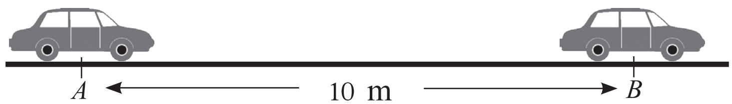
22 Rate

By studying this lesson you will be able to

- solve problems related to distance, time and speed
- represent information related to distance and time graphically
- solve problems related to liquid volumes, time and rate.

22.1 Speed

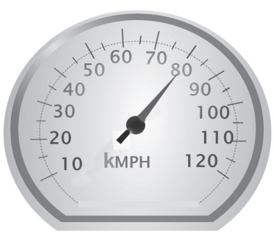


Let us assume that a battery operated toy car takes 5 seconds to travel from point *A* to point *B* which is 10 m away.

Then the distance that the car has travelled during 5 seconds is 10 m. If the distance that the car moves forward during each second is the same from the moment it starts, then the distance it travels during each second is $\frac{10}{5}$ metres, that is, 2 metres. Accordingly, as the car moves forward from *A*, the rate at which the distance changes with respect to time is 2 metres per second. We can define this value as the speed with which the car travels from *A* to *B*.

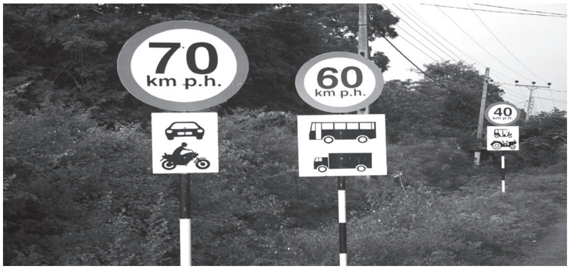
If the distance travelled by an object in motion is a constant per unit of time, then the object is said to be travelling with uniform speed. Further, the speed of the object is then the distance travelled per unit of time. From this point on, only objects which travel with uniform speed will be considered in this lesson.

However, in reality, vehicles that travel on the main road are usually unable to maintain a uniform speed throughout the whole journey due to the traffic on the road and various other reasons. The instrument called the speedometer gives the speed of a vehicle at any given instance.



The speed denoted by the speedometer in the figure can be written as 80 kmph. It can also be written as 80 km /h or as 80 kmh^{-1}.

As you travel along a main road, you may observe road signs with 40 kmph and 60 kmph written on them to indicate speed limits. Try to recall that heavy vehicles such as lorries carry a board at the back with 40 kmph written on it.



For an object that is moving with uniform speed, the relationship between the three quantities, namely the distance travelled, the time taken and the speed can be written as follows.

$$\boxed{\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}}$$

This relationship can also be written in the following simple form (without fractions).

$$\boxed{\text{Distance} = \text{Speed} \times \text{Time}}$$

Example 1

A feather floating on air with uniform speed, drifts 100 m in 20 seconds. Calculate the speed with which the feather drifts.

$$\text{Speed with which it drifts} = \frac{\text{Distance it drifts}}{\text{time}}$$

$$= \frac{100\text{ m}}{20\text{ s}}$$

$$= \underline{\underline{5\text{ ms}^{-1}}}$$

Example 2

Calculate the distance travelled in one minute by a bird that flies at a uniform speed of 5 ms^{-1}.

$$\text{Distance it flies} = \text{speed} \times \text{time}$$

$$= 5\text{ ms}^{-1} \times 60\text{ s}$$

$$= \underline{\underline{300\text{ m}}}$$

Example 3

Calculate the time it takes for a car to travel 150 km on a highway, at a uniform speed of 60 kmh^{-1}.

$$\text{Time taken} = \frac{\text{Distance}}{\text{Speed}}$$

$$= \frac{150\text{ km}}{60\text{ kmh}^{-1}}$$

$$= \underline{\underline{2\frac{1}{2}\text{h}}}$$

Example 4

How far does a motorcycle travel along a main road in 5 seconds, if its speedometer displays a constant speed of 36 kmh^{-1} during this period?

Here, the speed has been given in kilometres per hour. Let us convert it to metres per second.

Since the speed is 36 kmh^{-1},

distance travelled during an hour = 36 km

$= 36 \times 1000$ m

However, 1 hour = 60×60 seconds

$\therefore$ Distance travelled in 60×60 seconds = 36×1000 m

Distance travelled in 1 second = $\dfrac{36 \times 1000}{60 \times 60}$ m

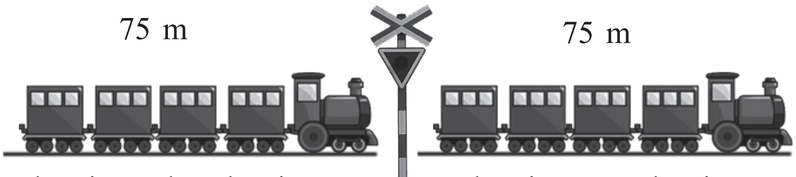
$\therefore$ Distance travelled by the motorcycle in one second = 10 m

$\therefore$ Distance travelled in 5 seconds = 10×5 m

$= \underline{\underline{50\text{ m}}}$

Example 5

How long does it take a train which is 75 m long to pass a signpost, if it is travelling at a uniform speed of 60 kmh^{-1}?



When it reaches the signpost   When it passes the signpost

The distance travelled by the train as it passes the signpost = 75 m

First, let us find the speed in terms of metres per second.

The speed of the train is 60 kmh^{-1}.

$\therefore$ Distance travelled in one hour = 60 km

Distance travelled in one hour = 60×1000 m

Distance travelled in one second = $\dfrac{60 \times 1000}{60 \times 60}$ m

$= \dfrac{50}{3}$ m

$\therefore$ Speed of the train $= \dfrac{50}{3}\, ms^{-1}$

Since time $= \dfrac{\text{distance}}{\text{speed}}$,

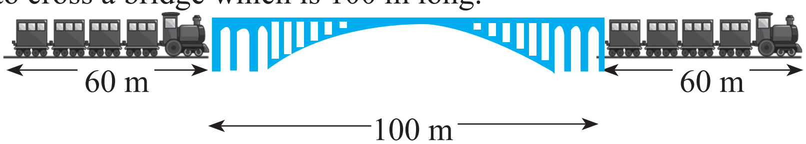
time taken by the train to pass the signpost $= 75 \div \dfrac{50}{3}$ seconds

$= 75 \times \dfrac{3}{50}$ seconds

$= \underline{\underline{4.5 \text{ seconds}}}$

Example 6

Find the time it takes for a train of length 60 m travelling at a uniform speed of 72 kmh^{-1} to cross a bridge which is 100 m long.



Here, the time taken for the train to travel a distance of 160 m needs to be found. For this, let us first find the speed in metres per second.

$72\, kmh^{-1} = \dfrac{72 \times 1000}{60 \times 60}\, ms^{-1}$

$= 20\, ms^{-1}$

The total distance travelled in crossing the bridge = 100 m + 60 m

= 160 m

Distance travelled by the train in 1 second = 20 m

That is, time taken to travel 20 m = 1 second

$\therefore$ Time taken to travel 160 m $= \dfrac{1}{20} \times 160$ seconds

$= \underline{\underline{8 \text{ seconds}}}$

Average Speed

A vehicle travelling along a main road is usually unable to maintain the same speed throughout the journey. The concept of average speed is important in such situations. The value obtained when the total distance travelled by an object is divided by the total time taken is called the average speed.

Example 1

An intercity bus took $\frac{1}{2}$ an hour to travel the first 25 km of a journey. If it took the bus 1 hour to cover the remaining 80 km of the journey, find the average speed of the bus.

$$\text{Total distance travelled by the bus} = 25 + 80 \text{ km}$$
$$= 105 \text{ km}$$
$$\text{Total time taken for the journey} = \frac{1}{2} + 1 \text{ h}$$
$$= 1\frac{1}{2} \text{ h}$$
$$\text{The average speed of the bus} = 105 \text{ km} \div 1\frac{1}{2} \text{ h}$$
$$= 105 \times \frac{2}{3} \text{ kmh}^{-1}$$
$$= \underline{\underline{70 \text{ kmh}^{-1}}}$$

Exercise 22.1

1. Calculate the speed of an aircraft which flies 1200 km in 4 hours with uniform speed.

2. If a child runs 200 m in 40 seconds at a uniform speed, find his speed in kilometres per hour.

3. On a certain day, an electric train moving at a uniform speed, took 6 hours to travel a distance of 300 km. On another day, the train took 8 hours to travel the same distance. Find the difference between the speeds at which the train travelled during the two days.

4. How long will it take an aircraft which travels at a uniform speed of 300 kmh^{-1} to fly 4500 km?

5. Find the distance in metres that a car which travels at a uniform speed of 48 kmh^{-1}, covers during 30 seconds.

6. A bus travels for 15 minutes at a speed of 40 kmh^{-1} and then it travels a further 30 minutes at a speed of 70 kmh^{-1}. Calculate the average speed of the bus.

7. If the time taken by a train to pass a signpost is 10 seconds when it is travelling at a uniform speed of 54 kmh^{-1}, find the length of the train.

8. Find the time it takes for a train of length 60 m travelling at a speed of 72 kmh^{-1} to pass a 100 m long platform.

9. A train leaves city *A* at 0800 h and travels at a uniform speed of 60 kmh^{-1} towards city *B*. Another train leaves city *B* at the same instance and travels at a uniform speed of 40 kmh^{-1} towards city *A*. If the distance between the two cities *A* and *B* is 100 km, calculate the time at which the two trains pass each other.

10. Two motorcyclists, who start their journeys at the same instance from two different cities, travel with uniform speeds of 40 kmh^{-1} and 50 km^{-1} respectively towards each other. If they meet each other $\frac{1}{2}$ an hour after commencing their journeys, find the distance between the two cities.

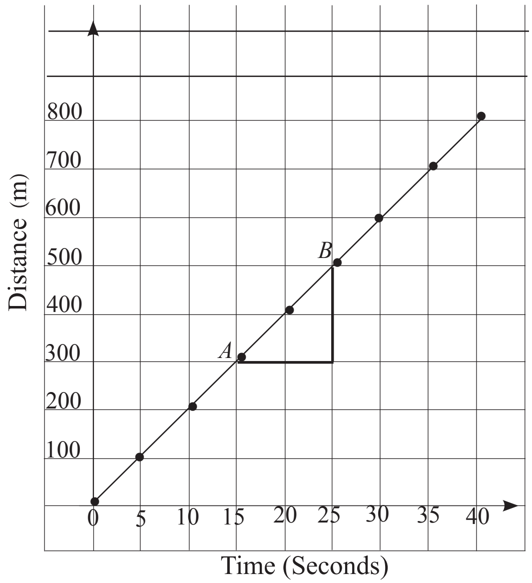
22.2 Distance - Time Graphs

A graph can be used to illustrate the change in the distance travelled by an object in motion, with respect to time. In such a graph, the *x* axis represents the time and the y axis represents the distance travelled. A graph of this form is called a **distance-time graph**.

A table prepared with the information collected by observing the motion of a satellite travelling with uniform speed is given below.

Time that has passed from the commencement of the journey (seconds)	5	10	15	20	25	30	35	40
Distance from the starting point (metres)	100	200	300	400	500	600	700	800

The distance-time graph drawn with this information is given below.



The speed of the satellite can be calculated by dividing the total distance travelled by the total time taken.

$$\text{Speed of the satellite} = \frac{800\text{ m}}{40\text{ s}} = 20\text{ ms}^{-1}$$

Observe that the gradient of the straight line *AB*

Since the satellite is travelling with uniform speed, the speed can also be obtained by considering the distance travelled per unit of time.

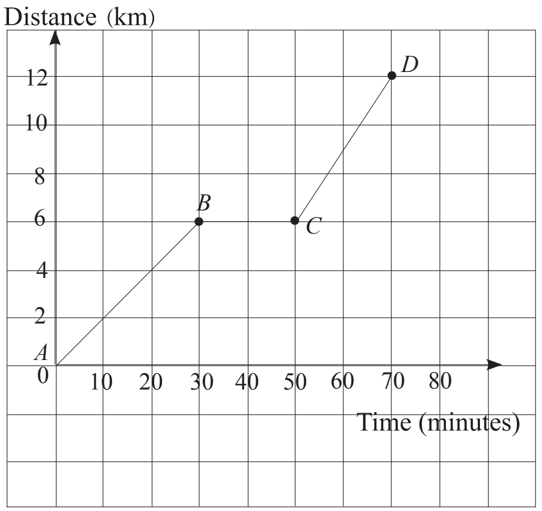
Accordingly, you can observe that the gradient of the graph and the speed of the satellite are equal. Therefore, for an object moving with uniform speed, a straight line is obtained as the distance-time graph, and the speed of the object can be obtained from the gradient of this line.

Gradient of the distance-time graph = Speed of the object in motion

Example 1

A distance-time graph illustrating the motion of Nimal who cycled to his friend's house and then returned back home after spending some time with his friend is given below.

(i) Calculate the speed at which Nimal cycled to his friend's house.
(ii) Calculate the speed at which Nimal returned home.



According to the above graph,

the distance from Nimal's house to his friend's house $= 6$ km

time taken by Nimal to cycle to his friend's house $= 30$ minutes

$= \frac{1}{2}$ h

$\therefore$ The speed at which Nimal cycled to his friend's house $= \dfrac{6 \text{ km}}{\frac{1}{2} \text{ h}}$

$= \underline{\underline{12 \text{ kmh}^{-1}}}$

The distance is the same during the period that Nimal spent time with his friend

Amount of time Nimal spent at his friend's house $=$ 20 minutes

Time taken for Nimal to return home $=$ 20 minutes

$= \frac{1}{3}$ h

Speed at which Nimal cycled back home $= \dfrac{6 \text{ km}}{\frac{1}{3} \text{ h}}$

$= \underline{\underline{18 \text{ kmh}^{-1}}}$

Exercise 22.2

1. The following table provides information on the distance travelled by a car moving at a uniform speed along a highway, and the time taken for the journey.

Time (hours)	0	1	2	3	4	5	6
Distance (km)	0	60	120	180	240	300	360

(i) Draw a distance-time graph with the above information.
(ii) Find the gradient of the graph.
(iii) Hence calculate the speed of the car.

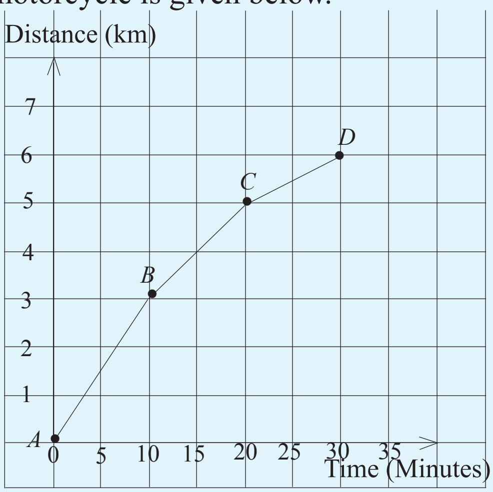
2. The change in distance with time of an object in motion is given in the following table.

Time (s)	0	2	4	6	8	10
Distance (m)	0	6	12	18	24	30

(i) Draw a distance-time graph with the above information.
(ii) Find the gradient of the graph.
(iii) Hence calculate the speed of the object.

3. A coach, moving with uniform speed from the commencement of its journey, travels a distance of 60 km in 2 hours. It then travels another 40 km in 2 hours, also with uniform speed, and reaches its destination. Represent the motion of the coach in a distance-time graph.

4. A distance-time graph of the motion of a man who travels from his home to the city on his motorcycle is given below.



(i) How far is it from his home to the city?
(ii) How long did it take him to reach the city?
(iii) Calculate his average speed.
(iv) Separately calculate the speeds at which he travelled from A to B, from B to C and from C to D.

22.3 Volume and Time

We defined speed as the distance travelled per unit of time. Another way of saying this is that speed is the rate of change of distance with respect to time. This idea of rate can also be used to describe various other processes that we come across in day to day life. Let us consider the example of water flowing out of a tap. If we collect the water that flows out from a tap during periods of one second each, and if by measuring we discover that the volume of water that flows out during each second is a constant, then we say that the water flows out at a uniform rate. Further, we call this constant value the rate at which water flows out from the tap.

When time is measured in seconds and the volume of water is measured in litres, the unit of the rate of flow is litres per second (ls^{-1}).

Suppose it takes 20 minutes for a tank of capacity 1000 l to be filled completely using a pipe through which water flows at a uniform rate.

Then, the volume of water that flowed out of the pipe during 20 minutes $= 1000\,l$

$\therefore$ The amount of water that flowed out during 1 minute $= \dfrac{1000\,l}{20}$

$= 50\ l$

Accordingly, the amount of water that flows out of the pipe per unit of time, that is, during one minute, is 50 litres. Therefore, we can express the rate at which water flows out of the pipe as 50 litres per minute.

$$\boxed{\text{Rate of change of volume} = \frac{\text{Change of volume}}{\text{Time}}}$$

This can also be represented as follows.

$$\boxed{\text{Change of volume} = \text{Rate of change of volume} \times \text{Time}}$$

Example 1

The time taken for 30 litres of petrol to be pumped into a car through a pump at a certain petrol shed was 60 seconds. Find the rate at which petrol flows out of the pump.

$$\text{Rate at which petrol flows out of the pump} = \frac{\text{Volume of petrol}}{\text{Time}}$$

$$= \frac{30\,l}{60\,s}$$

$$= \underline{\underline{\frac{1}{2}\,l\,s^{-1}}}$$

Example 2

The length, breadth and height of a cuboid shaped indoor water tank are 2 m, $1\frac{1}{2}$ m and 1 m respectively. On an occasion when the tank was completely filled with water, it took 50 minutes for the tank to be emptied by a pipe. Find the rate at which water flowed out through the pipe. (Assume that the water flowed through the pipe uniformly)

$$\text{Volume of the tank} = 2\text{ m} \times 1\frac{1}{2}\text{ m} \times 1\text{ m}$$

$$= 2 \times \frac{3}{2} \times 1\text{ m}^3$$

$$= 3\text{ m}^3$$

Since $1\text{ m}^3 = 1000\ l$,

the volume of water that can be filled into the tank $= 3 \times 1000\ l$
$= 3000\ l$

$$\therefore\ \text{Rate at which water flowed out through the pipe} = \frac{\text{capacity of the tank}}{\text{time}}$$

$$= \frac{3000\ l}{50\ \text{minutes}}$$

$$= \underline{\underline{\text{60 litres per minute}}}$$

Example 3

A saline solution was administered to a patient at a rate of 0.2 mls^{-1}. Calculate the time it takes for 450 ml of saline solution to be administered.

$$\text{Since rate} = \frac{\text{volume}}{\text{time}}$$

$$\text{Time} = \frac{\text{Volume of Saline}}{\text{Rate of administration}}$$

$$= \frac{450\text{ml}}{0.2\text{mls}^{-1}}$$

$$= 2250 \text{ seconds}$$

$$= \frac{2250}{60} \text{ minutes}$$

$$= 37\frac{1}{2} \text{ minutes}$$

Exercise 22.3

1. A cuboid shaped tank built to provide water to a housing scheme is of length 3 m, breadth 2m and height 1.5 m.

(i) Calculate the volume of the tank.

(ii) How many litres is the volume equal to?

(iii) How much time will it take to fill this tank completely using a pipe through which water flows at a uniform rate of 300 litres per minute?

2. If it took 40 minutes to completely fill a cube shaped tank of side length 2 m using a pipe, what is the rate at which water flows through the pipe in litres per minute? (Hint: 1 m^3 = 1000 l)

3. How long will it take to fill a fish tank of length 80 cm, breadth 60 cm and height 40 cm using a pipe through which water flows at a uniform rate of 6 *l* per minute? (Hint: 1 cm^3 = 1 ml)

4. The volume of a tank at a water distribution centre is 1800 m^3. If water is distributed from this tank at a rate of 500 ls^{-1}, how many minutes will it take to empty half the tank?

5. It took 40 minutes to fill an empty tank using a pump through which petrol flows at a uniform rate of 120 litres per minute. Find the capacity of the tank.

Summary

- Speed $= \dfrac{\text{Distance travelled by the object}}{\text{Time taken}}$

- Rate of change of volume $= \dfrac{\text{Change of volume}}{\text{Time}}$

Miscellaneous Exercise

1. A cylindrical water tank of cross-sectional area 0.5 m^2 is filled to a height of 70 cm in 1 minute and 10 seconds by a pipe through which water flows at a uniform rate. Calculate the rate at which water flows out of the pipe.

2. The distance between railway stations X and Y is 420 km. A train leaves station X at 7.00 p.m. and travels towards station Y with a uniform speed of 100 kmh^{-1}. An hour later, another train leaves station Y and travels towards station X with a uniform speed of 60 kmh^{-1}. At what time do the two trains pass each other?

3. The railway stations A and B are 300 km apart. A certain train takes 12 hours to travel from A to B and then back to A, after spending 2 hours at B. Another train leaves station A ten hours after the first train left A, and travels towards B at the same uniform speed. How far has the second train travelled when the two trains pass each other?