30 Probability

By studying this lesson you will be able to

- identify simple events and composite events
- find the probability of events which are not mutually exclusive
- find the probability of an event using a grid or a tree diagram

We know that we will get either heads or tails when we toss a coin. This observation is an example of a random experiment. The possible results are either 'heads' or 'tails'. However we cannot say with certainty what the outcome will be before observing it. Experiments in which we know the possible outcomes, but cannot say with certainty which outcome will occur are called **random experiments**. The set to which all the possible outcomes of a random experiment belong is called 'the sample space'. It is denoted by S.

The following table shows some examples for random experiments and the relevant sample spaces.

Random Experiment	Sample Space
1. Tossing a coin and recording the result	S = {Heads, Tails}
2. Rolling a die numbered from 1 to 6 and recording the number shown	$S = \{1, 2, 3, 4, 5, 6\}$
3. Throwing a ball at a target and recording the result.	S = {Hits the target, Misses the target}

Events

An event is a subset of the sample space. Consider the following examples.

Consider the random experiment of rolling a fair tetrahedronal die numbered from 1 to 4 and recording the result.

The sample space is $S = \{1, 2, 3, 4\}$.

Some subsets of this sample space are $\{3\}$, $\{2, 4\}$, $\{1, 2, 3\}$.

These subsets can be explained as follows:

$\{3\}$ denotes "The event of getting 3 as the result".

$\{2, 4\}$ denotes "The event of getting 2 or 4 as the result".

Also if "getting a number less than 4" is denoted by A, then it can be written as $A = \{1, 2, 3\}$.

An event is a subset of the sample space.

Simple events and composite events

Consider rolling an unbiased die numbered from 1 to 6. In this random experiment,

the sample space is $S = \{1, 2, 3, 4, 5, 6\}$.
Let us write some events relevant to the sample space.
$\{1\}, \{2\}, \{3\}, \{1, 3\}, \{2, 4\}, \{1, 3, 5\}, \{2, 3, 5\}, \{3, 4, 5, 6\}$

In the above events, $\{1\}$, $\{2\}$ and $\{3\}$ consist of only one outcome each. Such events are called **simple events**.

An event consisting of only one outcome is called a simple event.

Thus, $\{5\}$, $\{6\}$ are simple events.
Events which are not simple are called **composite events**. The events $\{1, 3\}$, $\{2, 4\}$, $\{1, 3, 5\}$ are composite events. These composite events can be further decomposed into subsets.

30.1 Equally likely outcomes

The sample space of tossing an unbiased coin is shown below.

S = {getting heads, getting tails}

Because the coin is unbiased, it is clear that the likelihood of getting either of these two outcomes is equal.

Let us consider another example.
There are 3 identical balls in a bag coloured red, white and black. Consider taking out one ball at random. The sample space is shown below.

S = {getting the red ball, getting the white ball, getting the black ball}

Because the balls are identical, it is clear that the likelihood of taking any one of the balls is equal.

If each of the outcomes in a random experiment has an equal likelihood of occuring, that experiment is called an experiment with **equally likely outcomes**.

Consider the experiment of "tossing an unbiased coin." As you have learnt in earlies grades, the probability of each equally likely outcome "getting heads" and "getting tails" in the sample space is $\frac{1}{2}$;

That is, the probability of heads occuring $= \frac{1}{2}$.

The probability of tails occuring $= \frac{1}{2}$.

Now let us consider a random experiment which is not an experiment with equally likely outcomes.

Amara planted a mango seed and observed whether a plant would grow within a week. The sample space is

$$S = \{\text{grow, not grow}\}.$$

Here there are no reasons to assume that the outcomes are equally likely. Here, taking the probability of a plant growing as $\frac{1}{2}$ is not correct.

In an instance where all the outcomes of a sample space are equally likely to occur, the probability of an event occuring is defined as below.

$$\boxed{\text{Probability of the event occuring} = \frac{\text{Number of elements in the event}}{\text{Number of elements in the sample space}}}$$

Let us denote the number of elements in the sample space S by $n\,(S)$ and the number of elements in the event A by $n\,(A)$. The probability of A occuring is denoted by $P(A)$. Then

$$\boxed{P\,(A) = \frac{n(A)}{n(S)}}$$

Example 1

With reference to an experiment in which a card is drawn randomly from a bag containing 5 identical cards numbered from 1 to 5,

(i) write the sample space and find $n(S)$.

(ii) write the elements of A and find $n(A)$ if A is the event of getting an even number

(iii) find the probability $P(A)$ that an even number is drawn.

It is clear that the outcomes are equally likely since the cards are identical.

(i) $S = \{1, 2, 3, 4, 5\}. \therefore\ n\,(S) = 5$

(ii) $A = \{2, 4\}. \therefore\ n\,(A) = 2$

(iii) $P(A) = \frac{n(A)}{n(S)}$

$= \underline{\underline{\frac{2}{5}}}$

Example 2

In an experiment of rolling an unbiased die with its faces marked 1, 2, 3, 4, 5, 6,
(i) find the probability that the number shown is 4.
(ii) find the probability that the number shown is odd.
(iii) find the probability that the number is greater than 2.

The sample space is $S = \{1, 2, 3, 4, 5, 6\}$. Thus $n(S) = 6$.

(i) Probability of getting 4 $= \frac{1}{6}$

(ii) Since there are three (1, 3 and 5) odd numbers, the probability of getting an odd number $= \frac{3}{6} = \frac{1}{2}$

(iii) Since there are four (3, 4, 5 and 6) numbers greater than 2, the probability of getting a number greater than 2 $= \frac{4}{6} = \frac{2}{3}$

Exercise 30.1

1. Write down the sample space for each of the following random experiments.
(i) Recording the card drawn from a pack of 10 identical cards numbered 1 to 10.
(ii) A circular disk is divided into three identical sectors, one of which is coloured red, another blue and the other yellow. A pointer is fixed to the center of the disk and the disk is spun. The colour of the sector at which it stops is recorded.
(iii) Recording the number of runs scored by a batsman in a single delivery in a cricket match.

2. Determine whether each event given below is a simple or a composite event.
(i) When rolling a tetrahedronal die numbered 1 to 4,
(*a*) getting the number 3
(*b*) getting a side with an odd number
(ii) In a pack of 5 identical cards labelled *A*, *B*, *C*, *D* and *E*,
(*a*) drawing the card labelled *A*
(*b*) drawing a card labelled with a vowel

3. When randomly taking a card from a bag containing 8 identical cards numbered 1 to 8,
(*a*) if the event of getting a card with a number greater than 4 is *A*, then write down the elements in *A*.
(*b*) Write 5 simple events in the event *A*.

4. There is a bag containing 10 identical pieces of paper numbered from 1 to 10. A piece of paper is drawn at random.

(i) Write the sample space.

(ii) If the event of drawing a square number is X, then write the elements of X and the value of $n(X)$.

(iii) Find the probability $P(X)$ of getting a square number.

5. 3 of 5 identical beads are blue and the remaining two are red. *A* bead is drawn randomly.

(i) Write the sample space.

(ii) Find the probability of drawing a red bead.

(iii) Find the probability of drawing a blue bead.

6. There are toffees of the same size and shape, but of different brands in a box. The table below gives information on them.

	Mango Flavoured	Orange Flavoured
Brand A	2	1
Brand B	3	2

A toffee is drawn randomly from the box. Find the probability of getting:

(i) an orange flavoured toffee
(ii) a toffee of brand A
(iii) a toffee of brand B
(iv) a mango flavoured toffee of brand A
(v) an orange flavoured toffee of brand B

30.2 The intersection and union of two events

If A and B are two events, then their intersection $A \cap B$ and their union $A \cup B$ are also events.

For example, suppose there are 5 identical balls numbered 1, 2, 3, 4, 5 and one is picked at random. Then,

the sample set $S = \{1, 2, 3, 4, 5\}$.

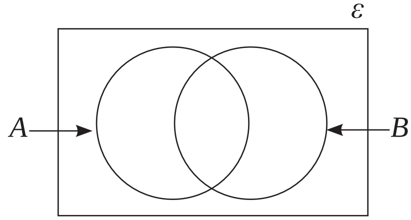
If we denote the event of picking a ball with a number greater than 2 as A,
$A = \{3, 4, 5\}$.

If we denote the event of picking an even numbered ball as B,
$B = \{2, 4\}$.

Then, $A \cap B = \{4\}$. Here, $A \cap B$ represents the event of picking a ball in both the sets A and B; that is, a ball with an even number greater than 2.

Furthermore, $A \cup B = \{2, 3, 4, 5\}$. Here, $A \cup B$ denotes the event of picking a ball in either set A or in set B; that is, a ball with either an even number or a number greater than 2.

Now let us consider a relationship between the events A, B, $A \cup B$ and $A \cap B$ where A and B are any two events in a sample space with equally likely outcomes.



From our knowlege of sets, we have the formula

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

Divide each term by $n(S)$ to get

$$\frac{n(A \cup B)}{n(S)} = \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)}.$$

Since the outcomes are equally likely, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Thus, for any two events A and B in a sample space with equally likely outcomes, we have

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

Therefore for the example discussed above, we have

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{5},$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{5},$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{5}$$

and $$P(A \cup B) = \frac{n(A \cup B)}{n(S)} = \frac{4}{5}.$$

Also, $P(A) + P(B) - P(A \cap B) = \frac{3}{5} + \frac{2}{5} - \frac{1}{5}$

$$= \frac{4}{5}$$

So the formula $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ is true for this example.

Mutually exclusive events

A fair tetrahedronal die with its sides numbered from 1 to 4 is rolled. Let us denote the event that it rests on an even numbered face by A and the event that it rests on an odd numbered face by B.

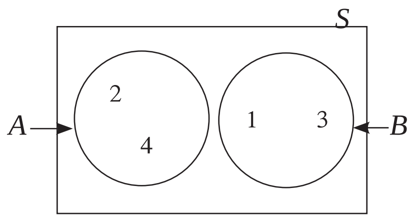
That is, $A = \{2, 4\}$ and $B = \{1, 3\}$.

Then $A \cap B = \emptyset$. This means that A and B have no common elements.

That is, these two events do not occur together. Such events are said to be **mutually exclusive events**.

If $A \cap B = \emptyset$. then A and B are mutually exclusive.

Now, let us find $A \cup B$ in the example we were talking about. Let us show the given information in a Venn diagram.



Then,

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{0}{4} = 0$$

Because $A \cap B = \emptyset$ when A and B are mutually exclusive, $P(A \cap B) = 0$

Therefore

If A and B are mutually exclusive events, $P(A \cup B) = P(A) + P(B)$

Complement of an event

There is a pack of 5 identical cards numbered from 1 to 5. Consider the random experiment of drawing a card randomly.

The sample space here is $S = \{1, 2, 3, 4, 5\}$.

If A is the event of drawing an even numbered card, then $A = \{2, 4\}$.

If the event of A not happening, that is, the event of not drawing an even numbered card is B, then, $B = \{1, 3, 5\}$.
In the above experiment, if the event of drawing an even numbered card is A, then the event of drawing a card which is not even numbered is the complement of A.
The complement of A is written as A' .
Therefore $A' = \{1, 3, 5\}$.

Here $A \cup A' = S$

Also $A \cap A' = \emptyset$.

Therefore, A and A' are mutually exclusive events.

These results are true for any event.

Accordingly $P(A \cup A') = P(A) + P(A')$

$$\therefore P(S) = P(A) + P(A')$$

$$\therefore 1 = P(A) + P(A') \qquad [\text{since } P(S) = 1]$$

$$\therefore P(A') = 1 - P(A)$$

$$\boxed{\text{For any event } A,\ P(A') = 1 - P(A)}$$

Example 1

For the events A and B of a random experiment,

$$P(A) = \frac{2}{7}\ P(B) = \frac{3}{7} \text{ and } P(A \cap B) = \frac{1}{14}.$$

Find (i) $P(A \cup B)$ (ii) $P(A')$ (iii) $P(B')$ (iv) $P[(A \cap B)']$ (v) $P[(A \cup B)']$

(i) Using the formula $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$P(A \cup B) = \frac{2}{7} + \frac{3}{7} - \frac{1}{14}$$

$$= \frac{4}{14} + \frac{6}{14} - \frac{1}{14} = \underline{\underline{\frac{9}{14}}}$$

(ii) Using the formula $P(A') = 1 - P(A)$

$$P(A') = 1 - \frac{2}{7}$$

$$= \frac{7}{7} - \frac{2}{7}$$

$$= \underline{\underline{\frac{5}{7}}}$$

(iii) Using the formula $P(B') = 1 - P(B)$

$$P(B') = 1 - \frac{3}{7}$$

$$= \frac{7}{7} - \frac{3}{7}$$

$$= \underline{\underline{\frac{4}{7}}}$$

(iv) $P[(A\cap B)'] = 1 - P(A\cap B)$

$$= 1 - \frac{1}{14}$$

$$= \frac{14}{14} - \frac{1}{14}$$

$$= \underline{\underline{\frac{13}{14}}}$$

(v) $P[(A\cup B)'] = 1 - P(A\cup B)$

$$= 1 - \frac{9}{14}$$

$$= \frac{14}{14} - \frac{9}{14}$$

$$= \underline{\underline{\frac{5}{14}}}$$

Example 2

X and Y are two mutually exclusive events of a random experiment.

$P(X) = \frac{1}{6}$ and $P(Y) = \frac{7}{12}$.

Find (i) $P(X\cap Y)$ (ii) $P(X\cup Y)$

(i) Since X and Y are mutually exclusive events, $P(X\cap Y) = 0$.

$$P(X\cup Y) = P(X) + P(Y)$$

$$= \frac{1}{6} + \frac{7}{12}$$

$$= \frac{2}{12} + \frac{7}{12} = \underline{\underline{\frac{9}{12}}} = \underline{\underline{\frac{3}{4}}}$$

Exercise 30.2

1. In a random experiment of rolling a fair die numbered from 1 to 6, let
A be the event that a prime number is obtained,
B be the event that a perfect square is obtained,
C be the event that a number greater than 4 is obtained and
D be the event that a multiple of 6 is obtained.

Select all pairs of mutually exclusive events.

2. Let X and Y be two events of a random experiment which are not mutually exclusive.

If $P(X) = \frac{1}{4}$, $P(Y) = \frac{5}{6}$ and $P(X\cap Y) = \frac{1}{6}$, find each of the following:

(i) $P(X\cap Y)$ (ii) $P(X')$ (iii) $P(Y')$ (iv) $P[(X\cap Y)']$ (v) $P[(X\cup Y)']$

3. A and B are two events of a random experiment.

$P(A) = \frac{2}{7}$ and $P(B') = \frac{1}{4}$. Find $P(A')$ and $P(B)$.

4. X and Y are two events of a random experiment. It is given that

$P(X) = \frac{1}{2}$, $P(Y) = \frac{1}{3}$ and $P(X \cup Y) = \frac{5}{6}$.

(i) Find $P(X \cap Y)$

(ii) There by show that X and Y are mutually exclusive.

5. X, Y and Z are three events of a random experiment. If

$P(X) = \frac{1}{6}$, $P(Y) = \frac{1}{9}$, $P(Z') = \frac{2}{3}$, $P(X \cap Y) = \frac{1}{18}$ and $P(X \cap Z) = \frac{1}{12}$.

Find the following:

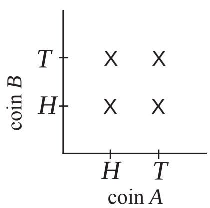
(i) $P(X)$ (ii) $P(Y')$ (iii) $P(Z)$ (iv) $P(X \cup Y)$ (v) $P[(X \cup Z)']$

30.3 Representation of the sample space in a grid

Consider a random experiment of tossing two unbiased identical coins A and B simultaneously. Let us denote the heads by H and the tails by T. The set of all possible outcomes of this experiment can be listed as follows:

Getting heads in both coins, (H, H)
Getting heads in coin A and tails in coin B, (H, T)
Getting tails in coin A and heads in coin B, (T, H)
Getting tails in both coins, (T, T)

Accordingly, the sample space can be written as $\{(H, H), (H, T), (T, H), (T, T)\}$. This sample space can be represented in a grid as follows:



Here, the outcomes are denoted by 'x'

The sample space of this experiment consists of four outcomes. Since the coins are unbiased and identical, all the possible outcomes are equally likely and thus the following probabilities are obtained:

(i) The probability that both coins show heads $= \frac{1}{4}$

(ii) The probability that coin A shows heads and coin B shows tails $= \frac{1}{4}$

(iii) The probability that one coin shows tails and the other coin shows heads $= \frac{2}{4}$

(iv) The probability that both coins show tails $= \frac{1}{4}$

Note: In the above random experiment all the outcomes were equally likely. Even though it is not compulsory to represent the outcomes in a grid, finding the probability by this method is not possible when the outcomes are not equally likely.

Example 1

Let us consider the experiment of tossing a coin and rolling a tetrahedronal die numbered from 1 to 4, and recording the faces touching the table.

(i) Show the sample space as a set of ordered pairs on a grid and then represent it on a grid.

(ii) Find the probability of getting

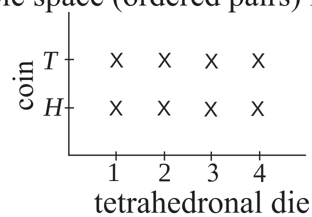
(*a*) 1 on the die.

(*b*) an even number on the die and tails on the coin.

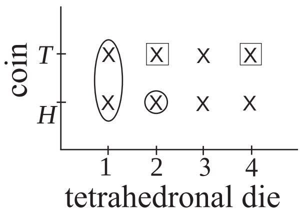
(*c*) 2 on the die and heads on the coin.

(i) $S = \{(1,H), (2,H), (3,H), (4,H), (1,T), (2,T), (3,T), (4,T)\}$

Let us show the sample space (ordered pairs) in a grid.



(ii) It is clear that all the results here are equally likely.



(*a*) In the above grid, the area marked by ( ) are the elements of the event of getting 1 on the die. There are 2 elements there. The total number of elements in the sample space is 8.

$\therefore$ The probability of getting 1 on the die $= \frac{2}{8} = \frac{1}{4}$

(*b*) In the above grid, the area marked by $\square$ are the elements of the event of getting an even number on the die and tails on the coin. There are 2 such elements.

$\therefore$ The probability of getting an even number on the die and tails on the coin $\Big\} = \frac{2}{8} = \frac{1}{4}$

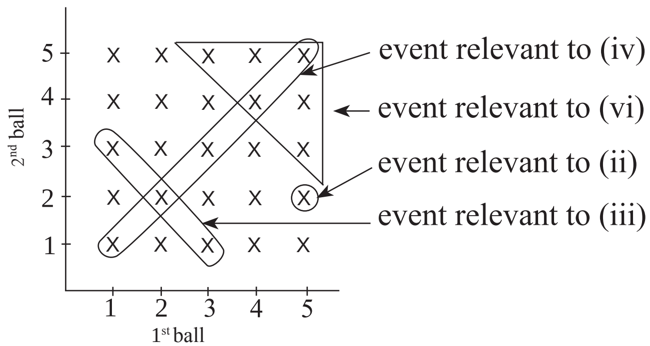
(*c*) In the above grid, the area marked by $\bigcirc$ are the elements of the event of getting two on the die and heads on the coin. There is one such element.

$\therefore$ The probability of getting two on the dice and heads on the coin $= \frac{1}{8}$

Example 2

There are 5 identical balls numbered from 1 to 5 in a bag. A ball is taken from the bag randomly and the number is recorded. Then the ball is put back in the bag (with replacement) and again a ball is randomly taken for a second time and this number is also recorded.

(i) Show the relevant sample space in a grid.



(ii) Find the probability that the first ball is numbered 5 and the second ball is numbered 2.

$$\frac{1}{25}$$

(iii) Find the probability that the sum of the numbers on the two balls is 4.

$$\frac{3}{25}$$

(iv) Find the probability that the same ball is taken on both occassions.

$$\frac{5}{25} = \frac{1}{5}$$

(v) Are the events in (ii) and (iv) mutually exclusive?
Yes. This is because the two events have no common elements.

(vi) Find the probability that the sum of the numbers on the two balls taken is greater than 7.

$$\frac{6}{25}$$

(vii) Are the events in (iii) and (iv) mutually exclusive?

No.The reason is that there is an element common to both events; i.e., (2,2).

Exercise 30.3

1. A fair die numbered from 1 to 6 is rolled and an unbiased coin is tossed at the same time. The side on top is recorded in each case. Consider this experiment.

(*a*) Show the sample space in a grid.

(*b*) Use it to find the probabilities of the following events.

(i) Getting 1 on the die and heads on the coin.

(ii) Getting an even number on the die and heads on the coin.

(iii) Getting tails on the coin.

2. Two fair dice numbered from 1 to 6 are rolled simultaneously and the side shown is recorded. Consider this experiment.

(*a*) Show the sample space in a grid.

(*b*) Use it to find the probabilities of the following events.

(i) The sum of the two numbers being 5.

(ii) The sum of the two numbers being greater than 10.

(iii) The two numbers being the same.

(iv) Getting 3 on the first die.

3. A bag contains identical beads. There are 3 red beads, one blue bead, and 2 yellow beads. These are named R_1, R_2, R_3, B, Y_1, Y_2. A bead is taken randomly, its colour is recorded and then put back in the bag. A bead is randomly taken from the bag again and its colour is recorded.

(*a*) Show the sample space in a grid.

(*b*) Use it to find the probabilities of the following events.

(i) The first bead being red and the second bead being yellow.

(ii) Both beads being red.

(iii) Both beads being the same colour.

(iv) Getting at least one blue bead.

(v) Write all the pairs of mutually exclusive events in the above questions (i) – (iv).

4. There are 5 roads labelled *A, B, C, D,* and *E* that meet at a junction. Here, it is possible to enter or exit from any road. Draw a grid showing all the possible ways in which a person can enter and exit and find the probabilities of the following events. (Assume that all the possible outcomes are equally likely.)

(i) Entering from *A* and exiting from *B*.

(ii) Entering from *A* or *B* and exiting from *D*.

(iii) Entering from *E*

(iv) Entering and exiting from different roads.

5. A plant has 4 red flowers and 3 yellow flowers of identical shape and size. 2 butterflies, *A* and *B*, come to the flowers to drink nectar. It is possible for both to drink nectar from the same flower at the same time. Draw a grid of the sample space, showing all the possible ways the butterflies can pick flowers to get nectar and find the probabilities of the following events. (Assume that the butterflies pick flowers randomly.)
(i) Butterfly *A* picking a red flower and butterfly *B* picking a yellow flower.
(ii) Both butterflies picking flowers of the same colour.
(iii) Both butterflies picking flowers of different colours.
(iv) Both butterflies picking the same flower.

30.4 Independent events

Consider the following random experiments.

(i) Two unbiased coins are tossed simultaneously and the sides shown are recorded. It is clear that whatever the result of one coin toss is, it will not affect the result of the other.

(ii) There are two bags containing some identical balls. One ball is taken randomly from each bag. It is clear that the ball taken from one bag will not affect what ball is taken from the other bag.

(iii) During the germination of a few planted seeds, germination of one seed does not have an impact on the germination of the other seeds.

In this way, if one event does not affect another, the two events are called **independent**.
The independence of two events is defined as follows.

$$\boxed{\text{If } P(A \cap B) = P(A).P(B) \text{ then } A \text{ and } B \text{ are independent.}}$$

We have learnt that two mutually exclusive events are two events that do not occur together. But what is meant by 2 events being independent is that the occurrance of one event does not affect the occurrance of the other event.

Example 1

X and *Y* are two independent events. $P(X) = \frac{1}{3}$ and $P(Y) = \frac{1}{4}$, Find $P(X \cap Y)$, and $P(X \cup Y)$.

Because *X* and *Y* are independent events

$$P(X \cap Y) = P(X).P(Y).$$

$$\therefore \quad P(X \cap Y) = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}.$$

Using the formula $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

$$P(X \cup Y) = \frac{1}{3} + \frac{1}{4} - \frac{1}{12}$$

$$= \frac{4+3-1}{12}$$

$$P(X \cup Y) = \frac{6}{12} = \frac{1}{2}$$

Example 2

The probability that candidate A will pass an examination is $\frac{1}{5}$ and the probability that candidate B will pass is $\frac{3}{10}$. Assume that these events are independent and find the probabilities of the events below.

(i) Both of them passing

(ii) One of them passing.

Let us denote the event of candidate A passing as A and candidate B passing as B.

(i) The probability of both candidates A and B passing is $P(A \cap B)$.

Because they are independent events,

$$P(A \cap B) = P(A).\ P(B)$$

$$= \frac{1}{5} \times \frac{3}{10} = \frac{3}{50}$$

(ii) The probability that one of the candidates pass is $P(A \cup B)$ Then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\therefore\ P(A \cup B) = \frac{1}{5} + \frac{3}{10} - \frac{3}{50}$$

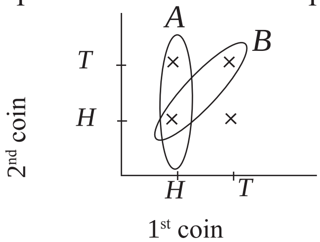
$$= \frac{10+15-3}{50}$$

$$= \frac{22}{50}$$

$$= \frac{11}{25}$$

Example 3

Two unbiased identical coins are tossed simultaneously. Let us represent the sample space of this random experiment in a grid.



Take the event of the first coin being heads as A. Take the event of both the coins showing the same side as B.

Here, the event of A or B does not affect the other event; therefore A and B are independent events.
Let us find the probabilities relevant to A and B.

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{4}$$

Further, $P(A).P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

That is, $P(A \cap B) = P(A).P(B)$

$\therefore$ A and B are independent events.

Exercise 30.4

1. X and Y are independent events. $P(X) = \frac{1}{2}$ and $P(X \cap Y) = \frac{1}{3}$
 (i) Find $P(Y)$.
 (ii) Find $P(X \cup Y)$.

2. An unbiased coin and a fair die numbered from 1 to 6 are tossed simultaneously.
 (*a*) Represent the sample space relevant to this experiment in a grid.
 (*b*) Take the event of the coin showing heads as A and the event of the die showing the number 4 as B. Show these events on the grid and find the probabilities of the events below.
 (i) $P(A)$ (ii) $P(B)$ (iii) $P(A \cap B)$ (iv) $P(A \cup B)$

3. A bag contains 3 red beads and 2 blue beads, all of which are otherwise identical. A bead is taken out randomly and its colour is recorded and then it is put back in the bag. A bead is randomly taken out again and its colour is recorded. Find the probabilities of the following events.

(i) Both of the beads being red.

(ii) The first bead being blue and the second bead being red.

(iii) The first bead being red and the second bead being blue.

(iv) Both beads being blue.

30.5 Tree Diagrams

Tree diagrams can be used to find the probabilities of events of a random experiment. Let us consider the following example in order to understand this method.

Example 1

An unbiased coin is tossed twice and in each case the outcomes is recorded. Draw the relevant tree diagram and find the probabilities of the following events.

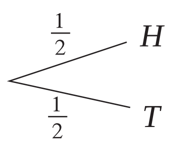
(i) Getting heads both times.

(ii) Getting the same side both times.

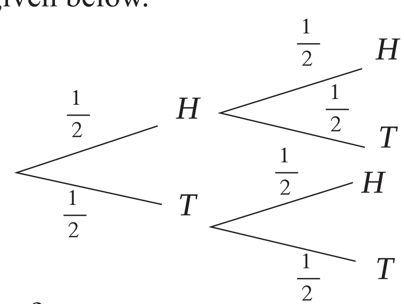
(iii) Getting tails at least once.

(vi) Getting heads the second time.

This experiment can be separated into two. That is, the first toss and the second toss. The two outcomes of the first toss can be represented in a tree diagram with two branches as shown below.



Here the corresponding probabilities are given on the branches. We know the probabilities are $\frac{1}{2}$ (since the coin is unbiased). For the second toss we can extend the tree diagram as given below.



Here too the probabilities are given on the branches. As the 1st and 2nd attempts are independent, both the probabilities are $\frac{1}{2}$ each. There are 4 paths from start to end. That is,

(i) Heads in the 1st attempt and heads in the 2nd attempt

(ii) Heads in the 1st attempt and tails in the 2nd attempt

(iii) Tails in the 1st attempt and heads in the 2nd attempt

(iv) Tails in the 1st attempt and tails in the 2nd attempt

All the possible outcomes are represented above.

As the events given by the 1st and the 2nd attempts are independent, to find the probability of each outcome, we can take the product of the relevant probabilities.

i.e., P (Heads in the 1st attempt, and heads in the 2nd attempt)

$$= \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{4}$$

Likewise,

P (Heads in the 1st attempt and tails in the 2nd attempt) = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

P (Tails in the 1st attempt and tails in the 2nd attempt) = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

P (Tails in the 1st attempt and heads in the 2nd attempt) = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Sample space of the experiment

$$S = \{(H,H), (H,T), (T,H), (T,T)\}$$

The probabilities in short are

$$P(H,H) = \frac{1}{4}$$

$$P(H,T) = \frac{1}{4}$$

$$P(T,H) = \frac{1}{4}$$

$$P(T,T) = \frac{1}{4}$$

Now let us answer the questions.

(i) P (getting heads both times) $= P\,(H, H)$

$= \dfrac{1}{4}$

(ii) P (getting the same side both times) $= P((H, H) \text{ or } T, T))$

$= P\,(H, H) + P\,(T, T)$

(Because the two events are mutually exclusive)

$= \dfrac{1}{4} + \dfrac{1}{4} = \dfrac{1}{2}$

(iii) Getting tails at least once $= 1 - P$ (Getting heads both times)

$= 1 - P(H, H)$

$= 1 - \dfrac{1}{4} = \dfrac{3}{4}$

(iv) Getting heads in the 2nd attempt $= P\,((H, H) \text{ or } (T, H))$

$= \dfrac{1}{4} + \dfrac{1}{4} = \dfrac{1}{2}$

(Because the two events are mutually exclusive)

Exercise 30.5

1. A box contains 4 red pencils and two black pencils. A pencil is taken from the box at random and returned back to the box after its color is recorded, and then another pencil is taken out of the box. Depict the sample space as a tree diagram and hence find the probability that

(i) both pencils are red.

(ii) the two pencils are of opposite colours.

(iii) the two pencils are of the same colour.

2. Sarath and Sunith work at the same workplace. They use the bus for their daily travel. The probability that Sarath is late to work is $\dfrac{1}{3}$ and that for Sunith is $\dfrac{1}{4}$. Depict the relevant sample space in a tree diagram and hence find the probability that

(i) both are not late

(ii) only one person is late.

3. The probability that the shooter in a netball team shoots the ball correctly is $\dfrac{3}{5}$. Draw the sample space of two shots in a tree diagram and find the probability of

(i) shooting correctly both times

(ii) shooting correctly once

Miscellaneous Exercises

1. In a survey of 25 students conducted to see who likes to drink tea and coffee, it was found that 17 students like to drink tea, 15 like to drink coffee and 10 like to drink both tea and coffee.

(*a*) Draw a Venn diagram illustrating this information.

(*b*) Use it to find the probability that a student

(i) likes to drink only tea

(ii) likes to drink only one of these two types

(iii) likes to drink either tea or coffee

(iv) does not like to drink either tea or coffee.

2. In a mixed school, each student who studies in the biological stream and in the mathematics stream has to take either Exam P_1 or P_2. Below is the actual classification of the 100 students in the two streams.

Type Of Exam	Sex	Biological Stream	Mathematics Stream
P_1	Girl	10	5
	Boy	20	5
P_2	Girl	30	10
	Boy	15	5

If a student is picked randomly, find the probability that this student is

(i) a girl

(ii) following the mathematics stream

(iii) doing Exam P_1

(iv) a boy following the mathematics stream and doing exam P_2

(v) given that the student is a girl, what is the probability of her following the biological stream?

3. Following is a quote from an advertisement:

"Every one in 7 of the tickets in this lottery is a winning ticket."

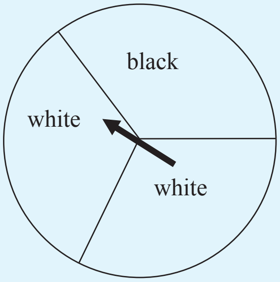
Hearing this, a person purchased two of the tickets of this lottery.

(*a*) Draw the relevant tree diagram.

(*b*) Find the probability of

(i) both tickets being winning tickets.

(ii) at least one of the tickets being a winning ticket.

4. 

A circular disk, as shown in the figure, is divided into three equal sectors. Two sectors are painted white and the third is painted black. A pointer at the centre is free to rotate. The pointer is given a spin and the colour of the sector at which the pointer stops is recorded.

Draw a tree diagram for two such spins and hence find the probability that the pointer stops in

(i) a white area in both instances

(ii) a black area at least once.

5. 10% of the candidates who applied for a job qualified through a competitive examination. Of those who qualified, 60% were selected in the first round. Find the probability that a randomly picked applicant is selected in the first round.

6. There are four choices for each question in a multiple choice question paper, out of which only one answer is correct. A student who is unsure of the answers for two of the questions chooses random answers. Draw a tree diagram and hence find the probability that

(i) the same choice number is selected for both questions.

(ii) both answers are correct

(iii) at least one answer is correct.

7. A and B are two government servants who work in the same office. They are entitled to a days leave on any one of the five working days in a week. Assuming that each of A and B gets his leave at random on a grid depict the sample space of all the possible ways in wihch they both can get leave during the five days of a week. Hence find the probability of each of the following events.

(i) A taking leave on Monday and B on Wednesday.

(ii) B taking leave on a day previous to A.

(iii) B taking leave on a day after A.

(iv) Both taking leave on the same day.