28 Constructions

By studying this lesson you will be able to

- construct the four basic loci
- construct parallel lines
- construct triangles with the given information.

28.1 Construction of the basic loci

The path of a point in motion is defined as its locus. Several examples of loci that can be observed in our environment are given below.

1. The path of a fruit which falls from a tree.
2. The path of the pointed end of a clock hand.
3. The path of a planet orbiting around the sun.
4. The path of a pendulum in a pendulum clock.
5. The path of a ball that is hit by a bat.

In this lesson we will only be considering loci in a plane.

Note:

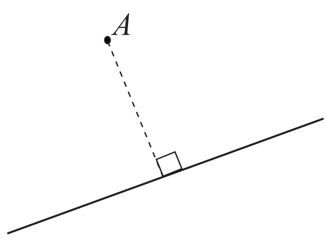
Before considering the construction of loci, direct your attention to the following facts.

1. Distance between two points:

Let us consider two points A and B that lie on a plane. What is meant by the distance between the two points is the length of the straight line segment joining the two points.

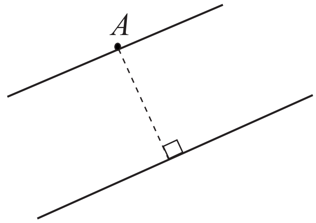
2. Distance from a point to a straight line:

Let us consider a given point A and a given straight line. What is meant by the distance from A to the straight line is the shortest distance from A to the straight line. This shortest distance is the perpendicular distance from A to the straight line.



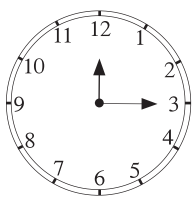
3. Distance between two parallel lines

Consider the following two parallel lines. Let us consider any point A on one of the lines. The perpendicular distance from A to the other straight line is said to be the distance between the two lines. Since the two lines are parallel, irrespective of where the point A is located on the line, this distance remains the same.



Now let us consider the 4 basic loci.

1. Constructing the locus of a point moving at a constant distance from a fixed point



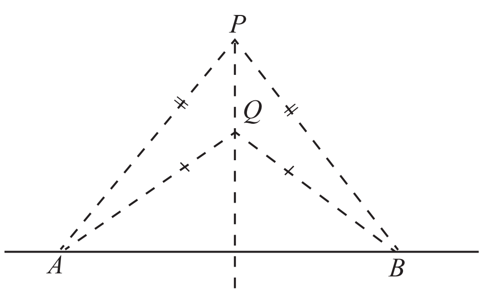
The pointed end of each hand on the clock face in the figure is always located at a constant distance from the centre of the clock, which is the location at which the hand is fixed to the clock. You will be able to observe when the clock is working, that the path of the pointed end of each hand is a circle. The point where the hands are fixed to the clock is the centre of these circles, and the radius of each circle is the length of the relevant hand. Observe here that the pointed end of each hand is travelling at a constant distance from a fixed point. That particular constant distance is the length of the hand.

The locus of a point moving at a constant distance from a fixed point is a circle.

Let us see how a circle is constructed.

Mark a point. Take the radius of the circle that you want to construct to the pair of compasses using the ruler and keep the point of the pair of compasses on the point you marked. Now draw the circle.

2. Constructing the locus of a point moving at an equal distance from two fixed points



As shown in the figure, the point P is at an equal distance from the two points A and B. Further, Q is another point which is at an equal distance from A and B. There are a large number of points such as these, which are at an equal distance from A and B. Observe what is obtained when all these points are joined together.

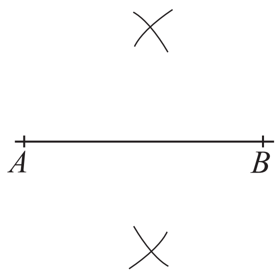
It is clear that the straight line that is obtained when all these points are joined together, passes through the midpoint of the line joining *A* and *B*, and is perpendicular to *AB*.

The locus of a point moving at an equal distance from two fixed points is the perpendicular bisector of the straight line joining the two points.

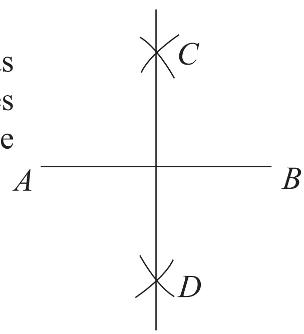
Now let us consider how this locus, that is, the perpendicular bisector of the line segment *AB* is constructed.

Mark two points and name them as *A* and *B*.

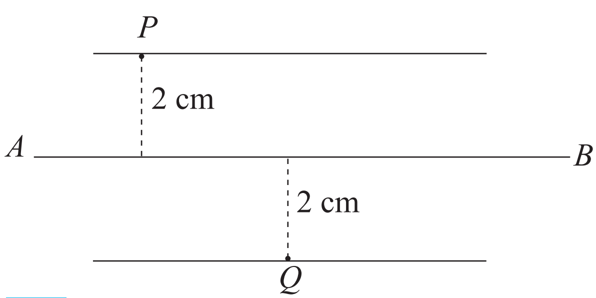
Step 1: Draw the line segment *AB*. On the pair of compasses, take a length which is a little more than half the length of *AB*, and taking *A* and *B* as the centres and the length on the pair of compasses as the radius, draw two arcs which intersect each other (as shown in the figure).



Step 2: Name the intersection points of the two arcs as *C* and *D* and draw the straight line which passes through these two points. This straight line is the required locus.



3. Constructing the locus of a point moving at a constant distance from a straight line



The figure illustrates a pair of straight lines drawn parallel to the straight line *AB* on opposite sides of *AB*. Each of these lines is at a constant distance of 2 cm from *AB*. Conversely, if a point lies at a distance of 2 cm from *AB*, then it is clear that this point must lie on one of the above two lines.

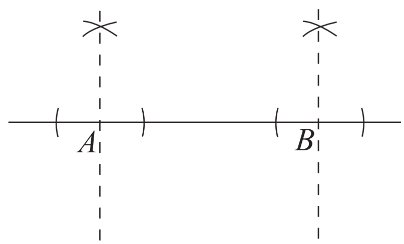
Accordingly, the locus of a point which lies 2 cm from the straight line AB is one of two straight lines which are parallel to AB and lie on opposite sides of AB 2 cm from it.

The locus of a point moving at a constant distance from a given straight line is a line parallel to the given straight line, at the given constant distance from the straight line, which may lie on either side of the straight line.

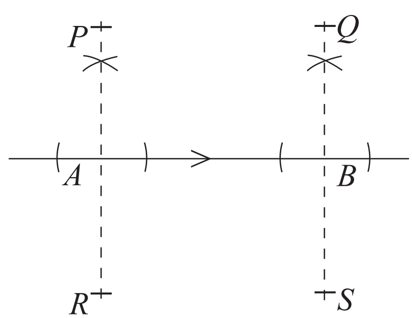
Now let us consider how a pair of lines parallel to a given straight line, which is the locus under consideration, is constructed.

Draw a straight line segment using a straight edge. Select two points A and B on this straight line.

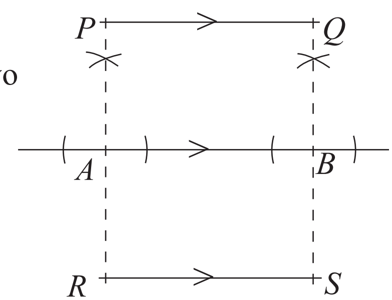
Step 1: At the points A and B, construct two lines perpendicular to the given line.



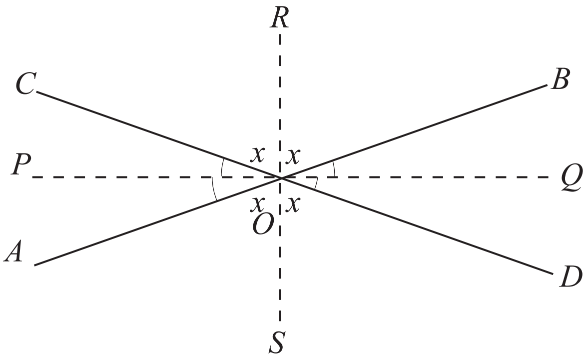
Step 2: On each of these two perpendicular lines, mark two points at the required distance (say 5 cm), on either side of the given straight line, and name them P, Q, R and S as shown in the figure.



Step 3: Draw the straight lines PQ and RS. These two straight lines are the required locus.



4. Constructing the locus of a point moving at an equal distance from two intersecting straight lines



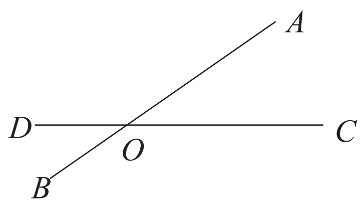
The straight lines AB and CD in the figure intersect at O. The straight line PQ has been drawn such that the angle $A\hat{O}C$ (and $B\hat{O}D$) is divided into two equal angles. The line PQ is called the bisector of the angle $A\hat{O}C$ (or $B\hat{O}D$). Similarly, the straight line RS has been drawn such that the angle $C\hat{O}B$ (and $A\hat{O}D$) is divided into two equal angles. The line RS is called the bisector of the angle $C\hat{O}B$ (or $A\hat{O}D$).

Can you see that the distance from any point on the line PQ to the lines AB and CD is equal? Understand that similarly, the distance from any point on the line RS to the lines AB and CD is also equal. Do you see that conversely, if a point is at an equal distance from the lines AB and CD, then it must lie on either PQ or RS?

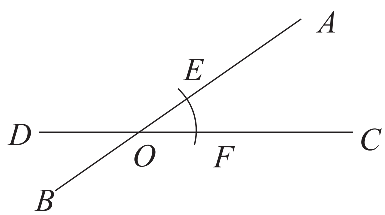
The locus of a point moving at an equal distance from two intersecting straight lines is a bisector of the angle formed at the intersection point of the lines.

Now let us consider how this locus is constructed.

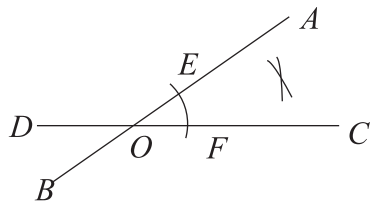
Let the two straight lines AB and CD intersect at the point O.



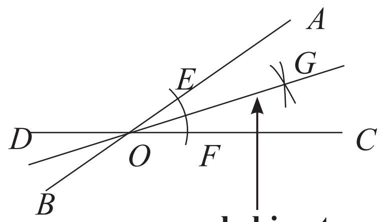
Step 1: Using the pair of compasses, draw an arc with centre O such that it intersects both BA and DC. Name the two points at which the arc intersects BA and DC as E and F respectively.



Step 2: Using the pair of compasses and taking E and F as centres, draw two intersecting arcs.



Step 3: Name the point of intersection of the two arcs as G, and draw the straight line which passes through the points O and G. Construct the other angle bisector in a similar manner.



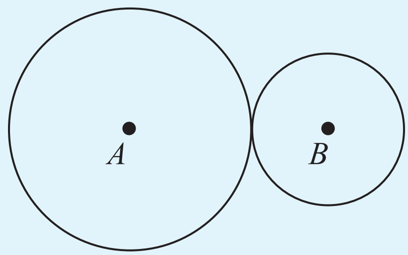
The required locus is one of these angle bisectors.

Exercise 28.1

1. If the length of the seconds hand of a clock is 3.5 cm, construct the path of the pointed end of this hand.

2. If the maximum distance between a cow and a tree to which the cow has been tied with a rope is 5 m, construct the path along which the cow can travel such that the distance between the tree and the cow will be at its maximum.

3. A is the centre of a fixed cogwheel of radius 3 cm, and B is the centre of a revolving cogwheel of radius 2 cm. Construct the locus of B as the smaller cogwheel revolves around the larger cogwheel of centre A.



4. (i) Construct a straight line segment PQ such that $PQ = 5$ cm. Construct two circles of radius 3 cm each with P and Q as centres.

(ii) Name the points of intersection of the two circles as X and Y and join XY.

(iii) Name the point of intersection of the straight lines PQ and XY as S and measure and write down the lengths of PS and QS.

(iv) Measure and write down the magnitudes of $P\hat{S}X$ and $Q\hat{S}X$.

(v) Describe the locus represented by XY.

5. Construct the straight line segment AB such that $AB = 7$ cm and divide it into four equal parts.

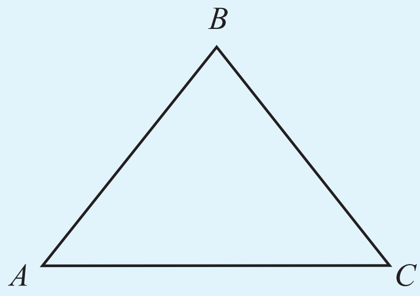
6. Draw the angle $B\hat{A}C$ such that $AB = 5$cm and $B\hat{A}C = 40^\circ$. Construct the locus of the points which are equi-distant from A and B and name the point of intersection of this locus and the straight line AC as D.

7. (i) Draw an acute triangle and name it ABC.

(ii) Construct the locus of a point which is equi-distant from A and C.

(iii) Construct the locus of a point which is equi-distant from A and B.

(iv) Name the point of intersection of these two loci as O. What can you say about the distance from O to the points A, B and C?



8. Draw a straight line segment KL. Construct the locus of a point which is 2.5 cm from this line.

9. Contruct a rectangle of length 5 cm and breadth 3 cm. Construct the locus of a point which lies outside the rectangle at a distance of 2 cm from the sides of the rectangle.

10. Using the protractor draw the following angles and construct their bisectors.

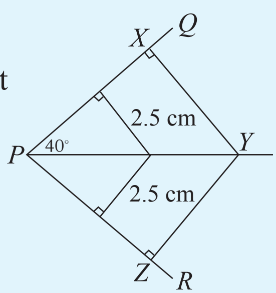
(i) 60° (ii) 90° (iii) 120°

11. Based on the information in the figure,

(i) name the locus of the points which are equi-distant from PQ and PR.

(ii) write down a relationship between XY and YZ.

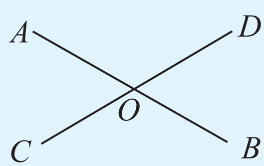
(iii) What is the magnitude of $R\hat{P}Y$?



12. The straight lines AB and CD in the figure intersect at O.

(i) Construct the locus of the points equi-distant from AB and CD.

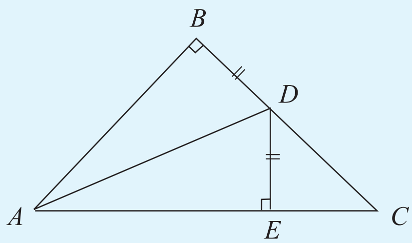
(ii) What is the magnitude of the angle between the two lines which form this locus?



13. In the given figure, $A\hat{B}C = A\hat{E}D = 90^{\circ}$ and $BD = DE$.

(i) Name the locus of the points which are equi-distant from AB and AC.

(ii) If $A\hat{C}B = 40^{\circ}$, what are the magnitudes of $B\hat{A}D$ and $C\hat{A}D$?



28.2 Construction of triangles

A triangle has three sides and three angles. The sides and the angles are called the elements of the triangle. Let us study three instances when a triangle can be constructed with the information given on the magnitude of three elements of a triangle.

1. When the lengths of the three sides of a triangle are given

Example 1

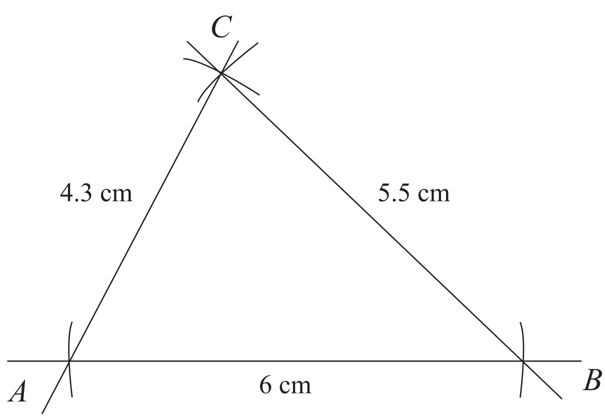
Construct the triangle ABC such that $AB = 6$ cm, $BC = 5.5$ cm and $AC = 4.3$ cm.

Step 1: Draw a straight line segment of length 6 cm and name it AB.

Step 2: Take B as the centre and draw a circular arc of radius 5.5 cm (of sufficient length).

Step 3: Draw another circular arc of radius 4.3 cm with centre A, such that it intersects the arc drawn in step 2 above.

Step 4: Name the point of intersection of the two arcs as C, and by joining AC and BC, complete the triangle ABC.



2. When the lengths of two sides and the magnitude of the included angle are given

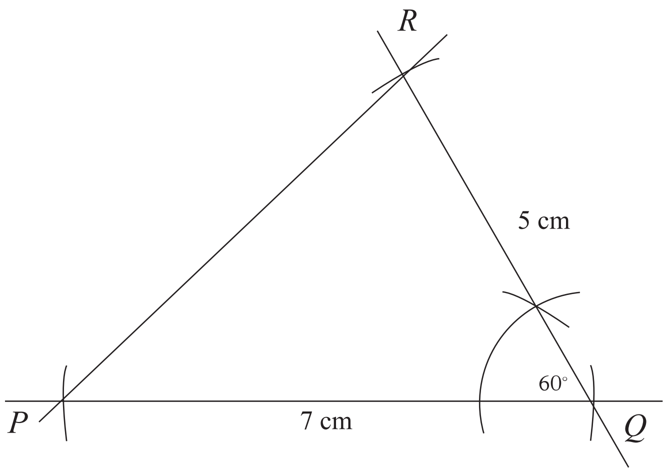
Example 2

Construct the triangle PQR such that $PQ = 7$ cm, $QR = 5$ cm and $P\hat{Q}R = 60^\circ$.

Step 1: Construct an angle of 60 degrees and name its vertex Q. The sides of the angle should be longer than the given lengths of the triangle.

Step 2: Mark a straight line segment QP of length 7 cm on one side of the angle, and a straight line segment QR of length 5 cm on the other side of the angle. (See the figure)

Step 3: Complete the triangle PQR by joining PR.



3. When the magnitudes of two angles and the length of a side are given

Example 3

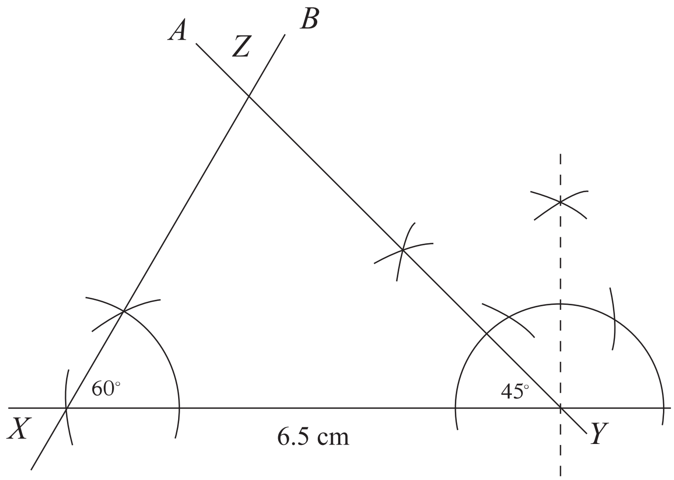
Construct the triangle XYZ such that $XY = 6.5$ cm, $X\hat{Y}Z = 45^\circ$ and $Y\hat{X}Z = 60^\circ$.

Step 1: Construct a straight line segment of length 6.5 cm and name it XY.

Step 2: Construct the angle $X\hat{Y}A$ at the point Y, such that $X\hat{Y}A = 45^\circ$

Step 3: Construct the angle $Y\hat{X}B$ at the point X, such that $Y\hat{X}B = 60^\circ$.

Step 4: Name the intersection point of YA and XB as Z. Then XYZ is the required triangle.



Exercise 28.2

1. Construct the equilateral triangle ABC of side length 6 cm.

2. Construct the isosceles triangle PQR, such that $PQ = 8$ cm and $PR = QR = 6$ cm.

3. (i) Construct the triangle KLM where $KL = 7.2$ cm, $LM = 6.5$ cm and $KM = 5$ cm.
(ii) Measure the magnitude of each angle in the triangle and write it down.

4. (i) Construct the triangle ABC where $AB = 6$ cm, $A\hat{B}C = 90^\circ$ and $BC = 4$ cm.
(ii) Measure and write down the length of the side AC.
(iii) Write down a relationship between the sides AB, BC and AC.
(iv) Thereby find an approximate value for $\sqrt{52}$.

5. (i) Construct the triangle XYZ such that $XY = 5$ cm, $X\hat{Y}Z = 75^\circ$ and $YZ = 6$ cm.
(ii) Measure and write down the length of the side XZ.
(iii) Measure and write down the magnitude of $Y\hat{X}Z$.

6. (i) Construct the triangle SRT such that $RS = 6.5$ cm, $S\hat{R}T = 120^\circ$ and $RT = 5$ cm.
(ii) Construct a straight line through T parallel to SR.

7. Construct the triangle DEF such that $DE = 6.8$ cm, $D\hat{E}F = 60^\circ$ and $E\hat{D}F = 90^\circ$.

8. (i) Construct the triangle ABC such that $AB = 6$ cm, $A\hat{B}C = 105^\circ$ and $BC = 4.5$ cm.
(ii) Thereby construct the parallelogram $ABCD$.
(iii) Measure the length of the diagonal AC and write it down.

9. (i) Construct the triangle PQR such that $QR = 7$ cm, $Q\hat{R}P = 60^\circ$ and $Q\hat{P}R = 75^\circ$
(ii) Construct the perpendicular from P to QR and name the foot of the perpendicular as S.
(iii) Measure and write down the length of PS.

10. (i) Construct the triangle KLM such that KL = 6.5 cm, $K\hat{L}M$ = 75° and LM = 5 cm.

(ii) Construct the quadrilateral $KLMN$ by finding the point N which is equidistant from K and M and is such that MN = 4 cm.

(iii) Measure and write down the magnitude of $L\hat{K}N$.

28.3 Constructions related to parallel lines

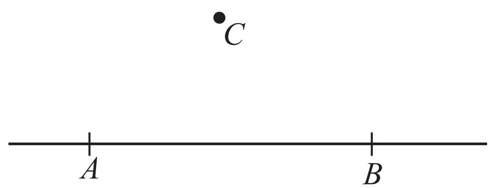
You have learnt in a previous grade how to construct parallel lines using a set square and a straight edge.

Now let us learn how to construct parallel lines using a straight edge and a pair of compasses.

1. Constructing a line parallel to a given straight line through an external point

Method 1

Let us assume that the straight line is AB and the external point is C.



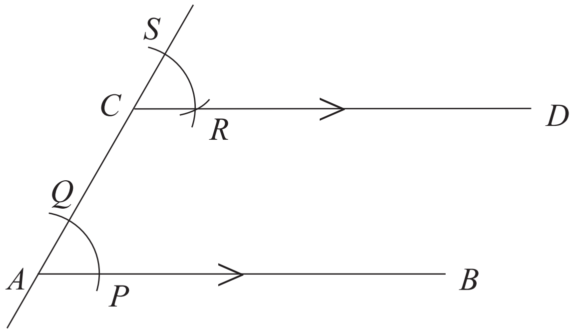
Step 1: Draw the straight line passing through the points A and C.

Step 2: Draw an arc on $B\hat{A}C$ taking A as the centre. Name this arc PQ.

Step 3: Taking the same radius, (that is, without changing the position of the pair of compasses), draw another arc with C as the centre, such that it intersects AC produced at S as shown in the figure.

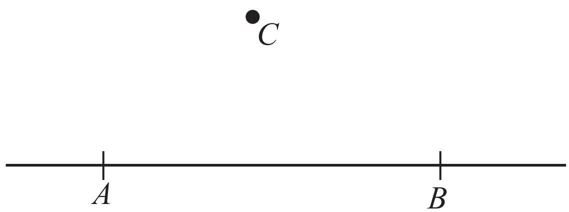
Step 4: Mark RS on the second arc as shown in the figure, such that it is equal in length to PQ.

Step 5: Draw the straight line CD such that is passes through the point R. Since the angle $R\hat{C}S$ which is then formed and $B\hat{A}C$ are corresponding angles which are equal to each other, the straight lines AB and CD are parallel to each other.



Method 2

Let us assume that the straight line is AB and the external point is C.



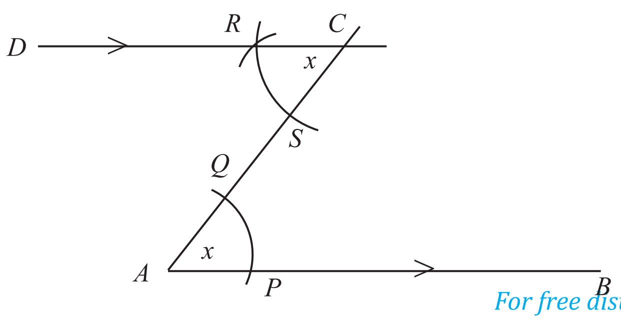
Step 1: Join AC.

Step 2: Draw an arc on $B\hat{A}C$, taking A as the centre. Name this arc PQ.

Step 3: Taking the same radius, draw another arc with C as the centre such that it intersects AC at the point S as shown in the figure.

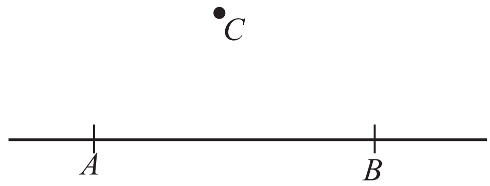
Step 4: Mark the point R on this arc such that RS is equal in length to PQ.

Step 5: Draw the straight line CD such that is passes through the point R. Since the angle $R\hat{C}S$ which is then formed and $B\hat{A}C$ are alternate angles which are equal to each other, the straight lines AB and DC are parallel to each other.



Method 3

Let us assume that the straight line is AB and the external point is C.



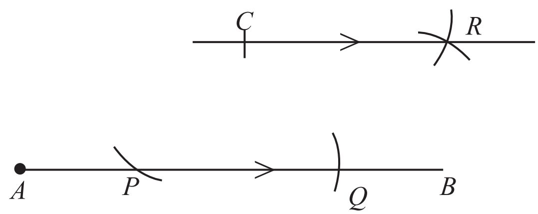
Step 1: Using a pair of compasses draw an arc with centre C such that it intersects AB. Name the point of intersection as P.

Step 2: Draw another arc with centre P and the same radius as that of the previous arc (i.e., keeping the radius CP unchanged), such that it intersects AB. Name the intersection point as Q.

Step 3: Draw another arc with centre Q and the same radius as before, in the direction of C.

Step 4: Now draw another arc with centre C and the same radius as before, such that it intersects the arc in step 3. Name the intersection point of the arcs as R.

Step 5: Join CR. Then CR is parallel to AB.



Activity

Do the following activity to further understand about constructions related to parallel lines.

1. Construct an angle of 60° and name the vertex as A. On one arm (side) of the angle mark point B such that $AB = 8$ cm. Mark point C on the other arm (side) such that $AC = 5$ cm. Now using the pair of compasses complete the parallelogram $ABDC$.

2. Draw two parallel lines such that the distance between the lines is 4 cm. Mark the points A and B on one line such that $AB = 7$ cm. Mark point D on the other line so that AD is 5 cm. Now complete the parallelogram $ABCD$.

3. Draw two parallel lines such that the distance between the lines is 4 cm. Mark the points A and B on one line such that AB is 7 cm. Mark point C on the other line such that $BC = 5$ cm. Now mark point D on the same line which C is on, such that $CD = 4$ cm. Then complete the quadrilateral $ABCD$ and observe that it is a trapezium.

Exercise 28.3

1. Draw an acute angle and name it $A\hat{B}C$. Construct a straight line segment which is parallel to AB and which passes through the point C.
2. Draw an obtuse angle and name it $P\hat{Q}R$. Construct a straight line segment which is parallel to PQ and which passes through the point R.
3. Construct a square of side length 6 cm.
4. Construct a rectangle of length 6.5 cm and breadth 4 cm. Name it as $ABCD$. Draw its diagonal AC and construct two straight line segments through the points B and D such that each is parallel to AC.
5. Construct the parallelogram $ABCD$ such that $AB = 6$ cm, $A\hat{B}C = 120^\circ$ and $BC = 5$ cm.
6. Construct the rhombus $KLMN$ such that $KL = 7$ cm and $K\hat{L}M = 60^\circ$.
7. (i) Construct a circle of radius 3 cm and name its centre O.
   (ii) Construct a chord of the above circle of length 4 cm and name it PQ.
   (iii) Join PO and produce it to meet the circle again at R.
   (iv) Construct a line through R parallel to PQ.