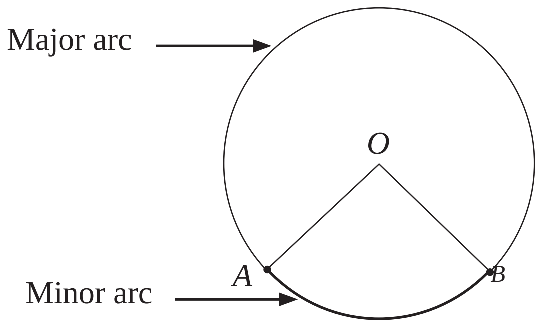
31 Angles in a Circle

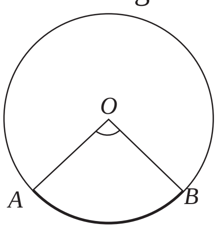
By studying this lesson you will be able to

identify and apply the theorems related to angles in a circle.

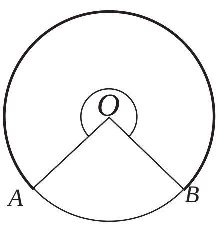
31.1 Angles subtended by an arc at the centre and on the circle



The circle in the figure is divided into two parts by the two points A and B on the circle. These two parts are called arcs. When the two points A and B are such that the straight line joining the two points passes through the centre of the circle, that is, when it is a diameter, then the two arcs are equal in length. When this is not the case, the two arcs are unequal in length. Then the shorter arc is called the **minor arc** and the longer arc is called the **major arc**.

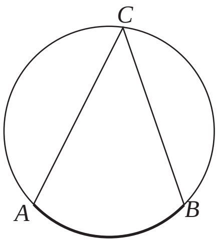


The angle $A\hat{O}B$ which is formed by joining the end points of the minor arc AB which is indicated by the thick line in the figure, to the centre, is defined as the angle subtended by the minor arc AB at the centre.



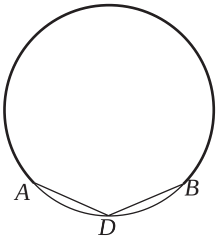
The reflex angle $A\hat{O}B$ which is formed by joining the end points of the major arc AB which is indicated by the thick line in the figure, to the centre, is defined as the angle subtended by the major arc AB at the centre.

Note: The angle subtended at the centre by a major arc is always a reflex angle.



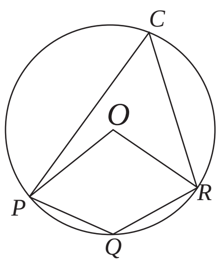
Let us assume that C is any point on the major arc AB.

The angle $A\hat{C}B$ is formed when the end points of the minor arc AB are joined to the point C on the major arc. That is, $A\hat{C}B$ is the angle that is subtended by the minor arc AB on the remaining part of the circle.



Similarly, the angle $A\hat{D}B$ in the given figure is defined as the angle that is subtended on the remaining part of the circle by the major arc AB.

Example 1



The centre of the circle in the given figure is O.

(a) Write down,

(i) the angle that is subtended by the minor arc PR on the remaining part of the circle.

(ii) the angle that is subtended at the centre by the minor arc PR.

(b) Write down,

(i) the angle that is subtended by the major arc PR on the remaining part of the circle.

(ii) the angle that is subtended at the centre by the major arc PR.

(a) (i) $P\hat{C}R$

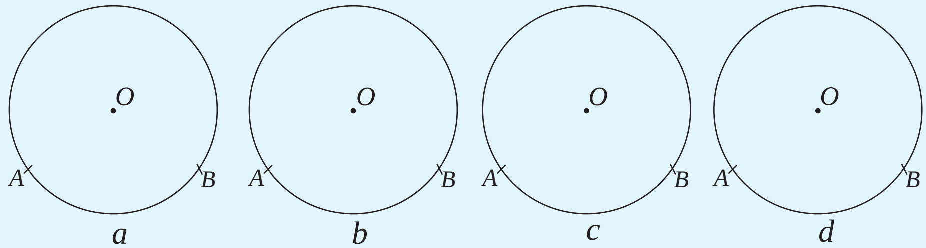
(ii) $P\hat{O}R$

(b) (i) $P\hat{Q}R$

(ii) reflex angle $P\hat{O}R$

Exercise 31.1

1. Copy the four circles in the figure given below onto your exercise book. The centre of each circle is denoted by O.



Mark each of the angles indicated below.

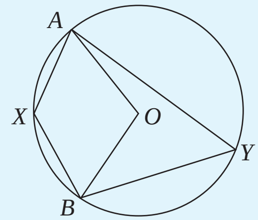
(i) An angle that is subtended on the remaining part of the circle by the minor arc in figure *a*.

(ii) The angle subtended at the centre by the minor arc in figure *b*.

(iii) An angle that is subtended on the remaining part of the circle by the major arc in figure *c*.

(iv) The angle subtended at the centre by the major arc in figure *d*.

2. According to the figure,



(i) write down for the minor arc AB,

(*a*) the angle subtended on the remaining part of the circle

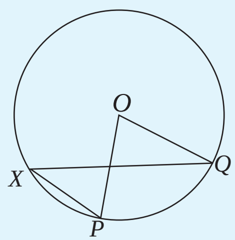
(*b*) the angle subtended at the centre.

(ii) write down for the major arc AB,

(*a*) the angle subtended on the remaining part of the circle

(*b*) the angle subtended at the centre.

3. The centre of the circle in the given figure is O. The point X is on the major arc PQ.



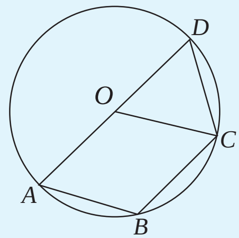
(i) Write down the angle that is subtended on the remaining part of the circle by the minor arc PQ.

(ii) Write down the angle that is subtended at the centre by the minor arc PQ.

(iii) Mark any point on the minor arc PQ and name it Y. Define the angle $P\hat{Y}Q$.

(iv) Write down the angle that is subtended at the centre of the circle by the major arc PQ.

4. The centre of the circle in the figure is O.



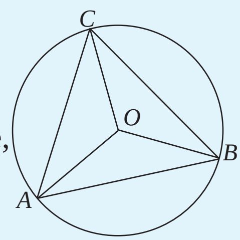
(i) Name an angle that is subtended on the remaining part of the circle by the minor arc AC.

(ii) Write down the angle that is subtended at the centre by the minor arc AC.

(iii) Write down an angle that is subtended on the remaining part of the circle by the major arc AC.

(iv) Write down the angle that is subtended at the centre by the major arc AC.

5. The centre of the circle in the figure is O.



(i) Write down for the minor arc AB,

(*a*) the angle subtended on the remaining part of the circle,

(*b*) the angle subtended at the centre.

(ii) Write down for the minor arc BC,

(*a*) the angle subtended on the remaining part of the circle

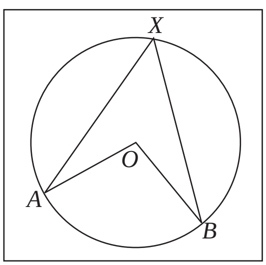
(*b*) the angle subtended at the centre.

31.2 The relationship between the angles subtended by an arc at the centre and on the circumference of a circle

Let us engage in the following activity to gain an understanding of the relationship between the angle subtended at the centre by an arc and the angle subtended on the remaining part of the circle by the same arc.

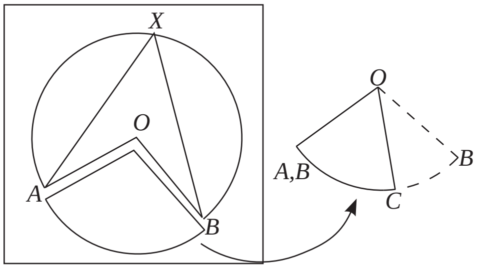
Activity

Draw a circle on a tissue paper and name its centre O. Mark two points on the circle such that a minor arc and a major arc are obtained. Name these two points as A and B.



Mark a point on the major arc and name it X.

Identify the angle subtended at the centre by the arc AB. This angle is $A\hat{O}B$. Cut out the sector of the circle AOB as shown in the figure.



- Fold the sector of the circle AOB into two such that OA and OB coincide, to obtain an angle which is exactly half the size of $A\hat{O}B$.
- Place this folded sector on $A\hat{X}B$ such that O overlaps with X and examine it.

You would have established the fact that the angle $A\hat{O}B$ subtended at the centre by the minor arc AB is twice the angle $A\hat{X}B$ subtended by this arc on the remaining part of the circle. In the same manner, mark arcs of different lengths on circles of varying radii and repeat this activity.

Through these activities you would have observed that the angle subtended at the centre by an arc is twice the angle subtended on the remaining part of the circle by the same arc.

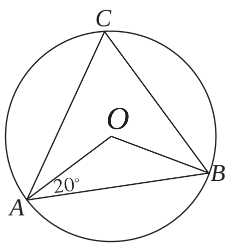
This result is given as a theorem below.

Theorem

The angle subtended by an arc at the centre of a circle is twice the angle subtended by the same arc on the remaining part of the circle.

Now, by considering the following examples, let us see how calculations are performed using the above theorem.

The points A, B and C are on a circle of centre O. If $O\hat{A}B = 20^\circ$, let us find the magnitude of $A\hat{C}B$.



$OA = OB$ (The radii of a circle are equal)

$\therefore$ OAB is an isosceles triangle.

In an isosceles triangle, since angles opposite the equal sides are equal,

$$O\hat{A}B = O\hat{B}A$$

$$\therefore \; O\hat{B}A = 20^\circ. \qquad (\text{Since } O\hat{A}B = 20^\circ)$$

Since the sum of the interior angles of a triangle is equal to 180°,

$$A\hat{O}B + O\hat{A}B + O\hat{B}A = 180^\circ.$$

$$A\hat{O}B + 20^\circ + 20^\circ = 180^\circ$$

$$A\hat{O}B + 40^\circ = 180^\circ$$

$$A\hat{O}B = 180^\circ - 40^\circ$$

$$A\hat{O}B = 140^\circ$$

The angle subtended at the centre by the minor arc AB is $A\hat{O}B$. Since $A\hat{C}B$ is an angle subtended by this arc on the remaining part of the circle, according to the theorem,

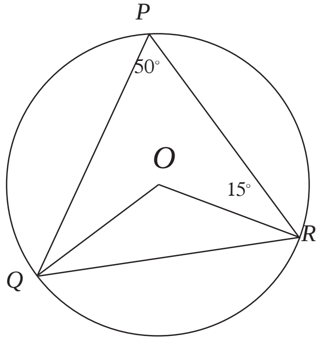
$$2\,A\hat{C}B = A\hat{O}B$$

$$\therefore A\hat{C}B = \frac{140^\circ}{2}$$

$$\therefore \underline{\underline{A\hat{C}B = 70^\circ}}$$

Example 1

The centre of the circle in the given figure is O. Using the information in the figure find $P\hat{Q}R$.



$Q\hat{O}R = 2\,Q\hat{P}R$ (The angle subtended at the centre by an arc of a circle is twice the angle subtended by the arc on the remaining part of the circle)

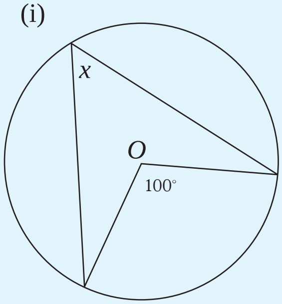
$$\therefore Q\hat{O}R = 2 \times 50^\circ$$
$$= 100^\circ$$
$$O\hat{Q}R + O\hat{R}Q + Q\hat{O}R = 180^\circ$$ (The sum of the interior angles of a triangle is 180°)
$$\therefore O\hat{Q}R + O\hat{R}Q + 100 = 180^\circ$$
$$O\hat{Q}R + O\hat{R}Q = 80^\circ \text{ —— } ①$$
$$OQ = OR$$ (The radii of a circle are equal)
$$\therefore O\hat{Q}R = O\hat{R}Q$$ (In an isosceles triangle, the angles opposite equal sides are equal)

According to ① $2\,O\hat{R}Q = 80^\circ$
$$O\hat{R}Q = \frac{80^\circ}{2}$$
$$O\hat{R}Q = 40$$
Now, $P\hat{R}Q = P\hat{R}O + O\hat{R}Q$
$$P\hat{R}Q = 15^\circ + 40^\circ$$
$$P\hat{R}Q = 55^\circ$$
$$P\hat{Q}R + Q\hat{P}R + P\hat{R}Q = 180^\circ$$ (The sum of the interior angles of a triangle is 180°)
$$P\hat{Q}R + 50^\circ + 55^\circ = 180^\circ$$
$$P\hat{Q}R + 105^\circ = 180^\circ$$
$$P\hat{Q}R = 180^\circ - 105^\circ$$
$$P\hat{Q}R = \underline{\underline{75^\circ}}$$

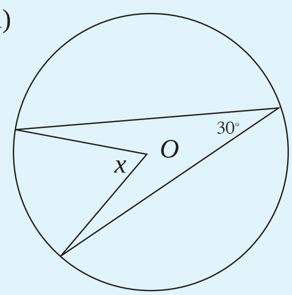
Exercise 31.2

1. The centre of each of the circles given below is O. Find the value of x based on the given data.

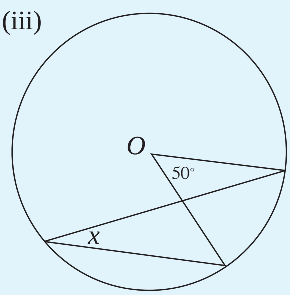
(i)



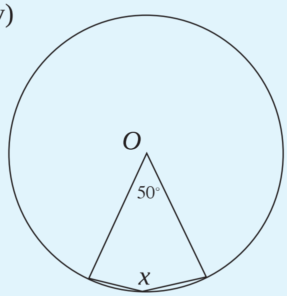
(ii)



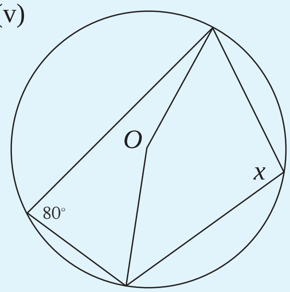
(iii)



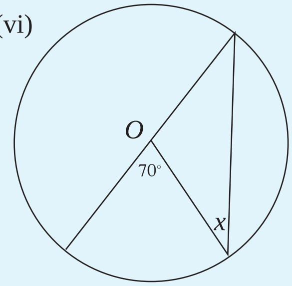
(iv)



(v)

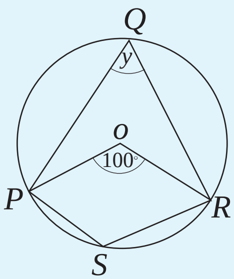


(vi)

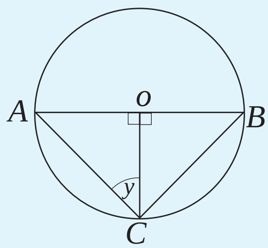


2. The centre of each of the following circles is denoted by O. Providing reasons, find the value of y based on the given data.

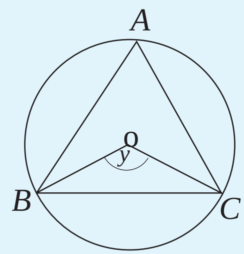
(i)



(ii)

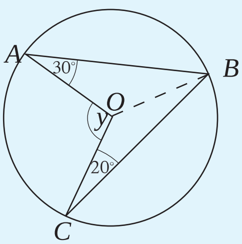


(iii)



$A\hat{B}C = 70°$, $A\hat{C}B = 60°$

(iv)



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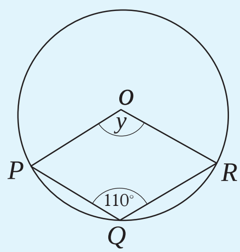

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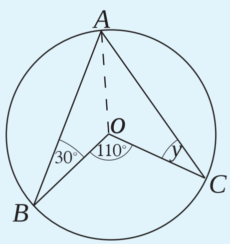
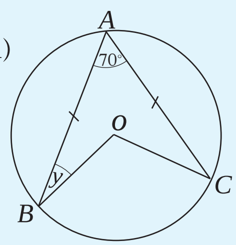
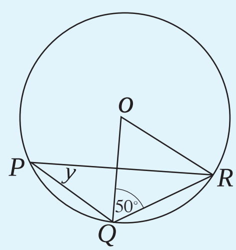


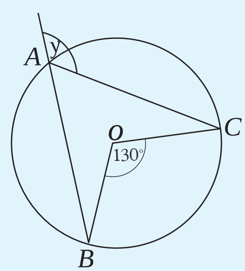
(vii)



(viii)

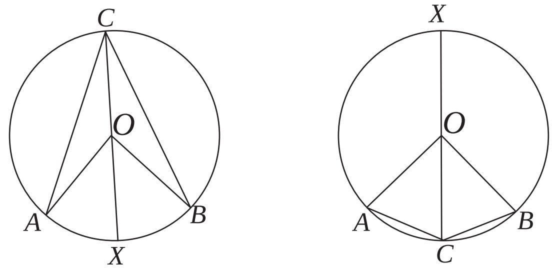


(ix)



31.3 Formal proof of the theorem

"The angle subtended at the centre by an arc of a circle is twice the angle subtended by the same arc on the remaining part of the circle".



Data : The points A, B and C lie on the circle of centre O.

To be proved : $A\hat{O}B = 2\,A\hat{C}B$.

Construction : The straight line CO is produced up to X

Proof : $OA = OC$ (Radii of the same circle)

$\therefore\ O\hat{A}C = O\hat{C}A$ —— ① (Since the angles opposite equal sides of an isosceles triangle are equal)

$O\hat{A}C + O\hat{C}A = X\hat{O}A$ —— ② (Exterior angle formed by producing a side of a triangle is equal to the sum of the interior opposite angles)

From ① and ②, $X\hat{O}A = 2\,O\hat{C}A$ —— ③

Similarly, $X\hat{O}B = 2\,O\hat{C}B$ —— ④

From ③ and ④, $\underbrace{X\hat{O}A + X\hat{O}B} = 2\ O\hat{C}A + O\hat{C}B$

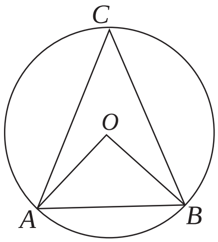
$$A\hat{O}B = 2\ \underbrace{(O\hat{C}A + O\hat{C}B)}$$

$$\underline{\underline{A\hat{O}B = 2\ A\hat{C}B}}$$

Now let us consider how the theorem which has been proved above can be used to prove other results (riders).

Example 1

The points A,B and C lie on a circle of centre O. If $A\hat{C}B + A\hat{B}C = A\hat{O}B$, show that $AC = AB$.



Proof: $A\hat{C}B + A\hat{B}C = A\hat{O}B$ —— 1 (Given)

$2\,A\hat{C}B = A\hat{O}B$ —— 2 (The angle subtended at the centre by an arc of a circle is twice the angle subtended by the same arc on the remaining part of the circle)

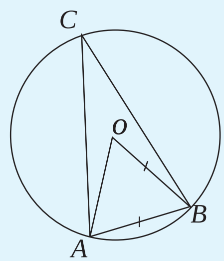
By 1 and 2

$$2\,A\hat{C}B = A\hat{C}B + A\hat{B}C$$
$$2\,A\hat{C}B - A\hat{C}B = A\hat{B}C$$
$$A\hat{C}B = A\hat{B}C$$
$$\underline{\underline{AC = AB}}$$

(In an isosceles triangle sides opposite equal angles are equal)

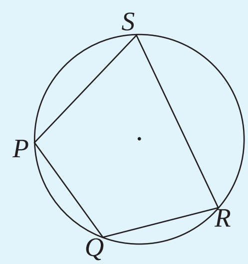
Exercise 31.3

1. The points A, B and C lie on a circle of centre O. If $OB = AB$, show that $A\hat{C}B = 30^\circ$.

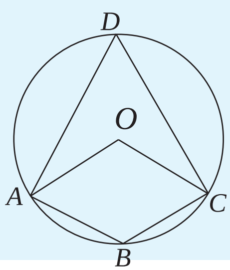


2. P, Q, R and S are points on a circle.

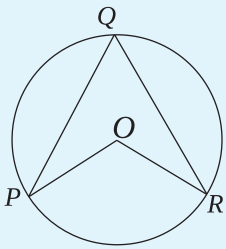
Prove that $P\hat{Q}R + P\hat{S}R = 180^\circ$.



3. A, B, C and D are points on a circle of centre O. If $A\hat{O}C = A\hat{B}C$, show that $A\hat{D}C = 60^\circ$.

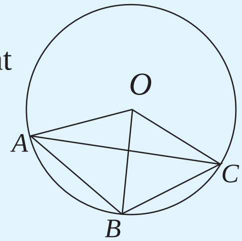


4.

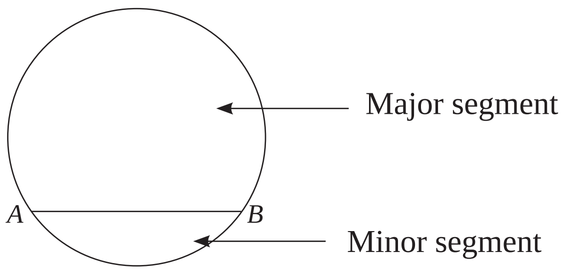


P, Q and R are points on the circumference of the circle of centre O. If $O\hat{P}Q = O\hat{R}Q$, show that $P\hat{O}R = 4\,O\hat{R}Q$. (Join O and Q)

5. The points A, B and C lie on a circle of centre O. Show that $A\hat{O}C = 2\,(B\hat{A}C + B\hat{C}A)$.

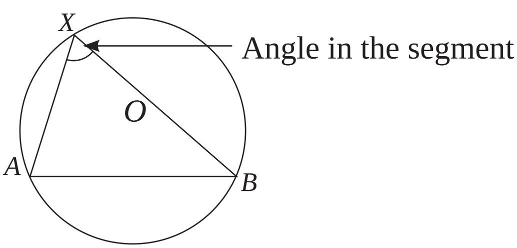


31.4 Relationship between the angles in the same segment

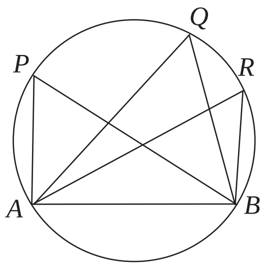


A circle and a chord AB of the circle are shown in the figure. The circle is divided into two regions by the chord.

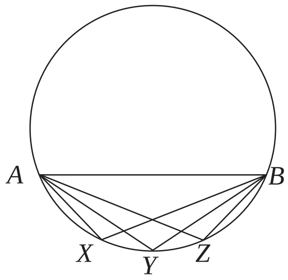
One is the region bounded by the chord and the major arc which is called the **major segment.** The other is the region bounded by the chord and the minor arc called the **minor segment.**



The angle formed by joining the end points of the chord AB to a point on the arc of a segment is defined as an angle in the segment. $A\hat{X}B$ is an angle in the segment AXB .



$A\hat{P}B$, $A\hat{Q}B$ and $A\hat{R}B$ in the figure are angles in the major segment. Therefore, $A\hat{P}B$, $A\hat{Q}B$ and $A\hat{R}B$ are called angles in the same segment.

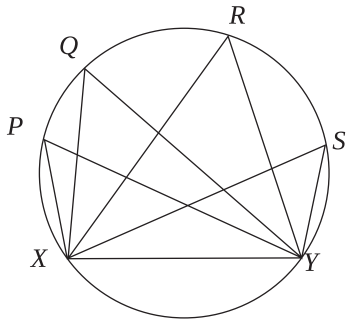


The angles $A\hat{X}B$, $A\hat{Y}B$ and $A\hat{Z}B$ in the figure are angles in the minor segment and hence belong to the same segment.

Let us identify the relationship between angles in the same segment through the following activity.

Activity

- Draw a circle on a piece of paper. Mark the points X and Y on the circle and draw the chord XY.
- Mark the points P, Q, R and S on the arc XY of the major segment.
- Join these points to the two end points of the chord XY. Then the angles $X\hat{P}Y$, $X\hat{Q}Y$, $X\hat{R}Y$ and $X\hat{S}Y$ which are angles in the same segment are obtained.

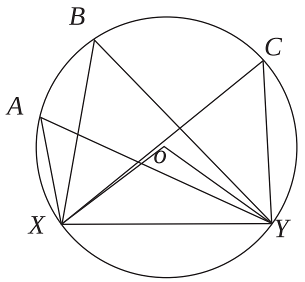


- Using a protractor, measure the angles that you have drawn in the same segment. Examine the magnitude of each angle.
- In the same manner, draw several angles in the minor segment, measure them and examine the values you obtain.

Through these activities you would have identified that angles in the same segment are equal in magnitude. This is given below as a theorem.

Theorem: The angles in the same segment of a circle are equal.

Let us establish this theorem through a geometric proof.



Data : The points A, B and C lie on the circle of centre O, on the same side of the chord XY.

To be proved : $X\hat{A}Y = X\hat{B}Y = X\hat{C}Y$

Construction : Join XO and YO.

Proof : The angle subtended at the centre by an arc of a circle is twice the angle subtended by the same arc on the remaining part of the circle.

$$\therefore X\hat{O}Y = 2\, X\hat{A}Y \text{ ——— } (1)$$

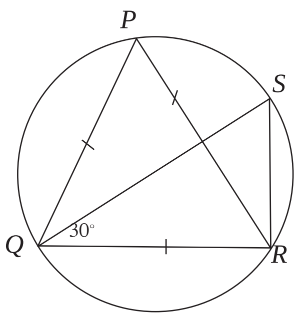
$$X\hat{O}Y = 2\, X\hat{B}Y \text{ ——— } (2)$$

$$X\hat{O}Y = 2\, X\hat{C}Y \text{ ——— } (3)$$

From (1), (2) and (3), $2\, X\hat{A}Y = 2\, X\hat{B}Y = 2\, X\hat{C}Y$

$$\therefore \underline{\underline{X\hat{A}Y = X\hat{B}Y = X\hat{C}Y}}$$

Let us consider how calculations are done using the above theorem. Find $Q\hat{R}S$ using the information in the figure.



In the above figure, $PQ = QR = PR$ and $R\hat{Q}S = 30^\circ$. Let us find $Q\hat{R}S$.

Since $PQ = QR = PR$, (the triangle PQR is an equilateral triangle)

$$Q\hat{P}R = 60^\circ$$

$$Q\hat{P}R = Q\hat{S}R \text{ (Angles in the same segment are equal)}$$

$$\therefore Q\hat{S}R = 60^\circ$$

Since the sum of the interior angles of a triangle is 180°,

$$Q\hat{R}S + R\hat{Q}S + Q\hat{S}R = 180^\circ$$

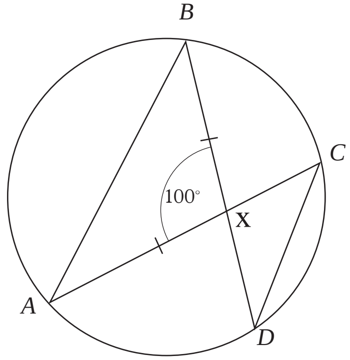
$$Q\hat{R}S = 180^\circ - (30^\circ + 60^\circ)$$

$$Q\hat{R}S = 180^\circ - 90^\circ$$

$$\underline{\underline{Q\hat{R}S = 90^\circ}}$$

Example 1

Find the magnitude of $B\hat{D}C$ using the information in the figure.



XAB is an isosceles triangle since *XB* = *XA*.

$\therefore \quad X\hat{B}A = X\hat{A}B$ —— (1) (In an isosceles triangle, the angles opposite equal sides are equal)

In the triangle *ABX*,

$X\hat{B}A + X\hat{A}B + A\hat{X}B = 180^\circ$ (The sum of the interior angles of a triangle is 180°)

$$X\hat{B}A + X\hat{A}B + 100^\circ = 180^\circ$$

$$X\hat{B}A + X\hat{A}B = 180^\circ - 100^\circ$$

$$X\hat{B}A + X\hat{A}B = 80^\circ$$

From (1), $2\,X\hat{A}B = 80^\circ$ (Since $X\hat{B}A = X\hat{A}B$)

$$\therefore X\hat{A}B = 40^\circ$$

Since the angles in the same segment are equal,

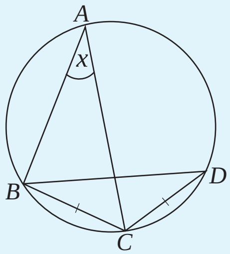
$$B\hat{D}C = X\hat{A}B$$

$$\therefore \underline{\underline{B\hat{D}C = 40^\circ}}$$

Exercise 31.4

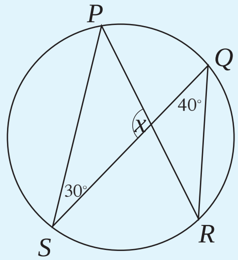
Find the value of x in the following exercises.

1.

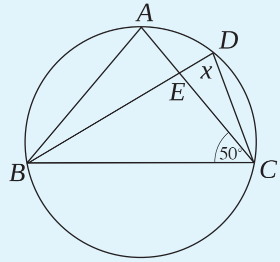


$B\hat{C}D = 110^\circ$

2.

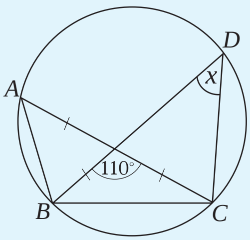


3.

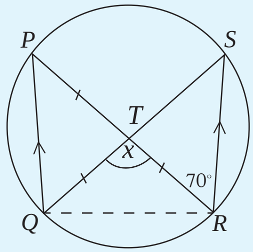


$AB = AC$

4.



5.



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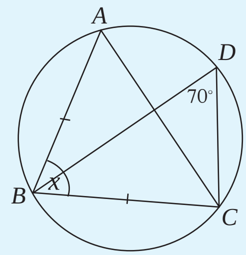

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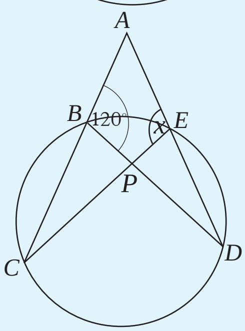

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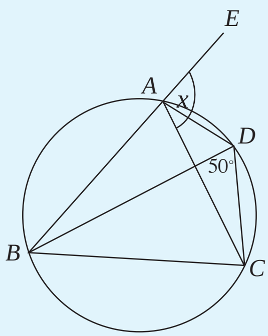
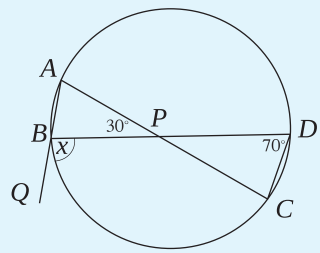


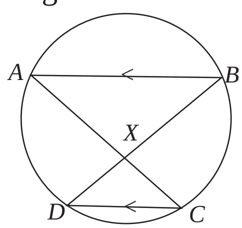
9.



31.5 Proving riders using the theorem "Angles in the same segment of a circle are equal."

Example 1

Prove that $AC = BD$ using the information in the figure.



Proof: $A\hat{B}D = B\hat{D}C$ ($AB//DC$, alternate angles)

$A\hat{B}D = A\hat{C}D$ (Angles in the same segment)

$\therefore B\hat{D}C = A\hat{C}D$

Since the sides opposite equal angles in a triangle are equal, in the triangle XCD,

$XD = XC$

$B\hat{A}C = A\hat{C}D$ ($AB//CD$, alternate angles)

$A\hat{B}D = A\hat{C}D$ (Angles in the same segment)

$\therefore B\hat{A}C = A\hat{B}D$

Since the sides opposite equal angles in a triangle are equal,

$XA = XB$

$XC = XD$ (Proved)

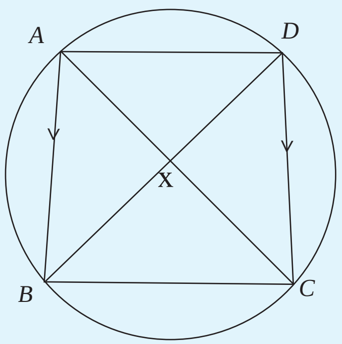
Using the axioms,

$$\underbrace{XA + XC} = \underbrace{XB + XD}$$

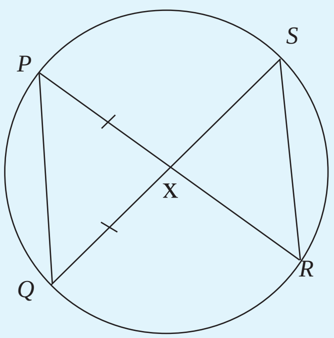
$$\therefore \underline{\underline{AC = BD}}$$

Exercise 31.5

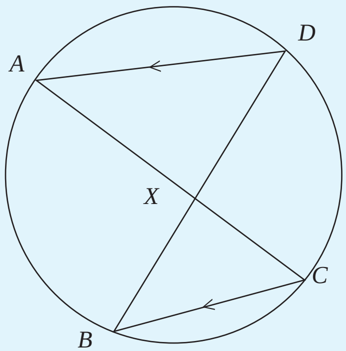
1. If $AB//DC$, show that .



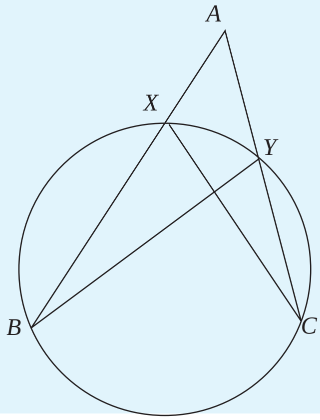
2. If $PX = QX$, show that $PQ//SR$.



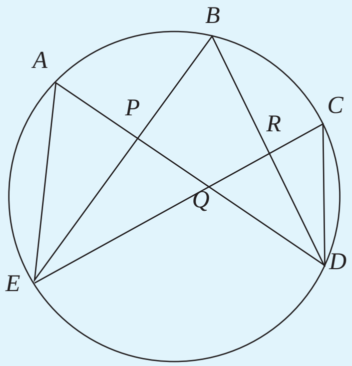
3. If $AD// BC$, show that $AX = DX$.



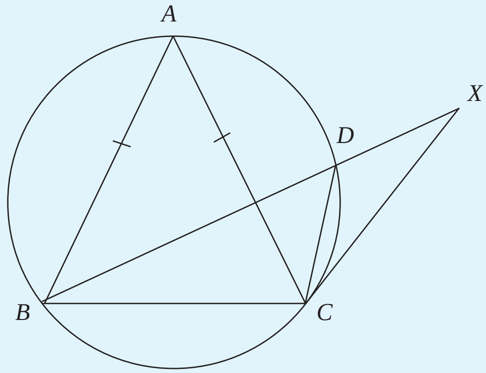
4. Show that $A\hat{X}C = A\hat{Y}B$.



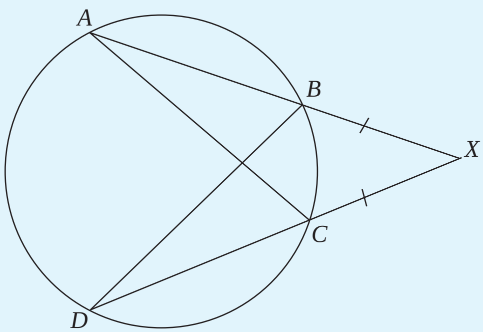
5. If $B\hat{P}Q = B\hat{R}Q$, show that BE is the bisector of $A\hat{E}C$.



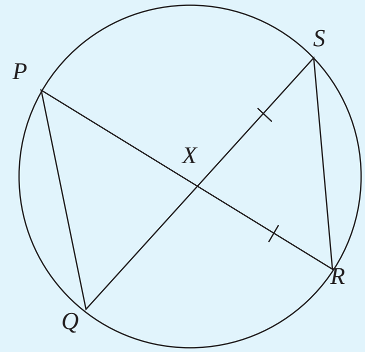
6. If $AB = AC$, show that $C\hat{D}X = 2\,A\hat{B}C$.



7. If $XB = XC$, show that $AC = BD$.

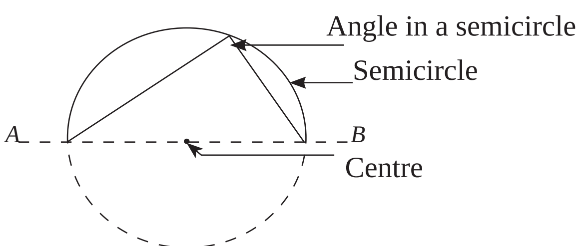


8. If $XS = XR$, show that $XP = XQ$.



31.6 Angles in a semicircle

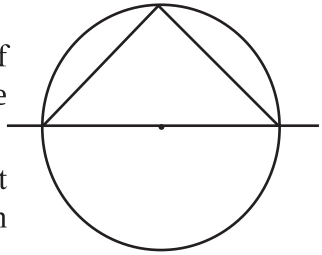
An arc of a circle which is exactly half a circle is defined as a semicircle.



By drawing a line through the centre of a circle, the circle is divided into two semicircles. The angle formed by joining a point on a semicircle to its end points is called an angle in the semicircle.

Let us engage in the following activity to identify the properties of angles in a semicircle.

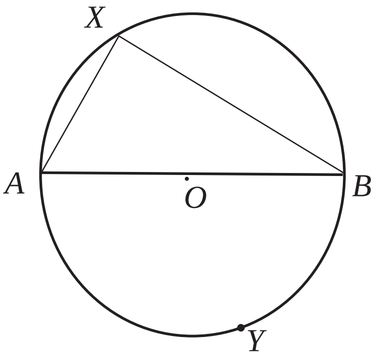
Activity 31.3



- Draw a circle on a piece of paper using a pair of compasses. Then draw a diameter of the circle. Now, the circle is divided into two semicircles.
- Mark a point on one of the semicircles. Join this point to the two end points of the semicircle. Then an angle in a semicircle is obtained.
- Using the protractor, measure the angle.

You would have observed that the angle in the semi-circle is 90°. In the above manner, draw several more circles and draw and measure angles in a semi-circle for these circles too. You will be able to identify through this activity that the angle in a semi-circle is always a right angle.

Let us establish the above relationship through a geometric proof.



Data: As shown in the figure, X and Y are points on the circle with centre O and AB is a diameter of the circle.

To be proved: $A\hat{X}B$ is a right angle.

Proof: $A\hat{O}B$, is the angle subtended at the centre by the arc AYB.

Since it is a semicircle, AOB is a diameter.

$A\hat{O}B$ = 2 right angles ———— ①

$A\hat{X}B$ is an angle subtended on the remaining part of the circle by the chord AYB.

Since the angle subtended at the centre by an arc of a circle is twice the angle subtended by the same arc on the remaining part of the circle,

$A\hat{O}B = 2\,A\hat{X}B$ ———— ②

By ① and ②,

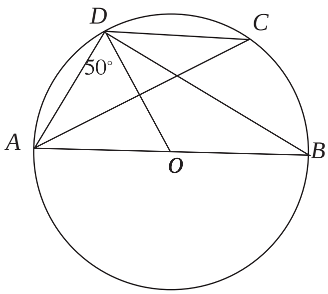
$$2\ A\hat{X}B = 2 \times 2 \text{ right angles}$$
$$\therefore A\hat{X}B = 1 \text{ right angle}$$

The relationship which has been established through the above proof is given below as a theorem.

Theorem: An angle in a semicircle is a right angle.

Let us identify how calculations are performed using the above theorem by considering the following examples.

Let us find the magnitude of $A\hat{C}D$ using the data in the figure of a circle with centre O.

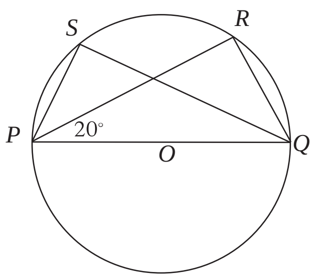


$$A\hat{D}B = 90^\circ \quad \text{(Angle in a semicircle)}$$
$$A\hat{D}B = A\hat{D}O + O\hat{D}B$$
$$\therefore A\hat{D}O + O\hat{D}B = 90^\circ$$
$$50^\circ + O\hat{D}B = 90^\circ$$
$$O\hat{D}B = 90^\circ - 50^\circ$$
$$O\hat{D}B = 40^\circ$$

Since they are radii of the same circle,
$$DO = OB$$

Since the angles opposite equal sides of a triangle are equal,
$$D\hat{B}O = O\hat{D}B$$
$$\therefore D\hat{B}O = 40^\circ$$
$$D\hat{B}O = A\hat{C}D \quad \text{(Angles in the same segment)}$$
$$\therefore \underline{\underline{A\hat{C}D = 40^\circ}}$$

Example 1



PQ is a diameter of the circle $PQRS$.

If $Q\hat{P}R = 20^\circ$ and $PS = QR$, find the magnitude of $R\hat{P}S$.

$P\hat{R}Q = 90^\circ$ (Angle in a semicircle)

$P\hat{Q}R + Q\hat{P}R + P\hat{R}Q = 180^\circ$ (The sum of the interior angles of a triangle is 180°)

$P\hat{Q}R + 20^\circ + 90^\circ = 180^\circ$

$P\hat{Q}R = 180^\circ - 110^\circ$

$P\hat{Q}R = 70^\circ$

Since PQ is a diameter,

$P\hat{S}Q = 90^\circ$ (Angle in a semicircle)

$P\hat{R}Q = 90^\circ$ (Angle in a semicircle)

$\therefore$ The triangles PSQ and PRQ are right angled triangles.

$\therefore$ In the triangles PSQ and PRQ,

$SP = RP$ (Given)

PQ is a common side.

$\therefore \Delta PSQ \equiv \Delta PRQ$ (RHS)

$\therefore S\hat{P}Q = P\hat{Q}R$ (Corresponding angles of congruent triangles)

$\therefore S\hat{P}Q = 70^\circ$

$R\hat{P}S + Q\hat{P}R = 70^\circ$

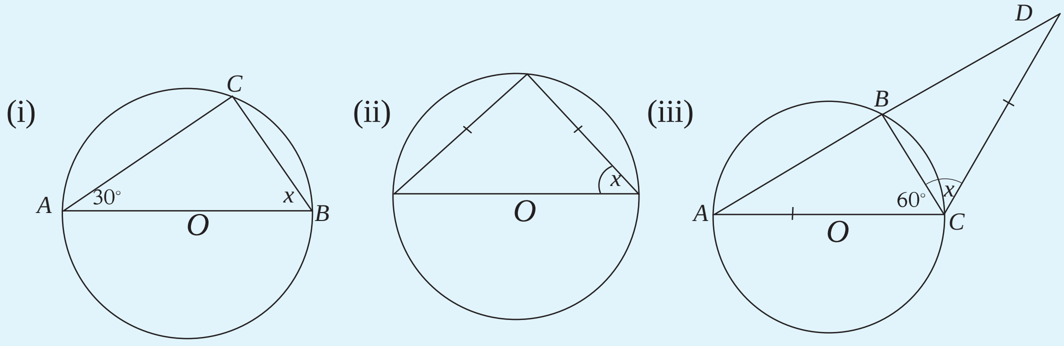
$R\hat{P}S + 20^\circ = 70^\circ$

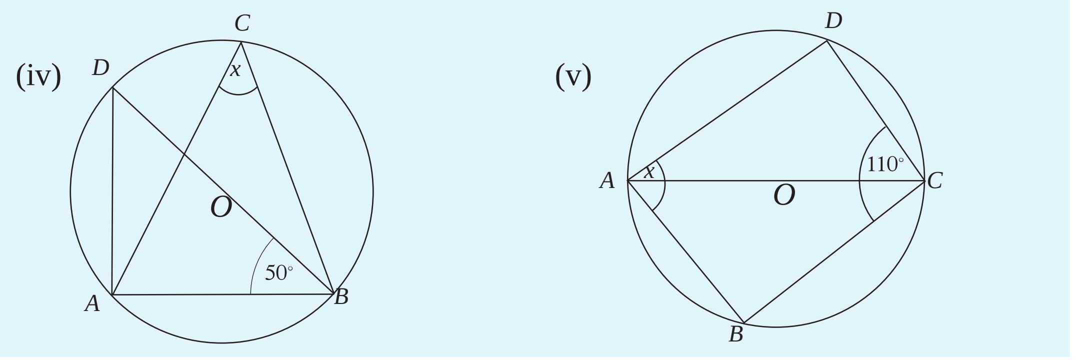
$R\hat{P}S = 70^\circ - 20^\circ$

$\underline{\underline{R\hat{P}S = 50^\circ}}$

Exercise 31.6

1. The centre of each of the following circles is denoted by O. Find the value of x based on the data in the figure.

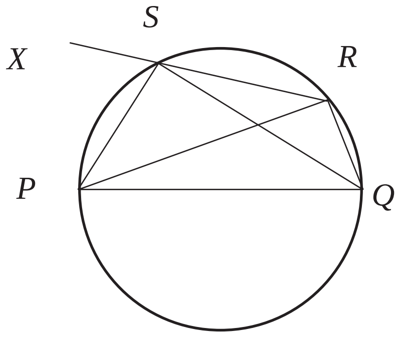




31.7 Proving riders using the theorem '' The angle in a semicircle is a right angle''

Example 1

PQ is a diameter of the circle *PQRS*. The chord *RS* has been produced to *X*. Prove that $R\hat{P}Q + P\hat{S}X = 90^{\circ}$.



Proof:

$Q\hat{S}R + P\hat{S}Q + P\hat{S}X = 180^{\circ}$ (Sum of the angles on a straight line is 180°)

$P\hat{S}Q = 90^{\circ}$ (Angle in a semicircle is 90°)

$\therefore\ Q\hat{S}R + 90^{\circ} + P\hat{S}X = 180^{\circ}$

$Q\hat{S}R + P\hat{S}X = 180^{\circ} - 90^{\circ}$

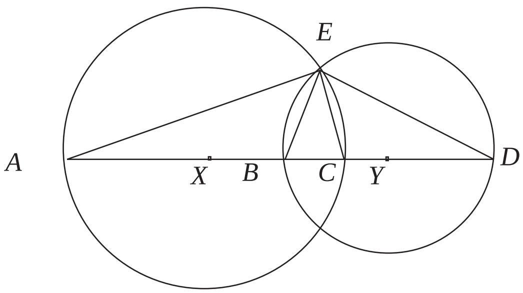
$Q\hat{S}R + P\hat{S}X = 90^{\circ}$

$Q\hat{S}R$ and $R\hat{P}Q$ are angles in the segment *PSRQ*.

$\therefore\ Q\hat{S}R = R\hat{P}Q$

$\therefore\ \underline{\underline{R\hat{P}Q + P\hat{S}X = 90^{\circ}}}$

Example 2



The centres of the given two circles are X and Y. Prove that $A\hat{E}B = C\hat{E}D$.

Proof:

Since AC passes through X, AC is a diameter of the circle with centre X.

$\therefore$ The arc AEC is a semicircle.

$\therefore A\hat{E}C = 90^\circ$ (Since the angle in a semicircle is a right angle)

$\therefore A\hat{E}B + B\hat{E}C = 90^\circ$ ——— ①

Since BD passes through the centre Y, BD is a diameter of the circle with centre Y.

$\therefore$ The arc BED is a semicircle.

$\therefore B\hat{E}D = 90^\circ$ (Since the angle in a semicircle is a right angle)

$C\hat{E}D + B\hat{E}C = 90^\circ$ ——— ②

$A\hat{E}B + B\hat{E}C = C\hat{E}D + B\hat{E}C$

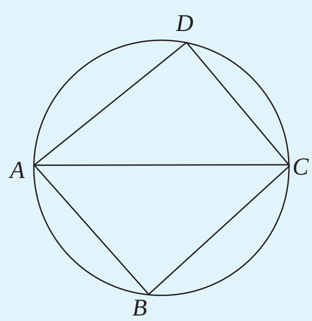
Subtracting $B\hat{E}C$ from both sides.

$\underline{\underline{A\hat{E}B = C\hat{E}D}}$

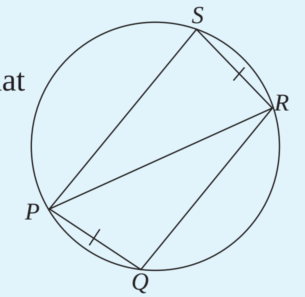
Exercise 31.7

1. AC is a diameter of the circle $ABCD$.

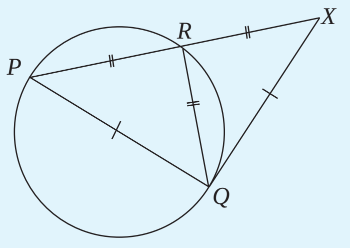
Show that $B\hat{A}D + B\hat{C}D = 180^\circ$.



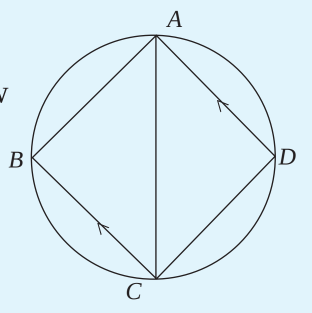
2. PR is a diameter of the circle $PQRS$. If $PQ = RS$, show that $PQRS$ is a rectangle.



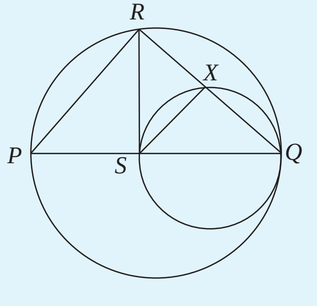
3. PQ is a diameter of the circle PQR. If $PQ = QX$ and $PR = QR = RX$, then show that $P\hat{Q}X = 90°$.



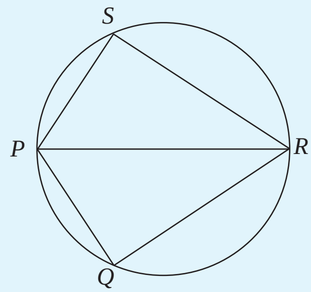
4. AC is a diameter of the circle $ABCD$. If $BC // AD$, show that $ABCD$ is a rectangle.

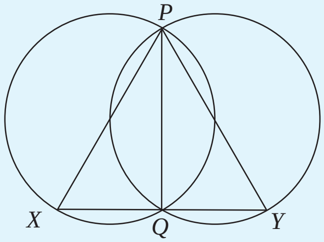


5. PSQ is a diameter of the larger circle and SQ is a diameter of the smaller circle. If RQ intersects the smaller circle at X, show that $P\hat{R}S = R\hat{S}X$.



6. *PR* is a diameter of the circle *PQRS*. If $S\hat{R}P = Q\hat{R}P$, show that $S\hat{P}R = Q\hat{P}R$.



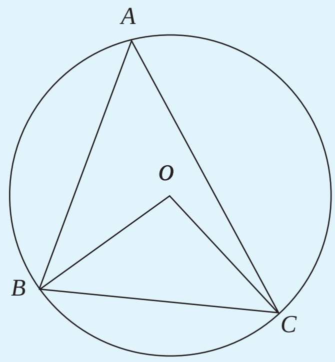


7. The two circles in the figure intersect at *P* and *Q*. *PX* and *PY* are diameters of the two circles. Show that *XQY* is a straight line.

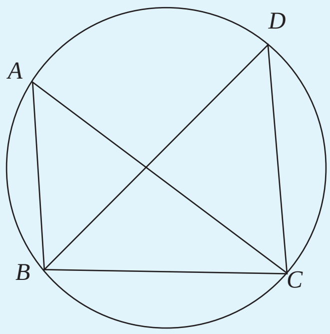
Miscellaneous Exercise

Mark the given data on the given figures and solve the problems.

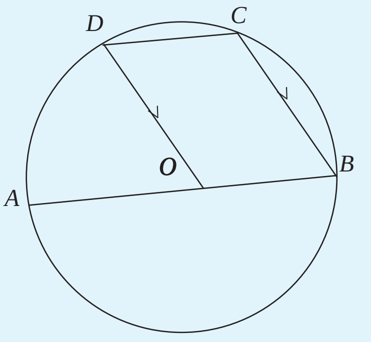
1. The centre of the circle *ABC* is *O*.
If $A\hat{B}O = O\hat{B}C$ and $A\hat{B}O = 40^\circ$,
find the magnitude of $A\hat{C}O$.



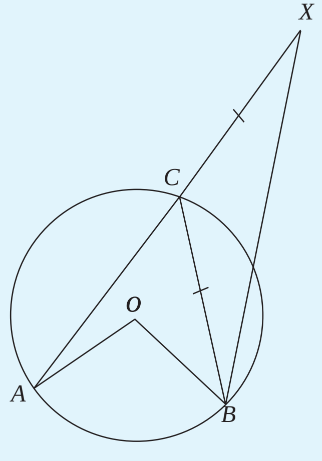
2. *BD* is a diameter of the circle *ABCD*. If *BC* = *CD* and $A\hat{C}B = 35^\circ$, find the magnitude of $A\hat{B}C$



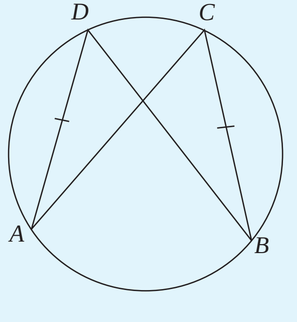
3. The centre of the circle $ABCD$ is O. If $BC//OD$ and $A\hat{B}C = 60°$, find the magnitude of the angle $B\hat{C}D$.



4. The centre of the circle ABC is O. AC has been produced to X such that $BC = CX$. Show that $A\hat{O}B = 4C\hat{B}X$.



5. The points A, B, C and D lie on the circle such that $AD = BC$. Show that $DB = CA$.



6. PQ is a diameter of the circle with centre O. Also, $QR//OS$. Show that $SR = SP$.