x > 30$$

(i) $\{31, 32, 33, 34, ...\}$

(ii)

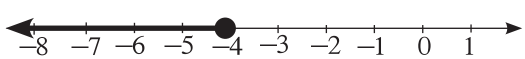
Example 2

Solve the inequality $-3x \geq 12$ and represent all the values that x can take on a number line.

$-3x \geq 12$ (When an inequality is divided by a negative number, the inequality sign changes)

$$\frac{-3x}{-3} \leq \frac{12}{-3}$$

$$x \leq -4$$



Review Exercise

1. Solve each of the following inequalities.

(i) $x+4>11$ (ii) $y+3\geq 0$ (iii) $p-5<2$ (iv) $p-3>-1$

(v) $a+5\leq 1$ (vi) $5y<12$ (vii) $-2x\geq 10$ (viii) $-3y<-9$

(ix) $\frac{-2x}{3}>6$

2. Solve each of the following inequalities and represent the solutions on a number line.

(i) $x+3\geq 1$ (ii) $y-4<-1$ (iii) $3x>-3$

(iv) $\frac{x}{2}\leq 0$ (v) $-5y>10$ (vi) $-4x\geq 12$

3. For each of the following inequalities, one of the values of x which satisfies the inequality is given within the brackets. Select that value and underline it.

(i) $x+3>7\ (4,7)$ (ii) $x-3<2\ (1,6)$ (iii) $3x>7\ \left(2.3,\frac{8}{3}\right)$

(iv) $-2x<8\ (-5,3)$ (v) $5-x>6\ (12,-2)$

4. (i) Solve the inequality $x+1>-2$ and write down the smallest integral value that x can take.

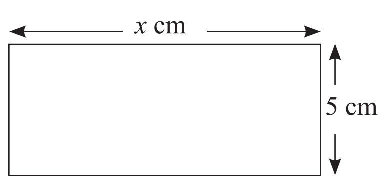
(ii) Solve the inequality $-3y>15$ and write the largest integral value that y can take.

5. Solve the inequalities $x+3>1$ and $2x\leq 12$ and represent all the solutions on a number line.

25.1 Inequalities of the form $ax+b\gtreqless c$

Example 1

Nimal who constructed a rectangular structure of breadth 5 cm as shown in the figure using a 30 cm long piece of wire, saved a small piece of the wire.



If the length of the rectangle is taken as x, an inequality in terms of x, involving the perimeter of the rectangular structure is given by $2x+10<30$. On a number line, represent all possible values that x can take if $x>5$.

$$2x + 10 < 30$$
$$2x + 10 - 10 < 30 - 10$$
$$2x < 20$$
$$\frac{2x}{2} < \frac{20}{2}$$
$$x < 10$$

Example 2

Solve the inequality $3 - 2x \leq 9$ and on a number line, represent all the possible values that x can take.

$$3 - 2x \leq 9$$
$$\cancel{3} - 2x - \cancel{3} \leq 9 - 3$$
$$-2x \leq 6$$
$$\frac{-2x}{-2} \geq \frac{6}{-2}$$
$$\underline{\underline{x \geq -3}}$$

Exercise 25.1

1. Solve each of the following inequalities.

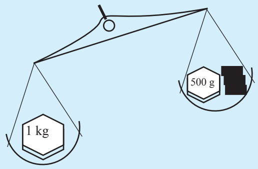
(i) $4x + 1 > 5$ (ii) $5x - 3 < 7$ (iii) $3 + 2p \geq 1$ (iv) $7x + 9 < -5$

(v) $-2y - 5 > 1$ (vi) $3 - 4x \geq 3$ (vii) $8 - 4y < 0$ (viii) $2(3 - x) > 10$

2. Solve each of the following inequalities and write down the set of integral solutions of the inequality.

(i) $5x + 1 > -4$ (ii) $3y - 1 \geq 2$ (iii) $-2p - 4 < 0$ (iv) $7 - 4p > 3$

3. Rs. 100 is sufficient to buy 3 mangoes and 2 mandarins. If the price of a mango is Rs. 20 and the price of a mandarin is taken to be Rs. y, then an inequality $60 + 2y \leq 100$ in y can be written. Solve this inequality and find the maximum value that the price of a mandarin can take.

4. 

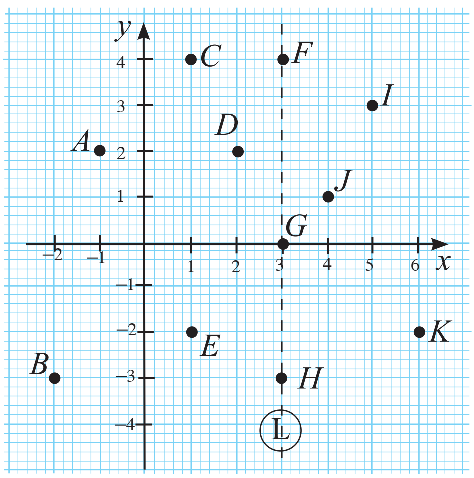
Nimal who placed a 1 kg standard weight in one pan of a balance scale, placed a 500 g standard weight and three cakes of soap of the same type on the other pan. He observed that the pan with the 1 kg standard weight dipped below the other pan.

If the mass of a cake of soap is taken as p grammes, an inequality $1000 > 500 + 3p$ in terms of p can be written. Find the maximum integral value that the mass of a cake of soap can be.

25.2 Regions represented by inequalities of the form $y \gtreqless a$ and $x \gtreqless b$

Regions which are separated by a line parallel to the y - axis

The points $A, B, C, D, E, F, G, H, I, J$ and K and the straight line L drawn parallel to the y axis are represented on the Cartesian plane given in the figure.



Consider the following tables and the related properties.

Points lying on the line L	**x coordinate**	**y coordinate**
F	3	4
G	3	0
H	3	–3

- The x coordinate of the points that lie on the line L is equal to 3.
- Therefore the straight line L is named $x = 3$.
- The x coordinate of any point that lies on the straight line $x = 3$ is equal to 3.

Points lying to the right of the line (L)	***x* coordinate**	***y* coordinate**
I	5	3
J	4	1
K	6	–2

- The x coordinate of each of the points that lie to the right of the straight line (L) is greater than 3.
- Therefor the region to the right of the straight line (L) is named $x > 3$.
- The x coordinate of any point that belongs to the region $x > 3$ is greater than 3.

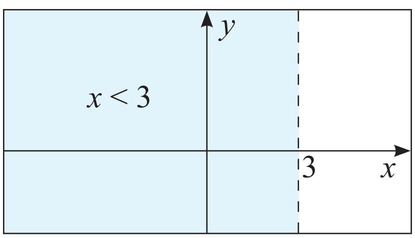
Points lying to the left of the line (L)	***x* coordinate**	***y* coordinate**
A	–1	2
B	–2	–3
C	1	4
D	2	2
E	1	–2

- The x coordinate of the points that lie to the left of the straight line (L) is less than 3.
- Therefore the region to the left of the straight line (L) is named $x < 3$.
- The x coordinate of any point that belongs to the region $x < 3$ is less than 3.

It is clear that the Cartesian plane illustrated in the above example is divided into three specific regions, namely $x < 3$, $x = 3$ and $x > 3$ by the straight line $x = 3$.

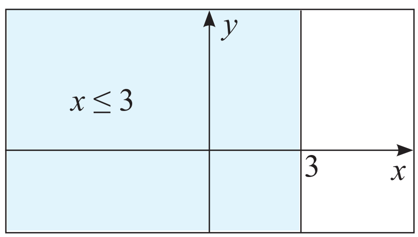
Now let us see how these regions are represented in a Cartesian plane.

The region $x < 3$



The straight line $x = 3$ is indicated by a dashed line. This means that the points such that $x = 3$ do not belong to the region $x < 3$.

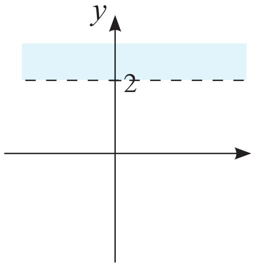
The region $x \leq 3$



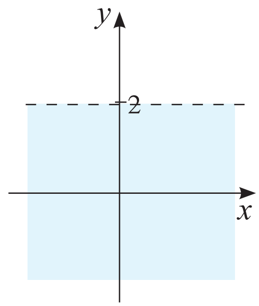
The line $x = 3$ has been indicated by a thick line. This means that both the regions $x < 3$ and $x = 3$ belong to the shaded region. Therefore this region is named $x \leq 3$.

A few more examples to illustrat the regions on a Cartesian plane separated by a line parallel to the x axis are given below.

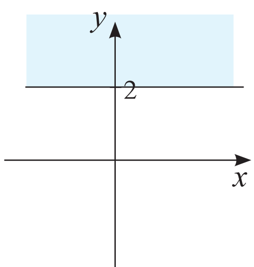
$y > 2$



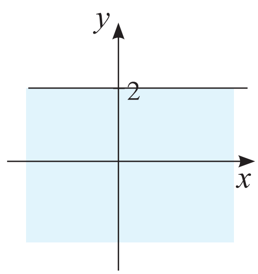
$y < 2$



$y \geq 2$



$y \leq 2$



Exercise 25.2

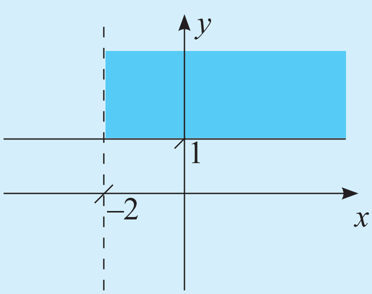
1. Write down the coordinates of three points belonging to the region $x < -2$.

2. Write down the coordinates of three points belonging to the region $x > -1$.

3. Write down the coordinates of three points belonging to both the regions $x > 1$ and $y < -2$.

4. Which of the following points belong to both the regions $x \leq -2$ and $y > 0$.
$A = (-3, 0)$ $B = (-2, 1)$ $C = (-1, 4)$

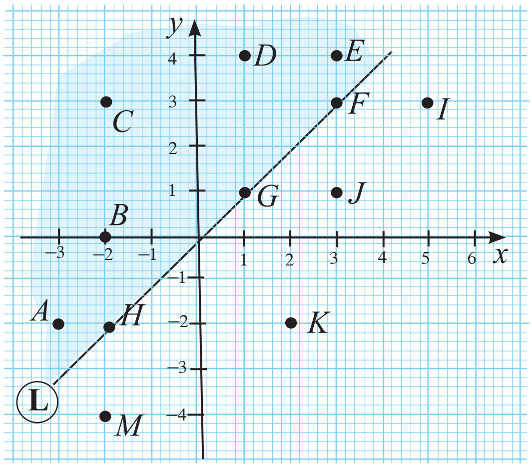
5. Write the two inequalities relevant to the shaded region.



6. In a Cartesian plane, shade the region satisfying the four inequalities $x > 1$, $x \geq 3$ $y \leq 2$, and $y > -1$.

25.3 Inequalities of the form $y \gtreqless x$

The points *A, B, C, D, E, F, G, H, I, J, K* and *M* and the straight line (L) are represented on the Cartesian plane given in the figure.



Points lying on the line (L)	x coordinate	y coordinate
F	3	3
G	1	1
H	-2	-2

- The y coordinate of each of the points that lie on the line (L) is equal to the corresponding x coordinate.
- Therefore the line (L) is named $y = x$.

Points belonging to the shaded region	x coordinate	y coordinate
A	-3	-2
B	-2	0
C	-2	3
D	1	4
E	3	4

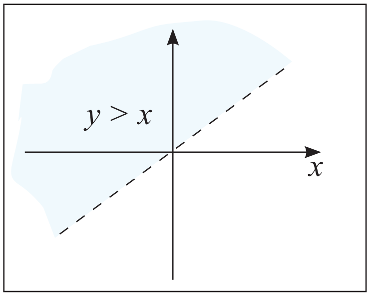
- The y coordinate of each of the points in the shaded region is greater than the corresponding x coordinate.
- Therefore the shaded region is named $y > x$.

Points belonging to the unshaded region	x coordinate	y coordinate
I	5	3
J	3	1
K	2	–2
M	–2	–4

- The y coordinate of each of the points in the unshaded region is less than the x coordinate.
- Therefore the unshaded region is named $y < x$.

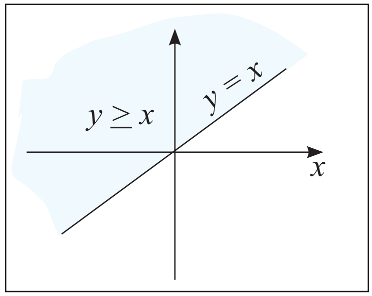
Now let us see how a few more inequalities are represented on a Cartesian plane.

(i) $y > x$



What is meant by indicating $y = x$ by a dashed line is that the points satisfying $y = x$ do not belong to the shaded region $y > x$.

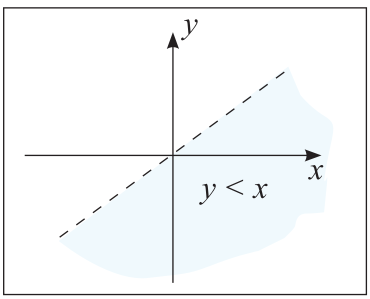
(ii) $y \geq x$



What is meant by indicating $y = x$ by a thick line is that the points satisfying $y = x$ belong to the shaded region $y \geq x$.

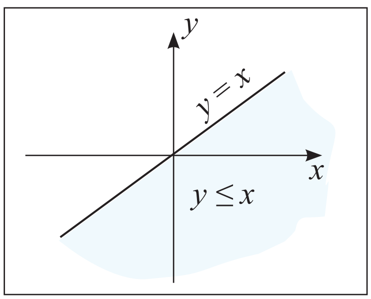
(iii)

$y < x$



(iv)

$y \leq x$



Exercise 25.3

1. Write down the coordinates of 3 points that belong to the region $y = x$.

2. Which of the following points belong to the region $y \geq x$?
$A = (5, 5)$ $B = (-3, -2)$ $C = (0, -1)$

3. Write down the coordinates of three points which satisfy both the inequalities $y < -2$ and $y > x$.

4. In a Cartesian plane, shade the area common to both the regions represented by the inequalities $x \geq 0$ and $y > x$.

5. Write down the coordinates of 3 points which satisfy the three inequalities $x < 3, y > 0$ and $y < x$.