23 Formulae

By studying this lesson you will be able to

- change the subject of a formula which involves squares and roots.
- find the value of an unknown term in a formula when the values of the other unknown terms are given.

You may recall that a formula expresses the relationship that exists between two or more physical quantities.
If the area of a rectangle is denoted by A, then A can be expressed in terms of the length a and breadth b of the rectangle as $A = a \times b$.

A is called the subject of this formula. The subject of a formula can be changed if required.
If the above formula is written in the form $b = \frac{A}{a}$, then the subject is b.

Do the following exercise to recall the facts you have learnt earlier about changing the subject of a formula.

Review Exercise

1. Make u the subject of the formula $v = u + at$.

2. Make F the subject of the formula $C = \frac{5}{9}(F - 32)$.

3. Consider the formula $l = a + (n-1)d$.
(i) Make a the subject of the formula.
(ii) Make d the subject of the formula.
(iii) Make n the subject of the formula.
(iv) Find the value of d when $l = 24$, $a = 3$ and $n = 8$.

4. Consider the formula $\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2}$.

(i) Make r_1 the subject of the above formula.
(ii) Find the value of r_1 when $R = 4$ and $r_2 = 6$.

23.1 Changing the subject of formulae containing squares and square roots

Given below is the formula for the area of a circle. Here A denotes the area and r denotes the radius of the circle.

$$A = \pi r^2$$

Let us consider how r is made the subject of this formula.

Let us first make r^2 the subject of the formula.

That is, $r^2 = \frac{A}{\pi}$.

Now, to make r the subject of the formula, let us take the square root of both sides.

This can be expressed as $r = \pm\sqrt{\frac{A}{\pi}}$.

Since $\sqrt{}$ denotes the positive square root value, remember that the signs + and – should be written in front of the symbol $\sqrt{}$ to denote the square root. In this example, since r represents the radius of the circle, it is positive. Therefore, we can ignore the negative value. However, when the meanings of the unknowns are not known (or have not been given), the correct form is to have both signs.

Now let us consider how the subject of a formula which contains a square root is changed. For this, let us consider the formula $t = 2\pi\sqrt{\frac{l}{g}}$.

Let us see how l is made the subject of this formula.

Let us first keep the term with the square root sign on one side of the equality symbol, and move all the other terms to the other side.

$$\frac{t}{2\pi} = \sqrt{\frac{l}{g}}$$

Let us now square both sides.

$$\left(\frac{t}{2\pi}\right)^2 = \sqrt{\left(\frac{l}{g}\right)}^2$$

Then $$\frac{t^2}{4\pi^2} = \frac{l}{g}$$

Now l can easily be made the subject of the formula.

$$\frac{gt^2}{4\pi^2} = l$$

That is,

$$l = \frac{gt^2}{4\pi^2}$$

Exercise 23.1

1. Make the unknown term within brackets the subject of the relevant formula.

(i) $v^2 - u^2 = 2as$ (u) (ii) $a^2 + b^2 = c^2$ (b)

(iii) $v = \frac{1}{3}\pi r^2 h$ (r) (iv) $v = \frac{a^2 h}{3}$ (a)

(v) $A = \pi\left(R^2 - r^2\right)$ (r) (vi) $E = \frac{1}{2}m\left(v^2 - u^2\right)$ (u)

2. Make the unknown term within brackets the subject of the relevant formula.

(i) $T = 2\pi\sqrt{\frac{l}{g}}$ (g) (ii) $\theta = \left(\frac{3rt}{m}\right)^{\frac{1}{2}}$ (m)

(iii) $4\sqrt{p} = q$ (p) (iv) $S = a + \sqrt{b}$ (b)

(v) $v = w\sqrt{a^2 - x^2}$ (a) (vi) $A = \pi r\sqrt{h^2 + r^2}$ (h)

23.2 Substitution

If the values of all the unknowns in a formula except for one are given, then by substituting these values in the formula, the value of the remaining unknown can be found.

Given below is the formula for the volume (v) of a cone, in terms of its radius (r) and its height (h).

$$v = \frac{1}{3}\pi r^2 h$$

Find the value of r when $v = 132$ and $h = 14$.

Let us first make r the subject of the formula.

$$\frac{3v}{\pi h} = r^2$$

$$\therefore \quad r = \sqrt{\frac{3v}{\pi h}}. \qquad \text{(since } r \text{ is positive)}$$

Now let us substitute the known values.

$$r = \sqrt{\frac{3 \times 132}{\frac{22}{\cancel{7}_1} \times \cancel{14}_2}}$$

$$r = \sqrt{9}$$

$$\underline{\underline{r = 3}}$$

To solve this problem, it is not necessary to first make r the subject of the formula.

The known values can be substituted first, and then r can be made the subject. This is done as follows.

$$v = \frac{1}{3}\pi r^2 h$$

$$132 = \frac{1}{3} \times \frac{22}{\cancel{7}_1} \times r^2 \times \cancel{14}_2$$

$$\frac{132 \times 3}{22 \times 2} = r^2$$

$$r^2 = 9$$

$$\underline{\underline{r = 3}}$$

The same answer is obtained by both methods. Therefore, either of the above two methods can be used to find the value of an unknown term.

However, there are many benefits of knowing how to change the subject of a formula. For example, when it is required to find the base radius of several cones of different volumes, if r is made the subject of the above formula for the volume of a cone, then it will be very easy to perform the required calculations to find the different radii.
Also, it is necessary to change the subject of the formula if these calculations are being done using a calculator or a computer.

Exercise 23.2

1. Consider the formula $v^2 = u^2 + 2as$.
(i) Find the value of a when $v = 10$, $u = 0$ and $s = 10$.
(ii) Find the value of s when $v = 10$, $u = 5$ and $a = 2$.
(iii) Find the value of u when $v = 10$, $a = 3$ and $s = 6$.

2. If $x = \sqrt{y + z}$,
(i) find the value of x when $y = 6$ and $z = 10$.
(ii) find the value of y when $x = 5$ and $z = 5$.

3. If $k^2 = lm$, find the value of k when $l = 9$ and $m = 4$.

4. Consider the formula $s = ut + \frac{1}{2}at^2$.
(i) Find the value of t when $u = 0$, $a = 5$ and $s = 250$.
(ii) Find the value of t when $u = 5$, $a = 10$ and $s = 30$.

5. Find the value of t in the formula $t = 2\pi\sqrt{\frac{l}{g}}$, when $l = 490$, $g = 10$ and $\pi = \frac{22}{7}$.

Miscellaneous Exercises

1. The relationship between the base radius r, the height h and the volume V of a cylinder is given by $V = \pi r^2 h$. If a cylindrical shaped water tank of base radius 50 cm is filled with water to a height of 70 cm, find the volume of water in the tank. Take $\pi = \frac{22}{7}$.

2. The surface area A of a sphere, expressed in terms of the radius r is given by the formula $A = 4\pi r^2$.

(i) Express the radius of the sphere in terms of the surface area.

(ii) If the surface area of a sphere is 616 cm^2, find its radius. Take $\pi = \frac{22}{7}$.

3. The kinetic energy E of an object of mass m travelling with velocity v is given by the formula $E = \frac{1}{2}mv^2$.

(i) Express the velocity of the object in terms of its kinetic energy and mass.

(ii) Find the kinetic energy of an object of mass 2.4 kg when it is travelling with a velocity of 3 ms^{-1}.

4. If the length of the hypotenuse of a right angled triangle is x, and the lengths of the other two sides are a and b respectively, then according to Pythagoras' theorem, $x = \sqrt{a^2 + b^2}$. Find b when x = 25 cm and a = 24 cm.

5. The energy that a moving object possesses is given by the formula $E = mgh + \frac{1}{2}mv^2$. In this formula, E denotes the energy of the object, m its mass, v its velocity and h the height of its position.

(i) Express the mass of the object in terms of the other quantities.

(ii) Express the velocity of the object in terms of the other quantities.

(iii) The energy possessed by an object of mass 3 kg when it is 5 m above the ground is 153 N. Find the velocity of this object at this moment. Take g = 10 ms^{-1}.