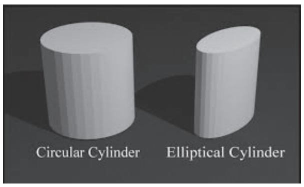
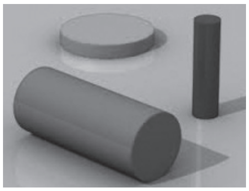
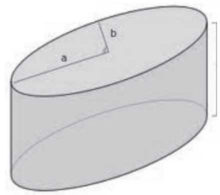
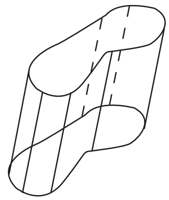
29 Surface Area and Volume

By studying this lesson you will be able to

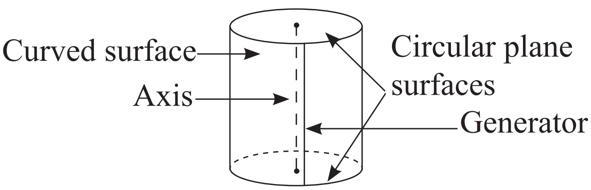
- calculate the surface area and volume of a right circular cylinder
- calculate the surface area and volume of a right triangular prism

The Cylinder



The cross sections of the solids given above are uniform, and the plane surfaces at the two ends are parallel to each other. Solids having such shapes are defined in general as cylinders.



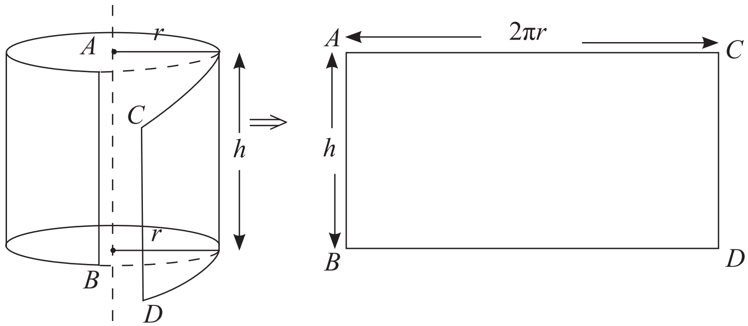
The cylinder in the figure has two circular plane surfaces at the top and the bottom. Apart from these, it also has a curved surface. The radii of the two circular plane surfaces are equal to each other. Therefore, the areas of these two plane surfaces are equal. The straight line joining the centres of these two circles is called the axis of the cylinder. Any straight line on the curved surface which is parallel to the axis of the cylinder is called a generator of the cylinder.

The axis of the cylinder is perpendicular to the two circular plane surfaces. Therefore, such a cylinder is called a **right circular cylinder**. (There are cylinders which are not right circular. However such cylinders will not be discussed here.) What is meant by the term “right” is that the axis of the cylinder is perpendicular to the two plane surfaces. What is meant by the term “circular” is that any cross section which is perpendicular to the axis is circular in shape.

The radius of a plane face of the cylinder is usually denoted by r and the length of the axis is usually denoted by h. The radius r of the circular faces is called the radius of the cylinder and the length of the axis h is called the height of the cylinder.

29.1 Surface area of a right circular cylinder

When the radius and the height of a cylinder are given, the areas of the three surface parts of the cylinder need to be added together to obtain the total surface area. The areas of the two circular faces at the two ends can be calculated using the formula for the area of a circle. A mechanism such as the following can be used to calculate the area of the curved surface



When the curved surface of the cylinder is cut along a generator as shown in the figure and opened out, a rectangle is obtained. The length of one side of this rectangle is equal to the height (h) of the cylinder while the length of the other side is equal to the perimeter of one of its circular plane faces.

The area of this rectangle is equal to the area of the curved surface of the cylinder. Accordingly, an expression for the curved surface of the cylinder can be developed in the following manner.

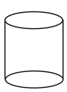
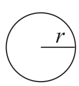
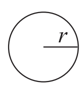
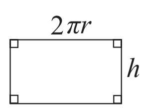
Area of the curved surface of the cylinder = Length of one side of the rectangle × Length of the other side of the rectangle

$= 2\pi r \times h$

$\therefore$ The area of curved surface of the cylinder is $\underline{\underline{2\pi rh}}$.

Now we can find the total surface area of the cylinder in the following manner.

Total surface area of the cylinder = Area of top face + Area of bottom face + Area of curved surface

 =  +  + 

$$A = \pi r^2 + \pi r^2 + 2\pi rh$$

$$\boxed{A = 2\pi r^2 + 2\pi rh}$$

Note:

(i) Exterior surface area of a cylinder without a lid $= \pi r^2 + 2\pi rh$

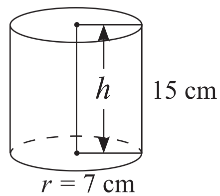
(ii) Exterior surface area of a cylinder without a base or a lid $= 2\pi rh$

Let us now consider several solved problems related to the surface area of a cylinder. In this lesson the value of π is taken as $\frac{22}{7}$ which is an approximate value for π.

Example 1

Determine the following for a cylindrical shaped solid log of base radius 7 cm and height 15 cm.

(i) Area of one plane face
(ii) Area of the curved surface
(iii) Total surface area

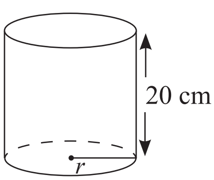


(i) Area of one plane face
$$= \pi r^2$$
$$= \frac{22}{7} \times 7 \times 7 \text{ cm}^2$$
$$= \underline{\underline{154 \text{ cm}^2}}$$

(ii) Area of the curved surface
$$= 2\pi rh$$
$$= 2 \times \frac{22}{7} \times 7 \times 15 \text{ cm}^2$$
$$= \underline{\underline{660 \text{ cm}^2}}$$

(iii) Total surface area
$$= 2\pi r^2 + 2\pi rh$$
$$= 2 \times (154) + 660 \text{ cm}^2$$
$$= 308 + 660 \text{cm}^2$$
$$= \underline{\underline{968 \text{ cm}^2}}$$

Example 2



The circumference of the base of a cylindrical vessel without a lid, of height 20 cm is 88 cm.

(i) Find the radius of the base.
(ii) Find the total exterior surface area.

Let us denote the base radius by r and the height by h.

(i) Circumference of the base $= 2\pi r$

$$\therefore 2\pi r = 88$$

$$\therefore \quad r = \frac{88}{2\pi} = \frac{88 \times 7}{2 \times 22}$$

$$\therefore \text{ The radius } \quad r = \underline{\underline{14 \text{ cm}}}$$

(ii) Total surface area $= \pi r^2 + 2\pi rh$

$$= \frac{22}{7} \times 14 \times 14 + 2 \times \frac{22}{7} \times 14 \times 20$$

$$= 616 + 1760$$

$$\therefore \text{ The total surface area } = \underline{\underline{2376 \text{ cm}^2}}$$

Example 3

The surface area of a solid metal cylinder is 2442 cm^2 while the sum of its radius and height is 37 cm.

(i) Find the radius of the cylinder.

(ii) Find the area of the curved surface of the cylinder.

Let us denote the radius of the cross section by r and the height by h.

(i) Sum of the radius and the height $= 37$ cm

That is, $\quad r + h = 37$ cm

The total surface area, $2\pi r^2 + 2\pi rh = 2442 \text{cm}^2$

$$\therefore 2\pi r(r+h) = 2442$$

$$\therefore 2\pi r(37) = 2442 \quad \text{(By substituting for } r+h\text{)}$$

$$\therefore r = \frac{2442 \times 7}{2 \times 22 \times 37}$$

$$= 10.5$$

$$\therefore \text{The radius } r = \underline{\underline{10.5 \text{ cm}}}$$

(ii)

$$r + h = 37 \text{ cm}$$

$$\text{Since } r = 10.5\text{cm}, \; h = 37 - 10.5 \text{ cm}$$

$$= 26.5 \text{ cm}$$

$$\therefore \text{The area of the curved surface} = 2\pi rh$$

$$= 2 \times \frac{22}{7} \times 10.5 \times 26.5 \text{ cm}^2$$

$$= \underline{\underline{1749 \text{ cm}^2}}$$

Exercise 29.1

1. The radius of a cylinder is 7 cm and its height is 12 cm.
 (i) Find the area of the two circular faces.
 (ii) Find the area of the curved surface.
 (iii) Find the total surface area.

2. Find the area of the metal sheet that is required to make 200 cylindrical tins of radius 3.5 cm and height 10 cm, without lids.

3. The total surface area of a cylindrical vessel with a lid is 5412 cm^2. If the area of the curved surface is 2640 cm^2,
 (i) find the total surface area of the two circular faces.
 (ii) find the radius of the cylinder.
 (iii) find the height of the cylinder.

4. The base of a cylindrical vessel with a lid, which has been produced using a thin metal sheet, has a circumference of 88 cm. If the area of the curved surface is 1078 cm^2, find the height of the vessel.

5. The area of the curved surface of a cylindrical tin with a lid is 990 cm^2.
 (i) If the height of the tin is 15 cm, find the base radius.
 (ii) Find the total area of the two circular faces.
 (iii) Find the total surface area.

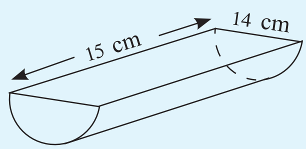
6. It is possible to paint an area of 13.5m^2 with one litre of a certain type of paint. The roof over the verandah of a certain house rests on 10 cylindrical pillars, each of height 3 m and diameter 28 cm. It has been decided to paint all these pillars.
 (i) Find the area of the curved surface of these 10 pillars to the nearest square metre.
 (ii) Find the required amount of paint in litres.
 (iii) If one litre of paint is Rs. 450, find the amount that has to be spent on the paint.

7. It is required to cover the total surface area of the curved surface of a right cylindrical shaped food container of radius 7 cm and height 10 cm with a label.

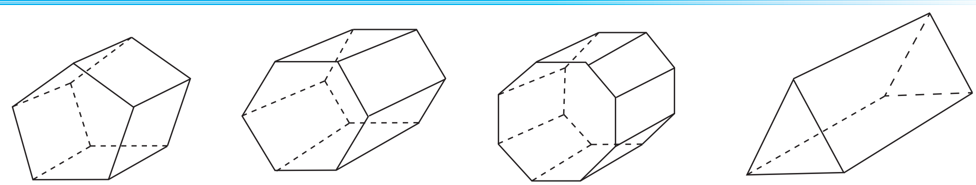
(i) How many labels can be cut out from a thin piece of paper of length 180 cm and breadth 90 cm such that the waste is minimized? Find the area of the piece of paper that goes waste.

(ii) Calculate how many such pieces of paper are required to cut out labels for 1200 containers of the above type.

8. The figure illustrates one half of a solid cylinder. Calculate the total surface area using the given information.

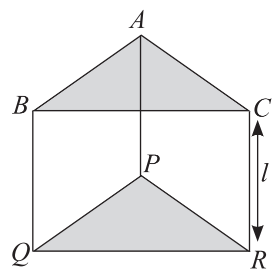


Prisms



The properties given below are common to the solids illustrated in the above figure.
- The cross section is uniform.
- The cross section takes the shape of a polygon.
- The side faces are rectangles.
- The faces at the two ends are perpendicular to the side faces.

Solids with these properties are called **right prisms**. From these right prisms, we will pay further attention to the one with a triangular cross section.



The figure illustrates a right prism with a triangular cross section. Here,

(1) ABC and PQR represent the two triangular faces at the two ends of the prism.

(2) the three rectangular side faces are represented by $BQRC$, $CRPA$ and $APQB$ (These faces are also called lateral faces).

(3) The distance between the two triangular faces is named the length or the height of a the prism and is denoted by l.

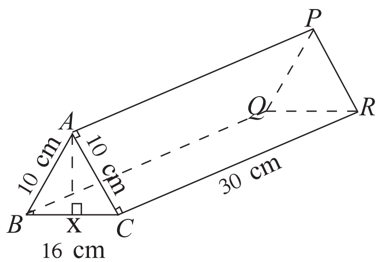
(4) The total surface area of the prism is the sum of the areas of the pair of triangular faces and the three rectangular faces.

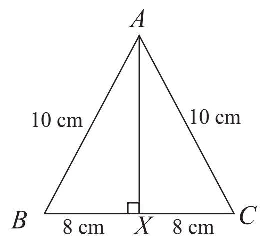
29.2 The surface area of a right prism with a triangular cross section

Example 1

Let us consider how the total surface area of the right prism with a cross section the shape of an isosceles triangle which is given below is found, using the given data.

Let us first find the area of the triangular face *ABC*. For this, let us find the perpendicular distance from A to *BC*.
According to the properties of isosceles triangles, if the midpoint of *BC* is *X*, then $AX \perp BC$. Now applying Pythogaras' theorem to the triangle *AXC*,





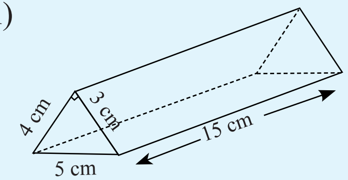
$$AC^2 = AX^2 + XC^2$$
$$10^2 = AX^2 + 8^2$$
$$100 - 64 = AX^2$$
$$\therefore\ 36 = AX^2$$
$$\therefore\ AX = \sqrt{36} \quad \text{(Since lengths cannot take negative values)}$$
$$\therefore\ AX = 6\text{ cm}$$

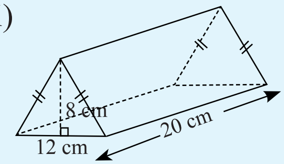
Accordingly, the area of the triangular face *ABC* $= \frac{1}{2} \times 16\text{ cm} \times 6\text{ cm} = 48\text{ cm}^2$

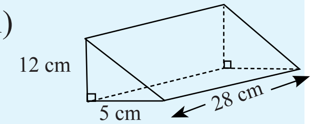
$\therefore$ The area of the two triangular faces *ABC* and *PQR* $= 2 \times 48\text{ cm}^2 = 96\text{ cm}^2$
The area of the rectangular face *ACRP* $= 10\text{ cm} \times 30\text{ cm} = 300\text{ cm}^2$
The area of the rectangular face *APQB* $= 10\text{ cm} \times 30\text{ cm} = 300\text{ cm}^2$
The area of the rectangular face *BCRQ* $= 16\text{ cm} \times 30\text{ cm} = 480\text{ cm}^2$
$\therefore$ The total surface area $= 96 + 300 + 300 + 480\text{ cm}^2$
$= \underline{\underline{1176\text{ cm}^2}}$

Exercise 29.2

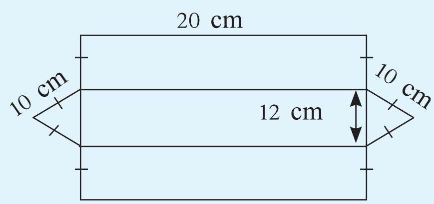
1. Find the total surface area of each of the following prisms.

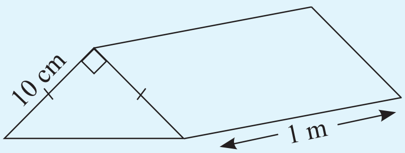
(i) 

(ii) 

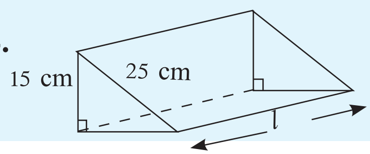
(iii) 

2. Find the total surface area of the right prism with a triangular cross section that can be made with the following net with the given measurements.



3. 

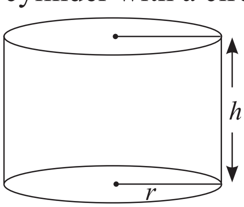
Find the surface area of the prism in the figure.

4. 

If the total surface area of the solid wooden prism in the figure is 2100cm^2, find the length of the prism (l).

29.3 Volume of a cylinder

Recall what you have learnt in previous grades about calculating the volume of solids with a uniform cross section. You calculated the volume by multiplying the area of the cross section by the height. We can calculate the volume of a right cylinder with a circular cross section in the same manner.



Let us consider a right circular cylinder of base radius r and perpendicular height h. Let us denote its volume by V.

The volume of the cylinder = Area of the cross section × Height

$$= \pi r^2 \times h = \pi r^2 h$$

$$\boxed{\text{Volume of the cylinder } (V) = \pi r^2 h}$$

Let us now consider the following solved problems related to the volume of a right circular cylinder.

Example 1

Find the volume of a right circular cylinder of radius 14 cm and height 20 cm.

Here $r = 14$ cm

$h = 20$ cm

$\therefore$ The volume of the cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 14 \times 14 \times 20 \text{ cm}^3$$

$$= \underline{\underline{12320 \text{ cm}^3}}$$

Example 2

The volume of a cylindrical vessel of base area 346.5 cm² is 6930 cm³.

(i) Find the radius of the cylinder.

(ii) Find the height of the cylinder.

(i)

Area of the base of a cylinder of radius r $= \pi r^2$

$$\therefore \pi r^2 = 346.5$$

$$\therefore r^2 = \frac{346.5}{22} \times 7$$

$$\therefore r^2 = 110.25$$

$$\therefore r = \pm 10.5 \quad \text{(Length cannot be negative)}$$

$$\therefore \text{The radius } (r) = \underline{\underline{10.5 \text{ cm}}}$$

(ii) Since the volume is 6930 cm^3

$$\pi r^2 h = 6\,930$$

$$346.5 \times h = 6\,930$$

$$\therefore h = \frac{6\,930}{346.5}$$

$$\therefore \underline{\underline{h = 20 \text{ cm}}}$$

Method 2

$$\pi r^2 h = 6930$$

$$\therefore \frac{22}{7} \times 10.5 \times 10.5 \times h = 6930$$

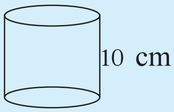
$$\therefore h = \frac{6930 \times 7}{22 \times 10.5 \times 10.5}$$

$$\therefore \text{Height} = \underline{\underline{20 \text{ cm}}}$$

Exercise 29.3

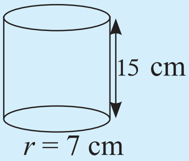
1. Find the volume of each of the following cylinders, based on the data that is given.

(i)

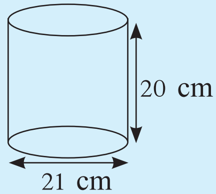


Cross sectional area 400 cm^2

(ii)



(iii)



2. (i) Complete the following table by finding the cross sectional area and volume of three cylinders, each of radius 7 cm and height 8 cm, 16 cm and 24 cm respectively.

Base radius	Cross sectional area	Height	Volume
(*a*) 7 cm (*b*) 7 cm (*c*) 7 cm		8 cm 16 cm 24 cm	

(ii) By considering the data in the above completed table, explain how the volume of a cylinder changes when the radius is a constant and the height is doubled and tripled.

3. (i) Complete the following table by finding the area of the cross section and the volume of three cylinders, each of height 20 cm and of radius 7 cm, 14 cm and 21 cm respectively.

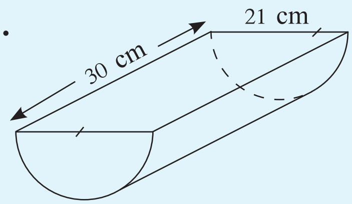
Base radius	Cross sectional area	Height	Volume
(*a*) 7 cm (*b*) 14 cm (*c*) 21 cm		20 cm 20 cm 20 cm	

(ii) By considering the data in the above completed table, explain how the volume of a cylinder changes when the height is a constant and the radius is doubled and tripled.

4. The diameter of a cylindrical shaped vessel is 28 cm. If the vessel contains a volume of 6160 cm^3 of water, find the height of the water level.

5. A rectangular metal sheet is of length 22 cm and breadth 11 cm. Draw the two cylinders that can be constructed with this sheet such that each side forms the curved edge, mark the measurements on the figure, and find the volume of each cylinder.

6. Find the volume of a right circular cylinder of diameter 20 cm and curved surface area 1000 cm^2.

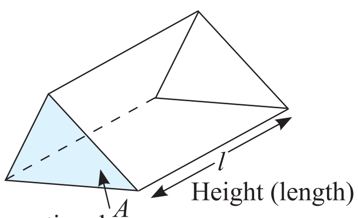
7. 

Calculate how many solid metal cylinders of height 21 cm and radius 3.5 cm can be made without wastage by heating the solid semi-cylindrical metal object with the measurements given in the figure.

8. A cylindrical vessel of radius 14 cm has been filled with water to a height of 30 cm. What is the minimum number of times that a cylindrical vessel of radius 7 cm and height 10 cm should be used to remove all the water in the given vessel?

29.4 Volume of a prism

Let us consider how the volume of a prism with a triangular cross section which you identified in 28.2 above is found.



We know that the volume of a right solid with a uniform cross section is equal to the product of the area of the cross section and the height (length).

We can use the above principle to find the volume of the right prism with a uniform triangular cross section given in the figure.

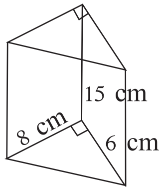
Then,

Volume of the prism = Area of the cross section × Perpendicular height (length)
$V = Al$

Note:
When the area A of the triangular cross section is not given directly, it is necessary to calculate it using the data related to the triangular cross section given in the problem.

Now consider the solved problems related to the volume of a prism given below.

Example 1



Based on the information in the figure,

(i) find the area of the cross section of the prism.

(ii) find the volume of the prism.

(i) Area of the cross section $= \frac{1}{2} \times 6 \times 8 = \underline{\underline{24 \text{ cm}^2}}$

(ii) Volume of the prism $=$ Area of the cross section $\times$ Height

$= 24 \text{ cm}^2 \times 15 \text{ cm}$

$= \underline{\underline{360 \text{ cm}^3}}$

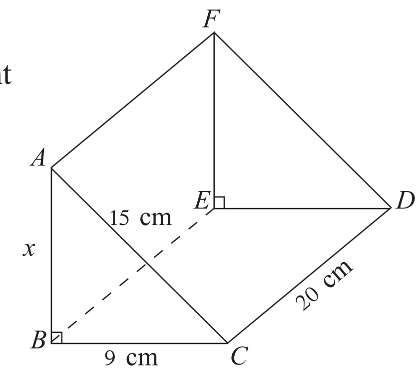
Example 2

A prism with a cross section the shape of a right angled triangle is shown in the figure.

(i) Find the length denoted by x in the cross section.

(ii) Find the area of the cross section.

(iii) Find the volume of the prism.



(i) By applying Pythagoras' theorem to the triangle ABC.

$$AC^2 = AB^2 + BC^2$$
$$15^2 = x^2 + 9^2$$
$$225 = x^2 + 81$$
$$225 - 81 = x^2$$
$$\sqrt{144} = x$$
$$x = \underline{\underline{12 \text{ cm}}}$$

(ii) Area of the cross section

$$= \frac{1}{2} \times 9 \times 12$$
$$= \underline{\underline{54 \text{ cm}^2}}$$

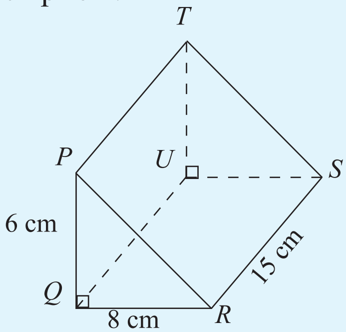
(iii) Volume of the prism

$$= 54 \times 20$$
$$= \underline{\underline{1080 \text{ cm}^3}}$$

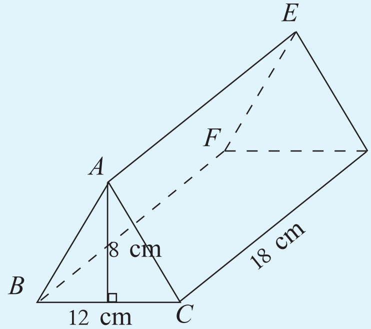
Exercise 29.4

1. Using the data that is marked on the prisms illustrated below find the volume of each prism.

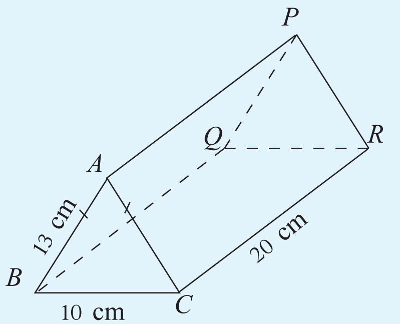
(i)



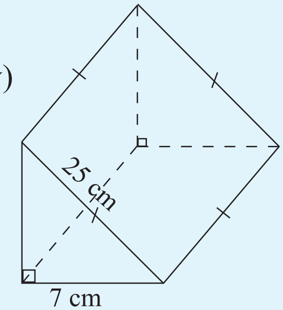
(ii)



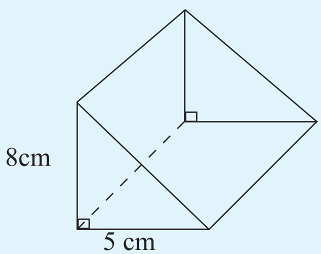
(iii)



(iv)

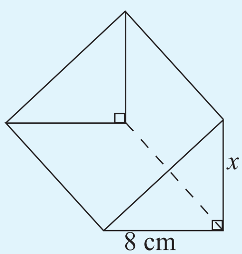


2. (i)

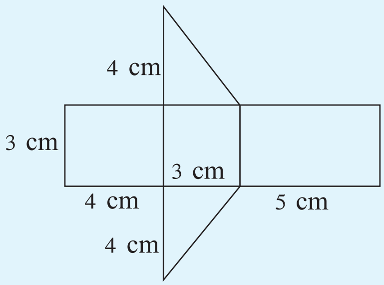


If the volume of the prism is 400 cm^3, find its length.

(ii)



Find the value of x if the height of the prism of volume 288 cm^3 given in the figure is 12 cm.

3. 

Find the volume of the prism that can be constructed using this net.

4. Water has been filled to a height of 8 cm of a cuboid shaped vessel of length 30 cm and breadth 20 cm. If the level of the water in the vessel rises by 2 cm when a right triangular prism of cross sectional area 60 cm^2 is dropped carefully into the water, find the perpendicular height of the prism.

5. A water tank, the shape of a prism with a triangular cross section of area 800 cm^2, is filled with water to a height of 30 cm. If this water is poured into a cuboid shaped vessel of length 60 cm and height 20 cm without wastage, to what height will the water level rise?

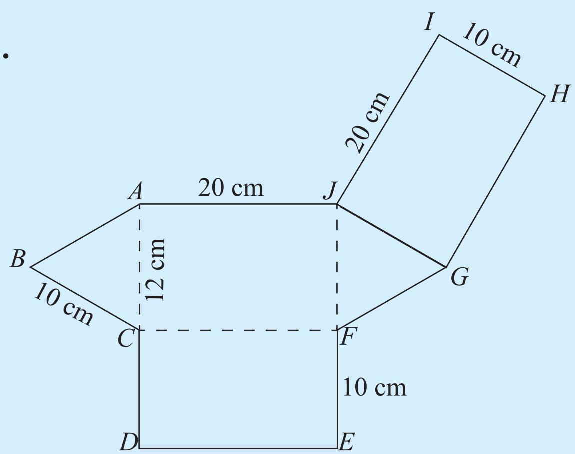
Summary

For a right circular cylinder of base radius r and height h,

- the total surface area $= 2\pi r^2 + 2\pi rh$
- the volume $= \pi r^2 h$

Miscellaneous Exercise

1. A cylindrical shaped log is of radius 14 cm and height 25 cm.
   (i) Find the total surface area.
   (ii) Find the volume.

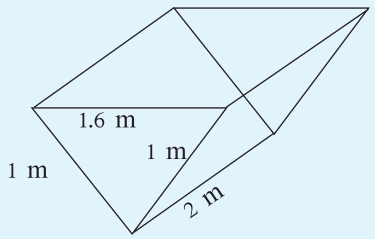
2. 

A sketch has been provided of a net with measurements, that has been drawn on a thick piece of paper such that it can be used to make a right triangular prism by folding it along the dotted lines.

(i) With which edge does the edge GH coincide?
(ii) With which vertex does the vertex H coincide?

(iii) Find the area of a triangular face of the prism that is made.
(iv) Find the total surface area and the volume of the prism.

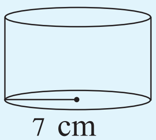
3.



A fish tank in the shape of a prism with a triangular cross section having the dimensions given in the figure has been made with cement in Dayan's garden.

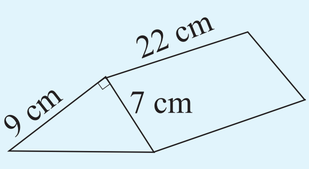
(i) Find the interior surface area of this tank.
(ii) Find in litres, the amount of water that is required to fill the tank completely.
(iii) If a pipe through which water flows at a rate of 20l per minute is used to fill the tank completely, find how much time is required to fill the tank.
(iv) Dayan now decides to build a new tank which takes the shape of a semi - circular cylinder of length 1 m, having the same volume as the above tank. Suggest suitable measurements for this tank.

4.



The volume of a cylinder of radius 7 cm and height h is 3080 cm^3.
i) Find the height of the cylinder.
ii) Find the surface area of the cylinder.

5.



A vessel, the shape of the prism in the figure is completely filled with water. All the water in this vessel is poured into a vessel the shape of a right cylinder of radius 7 cm. To what height does the water level rise in the cylinder?