21 Graphs

By studying this lesson you will be able to

- find the gradient of the graph of a straight line,
- draw the graph of a function of the form $y = ax^2 + b$.

Graph of a function of the form $y = mx + c$

The graph of a function of the form $y = mx + c$ is a straight line. The coefficient of x, which is m, represents the gradient of the line and the constant term c represents the intercept of the graph.

Review Exercise

1. Write down the gradient and intercept of the straight line represented by each of the following equations.

(i) $y = 3x + 2$ (ii) $y = -3x + 2$ (iii) $y = 5x - 3$

(iv) $y = 4x$ (v) $y = -5x$ (vi) $y = \frac{1}{2}x - 3$

(vii) $y = \frac{1}{2}x + 3$ (viii) $y = \frac{-2}{3}x - 1$ (ix) $2y = 4x + 5$

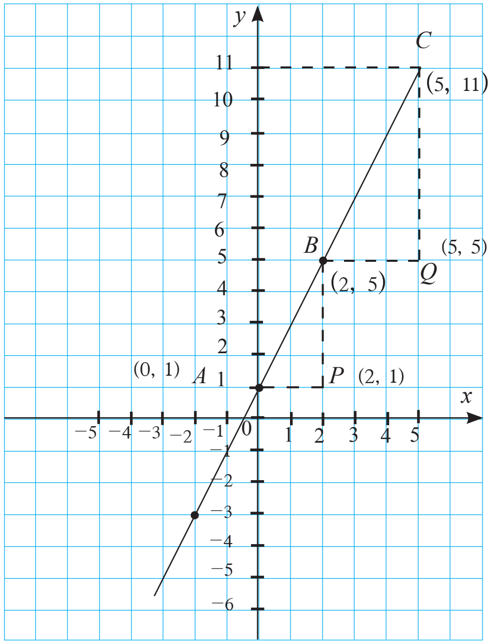
(x) $2y - x = 5$ (xi) $2y + 3 = 2x$ (xii) $\frac{1}{3}y - 5 = x$

21.1 Geometrical description of the gradient of a straight line

We defined the coefficient m of x in the equation $y = mx + c$ as the gradient of the straight line. Now by considering an example, let us see how the value of m is represented geometrically. To do this, let us consider the straight line given by $y = 2x + 1$. Let us use the following table of values to draw its graph.

x	-2	0	2
$y\ (= 2x + 1)$	-3	1	5

Let us mark any three points on the straight line. For example, let us take the three points as A (0, 1), B (2, 5) and C (5, 11).



First let us consider the points A and B.
Let us draw a line from A, parallel to the x – axis, and a line from B, parallel to the y – axis, and name the point of intersection of these lines as P. It is clear that the coordinates of the point P are (2, 1).

Also, length of $AP = 2 - 0$
$= 2$

length of $BP = 5 - 1$
$= 4$

Now for the points A and B, $\dfrac{\text{Vertical distance}}{\text{Horizontal distance}} = \dfrac{BP}{AP} = \dfrac{4}{2} = 2$.

We already know the gradiant of the straight line $y = 2x + 1$ is 2.

The quotient $\dfrac{\text{Vertical Distance}}{\text{Horizontal distance}}$ for the point A and B is also 2.

Now let us consider another case.
As the second case let us consider the points B and C.
Let us draw a line from B, parallel to the x - axis, and a line from C, parallel to the y - axis and name the point of intersection of these two lines as Q.

Then the coordinates of Q are (5, 5).

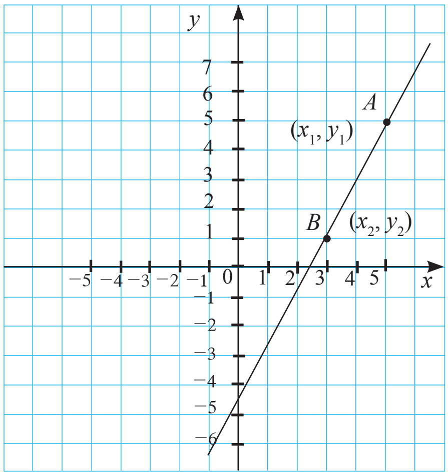
Length of $BQ = 5 - 2$
$= 3$

$$\text{Length of } CQ = 11 - 5$$
$$= 6$$

Now, for the points B and C, $\dfrac{\text{Vertical distance}}{\text{Horizontal distance}} = \dfrac{CQ}{BQ} = \dfrac{6}{3} = 2$.

In both instances, the ratio of the vertical distance to the horizontal distance between the two points under consideration is the gradient 2 of the straight line.

Accordingly, let us develop a formula to find the gradient of a straight line using its graph. Let us consider any straight line with equation $y = mx + c$.



Let us consider any two points $A\ (x_1, y_1)$ and $B\ (x_2, y_2)$ on the straight line.
Since these two points lie on the straight line,

$$y_1 = mx_1 + c \quad \text{——} \quad ①$$
$$y_2 = mx_2 + c \quad \text{——} \quad ②$$

From ① and ② $\quad y_1 - y_2 = mx_1 - mx_2$

$$\therefore\ y_1 - y_2 = m(x_1 - x_2)$$

$$\therefore\ \frac{y_1 - y_2}{x_1 - x_2} = m$$

$$\therefore\ m = \frac{y_1 - y_2}{x_1 - x_2}$$

$$\therefore\ \text{The gradient of the straight line} = \frac{y_1 - y_2}{x_1 - x_2}$$

Example 1

The coordinates of two points on a straight line are (3, 10) and (2, 6). Find the gradient of the straight line.

$$\text{Gradient of the straight line} = \frac{y_1 - y_2}{x_1 - x_2}$$

$$= \frac{10-6}{3-2}$$

$$= \frac{4}{1}$$

$$= \underline{\underline{4}}$$

Example 2

The coordinates of two points on a straight line are (6, 3) and (2, 5). Find the gradient of the straight line.

$$\text{Gradient of the straight line} = \frac{y_1 - y_2}{x_1 - x_2}$$

$$= \frac{3-5}{6-2}$$

$$= \frac{-2}{4}$$

$$= -\underline{\underline{\frac{1}{2}}}$$

Example 3

Find the gradient of the straight line that passes through the points (–2, 4) and (1, –2).

$$\text{Gradient of the straight line} = \frac{y_1 - y_2}{x_1 - x_2}$$

$$= \frac{4-(-2)}{-2-1}$$

$$= \frac{4+2}{-3}$$

$$= \frac{6}{-3}$$

$$= \underline{\underline{-2}}$$

Exercise 21.1

1. Calculate the gradient of the straight line which passes through each pair of points.

(i) (4, 6) (2, 2) (ii) (6, 2) (4, 3) (iii) (1, –2) (0, 7) (iv) (–2, –3) (2, 5)
(v) (4, 5) (–8, –4) (vi) (6, –4) (2, 2) (vii) (1, –4) (–2, –7) (viii) (4, 6) (–2, –9)

21.2 Finding the equation of a straight line when the intercept of its graph and the coordinates of a point on the graph are given

Example 1

The intercept of the graph of a straight line is 3. The coordinates of a point on the graph is (2, 7). Write the equation of the straight line.

The equation of a straight line graph with gradient m and intercept c is $y = mx + c$.

By substituing the value of the intercept and the coordinates of the point on the graph into the equation of the function we obtain,

$$y = mx + c$$
$$7 = 2m + 3$$
$$7 - 3 = 2m$$
$$4 = 2m$$
$$m = \frac{4}{2}$$
$$m = 2$$

By substituting $c = 3$ and $m = 2$ into the equation we obtain $y = 2x + 3$.

$$\underline{\underline{y = 2x + 3}}$$

Exercise 21.2

1. For each graph with the given intercept and passing through the given point, write down the equation of the corresponding function.

(i) Intercept = 1and (3, 10) (ii) Intercept = 2 and (3, 3)
(iii) Intercept = 5 and (2, 1) (iv) Intercept = 0 and (3, 12)
(v) Intercept = – 4 and (3, 8) (vi) Intercept = –5 and (–2, –9)

21.3 Finding the equation of a straight line which passes through two given points

Let us find the equation of the straight line which passes through the points (1, 7) and (3, 15). To obtain the equation, let us find the gradient and the intercept of the graph.
Let us first find the gradient of the straight line, using the coordinates (1, 7) and (3, 15) of the two points on the line.

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

$$m = \frac{7 - 15}{1 - 3}$$

$$m = \frac{-8}{-2}$$

$$m = 4$$

Let us substitute the value of m and the coordinates of one of the points into the equation $y = mx + c$. Thereby we can find the value of c.

$$x = 1 \quad y = 7 \quad m = 4$$

$$y = mx + c$$

$$7 = 4 \times 1 + c$$

$$7 - 4 = c$$

$$3 = c$$

$$c = 3$$

$m = 4$ and $c = 3$

The gradient of the graph is 4 and the intercept is 3.
Therefore, the required equation is $y = 4x + 3$.

Example 1

Find the equation of the straight line passing through the points (4, 3) and (2, –1).

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

$$m = \frac{3 - (-1)}{4 - 2}$$

$$m = \frac{4}{2}$$

$$m = 2$$

Now let us substitute the coordinates of the point (2, –1) and the gradient into the equation $y = mx + c$.

$$x = 2 \quad y = -1 \quad m = 2$$

$$\begin{aligned} y &= mx + c \\ -1 &= 2 \times 2 + c \\ -1 &= 4 + c \\ -1 - 4 &= c \\ -5 &= c \\ c &= -5 \end{aligned}$$

$\therefore$ The equation of the straight line is $y = 2x - 5$.

Exercise 21.3

1. Find the equation of each of the straight line graphs that passes through the given points.

(i) (1, 7) (2, 10) (ii) (3, –1) (–2, 9) (iii) (4, 3) (8, 4) (iv) (2, –5) (–2, 7)

(v) (–1, –8) (3, 12) (vi) (–5, 1) (10, –5) (vii) $\left(\frac{2}{3}, \frac{2}{3}\right)\left(1, 1\frac{1}{3}\right)$ (viii) (2, 2) (0, –4)

21.4 Graphs of functions of the form $y = ax^2$

Now let us identify several basic properties of graphs of functions of the form $y = ax^2$. Here a is a non-zero number. Here y is defined as the function. y can be considered as a function which is determined by ax^2.

First let us draw the graph of $y = x^2$.
To do this, let us proceed according to the following steps.

Step 1
Preparing a table of values to find the y values corresponding to given x values of the function.

$y = x^2$

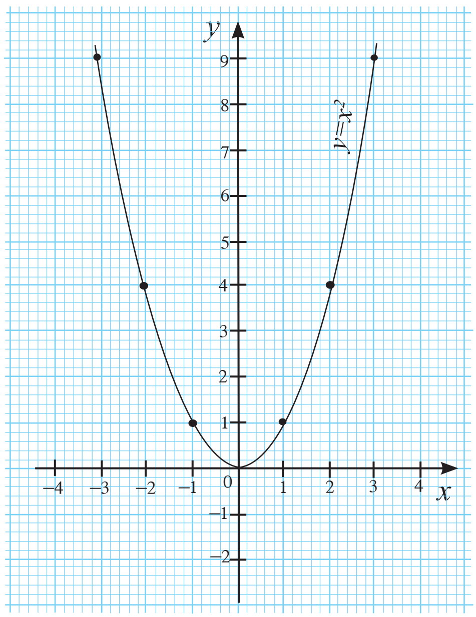
x	–3	–2	–1	0	1	2	3
x^2	9	4	1	0	1	4	9
y	9	4	1	0	1	4	9

Using the table of values, let us obtain the coordinates of the points required to draw the graph of the function.
(–3, 9), (–2, 4), (–1, 1), (0, 0), (1, 1), (2, 4), (3, 9)

Step 2
Preparing a Cartesian coordinate plane to mark the coordinates that were obtained.

The maximum value that the x coordinate takes in the pairs that were obtained is +3 and the minimum value is –3. The maximum value that the y coordinate takes is 9 and the minimum value is 0.
Let us draw the x and y axes according to a suitable scale, on the piece of paper that is to be used to draw the graph, so that the x - axis can be calibrated from –3 to 3 and the y - axis can be calibrated from 0 to 9.

Step 3
Drawing the graph of the function.
Let us mark the seven points on the coordinate plane that has been prepared.
Next let us join the points that have been marked so that a smooth curve is obtained. This smooth curve is the graph of the function $y = x^2$.



The curve that is obtained as the graph of a function of the form $y = ax^2$ is defined as a parabola.
Let us identify several properties of the graph of the function $y = x^2$ by considering the graph that was drawn.
For the function $y = x^2$,
- the graph is symmetric about the y - axis. Therefore, the y - axis is the axis of symmetry of the graph and the equation of the axis of symmetry is $x = 0$.
- when the value of x increases negatively (i.e., –3 to 0) the function decreases positively and when the value of x increases positively (i.e., 0 to +3) the function increases positively.

To identify the common properties of the graphs of functions of the form $y = ax^2$ where $a > 0$, let us draw the graphs of the functions $y = x^2$, $y = 3x^2$ and $y = \frac{1}{2}x^2$ on the same coordinate plane.

$y = 3x^2$

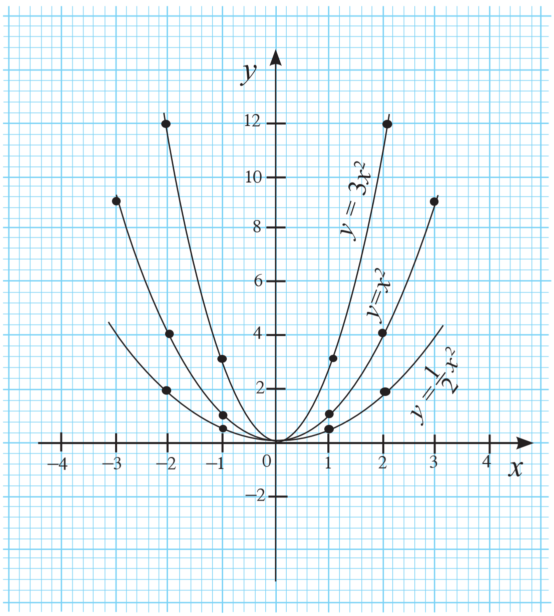
x	-2	-1	0	1	2
x^2	4	1	0	1	4
$3x^2$	12	3	0	3	12
y	12	3	0	3	12

$y = \frac{1}{2}x^2$

x	-2	-1	0	1	2
x^2	4	1	0	1	4
$\frac{1}{2}x^2$	2	$\frac{1}{2}$	0	$\frac{1}{2}$	2
y	2	$\frac{1}{2}$	0	$\frac{1}{2}$	2

$(-2, 12), (-1, 3), (0, 0), (1, 3), (2, 12)$

$(-2, 2), (-1, \frac{1}{2}), (0, 0), (1, \frac{1}{2}), (2, 2)$



Let us identify several common properties of the graphs of functions of the form $y = ax^2$, where $a > 0$, by considering the above graphs.

- The graph is a parabola with a minimum point.
- The coordinates of the minimum point are $(0, 0)$.
- The graph is symmetric about the y - axis.
- The equation of the axis of symmetry is $x = 0$.

- The minimum value of the function (i,e., value of y) is 0.
- The function decreases when the value of x increases negatively (increasing along negative values) and reaches the minimum value at $x = 0$.
- The function increases from 0 when the value of x increases positively (increasing along positive values).

To identify the common properties of graphs of functions of the form $y = ax^2$ where $a < 0$, let us draw the graphs of the functions $y = -x^2$, $y = -2x^2$ and $y = -\frac{1}{2}x^2$ on the same coordinate plane.

$y = -x^2$

x	-3	-2	-1	0	1	2	3
x^2	9	4	1	0	1	4	9
$-x^2$	-9	-4	-1	0	-1	-4	-9
y	-9	-4	-1	0	-1	-4	-9

$(-3, -9)$ $(-2, -4)$ $(-1, -1)$ $(0, 0)$ $(1, -1)$ $(2, -4)$ $(3, -9)$

$y = -2x^2$

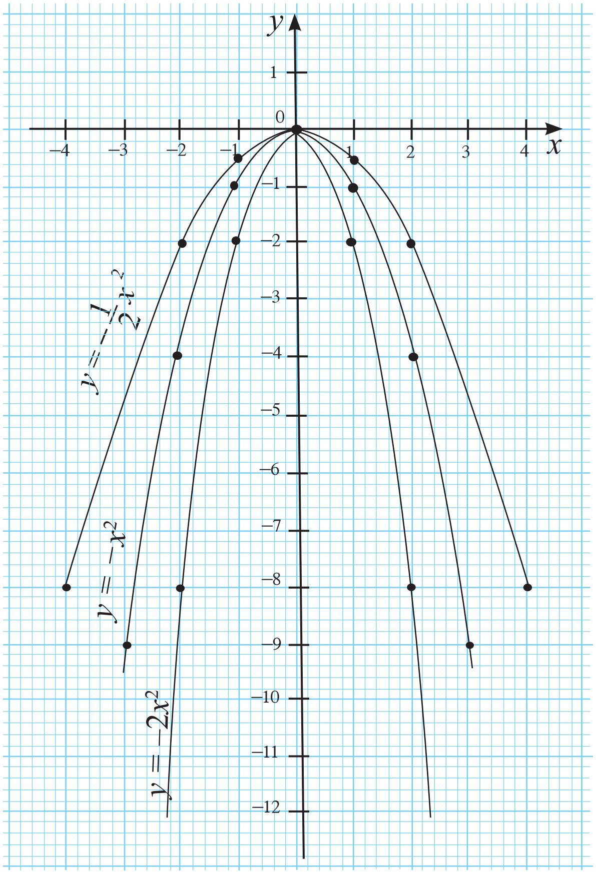
x	-2	-1	0	1	2
x^2	4	1	0	1	4
$-x^2$	-8	-2	0	-2	-8
y	-8	-2	0	-2	-8

$(-2, -8)$ $(-1, -2)$ $(0, 0)$ $(1, -2)$ $(2, -8)$

$y = -\frac{1}{2}x^2$

x	-4	-2	0	2	4
x^2	16	4	0	4	16
$-\frac{1}{2}x^2$	-8	-2	0	-2	-8
y	-8	-2	0	-2	-8

$(-4, -8)$, $(-2, -2)$, $(0, 0)$, $(2, -2)$, $(4, -8)$



Let us identify the common properties of the graphs of functions of the form $y = ax^2$, when a is negative ($a < 0$), by considering the above graphs.

- The graph is a parabola with a maximum point.
- The coordinates of the maximum point are (0, 0).
- The maximum value of the function is 0.
- The graph is symmetric about the y - axis.
- The equation of the axis of symmetry is $x = 0$.
- The function is increasing when x is negatively increasing and reaches the maximum point when $x = 0$.
- The function is decreasing when x is positively increasing.

Let us identify the basic properties of the graphs of functions of the form $y = ax^2$, by considering the graphs that have been drawn. Here a is any non-zero value.

For functions of the form $y = ax^2$,

- the graph is a parabola.
- the graph is symmetric about the y - axis.
  Therefore, the equation of the axis of symmetry of the graph is $x = 0$.
- the coordinates of the turning point (i.e., the maximum or minimum point) of the graph are (0, 0).
- when the coefficient of x takes a "positive" value, the graph is a parabola with a minimum point.
- when the coefficient of x takes a "negative" value, the graph is a parabola with a maximum point.

Example 1

By examining the function, write down for the graph of the function $y = \frac{2}{3}x^2$,

(i) the equation of the axis of symmetry,
(ii) the coordinates of the turning point,
(iii) whether the turning point is a maximum or a minimum point.

(i) The equation of the axis of symmetry is $x = 0$.
(ii) The coordinates of the turning point are (0, 0).
(iii) Since the coefficient of x^2 in the function is a positive value, the graph has a minimum point.

Example 2

By examining the function, write down for the graph of the function $y = -4x^2$,

(i) the equation of the axis of symmetry,
(ii) the coordinates of the turning point,
(iii) whether the turning point is a maximum or a minimum point.

Since the function is of the form $y = ax^2$,

(i) the equation of the axis of symmetry is $x = 0$.
(ii) the coordinates of the turning point are (0, 0).
(iii) since the coefficient of x^2 in the function is a negative value, the graph has a maximum point.

Exercise 21.4

1. Complete the following table by examining the function

Function	**Coordinates of the turning point**	**Minimum value of *y***	**Maximum value of *y***	**Equation of the axis of symmetry**
$y = 5x^2$				
$y = -\frac{1}{3}x^2$				
$y = -\frac{2}{3}x^2$				
$y = \frac{3}{4}x^2$				
$y = -7x^2$				

2. Incomplete tables of values prepared to draw the graphs of the functions $y = \frac{1}{3}x^2$ and $y = -\frac{1}{4}x^2$ are given below.

$y = \frac{1}{3}x^2$

x	-6	-3	0	3	6
y	12	___	0	3	___

$y = -\frac{1}{4}x^2$

x	-4	-2	0	2	4
y	-4	-1	0	___	___

(i) Complete the tables and draw the graphs separately.

(ii) For the functions, write down

(*a*) the equation of the axis of symmetry of the graph,

(*b*) the coordinates of the turning point of the graph,

(*c*) the maximum or minimum value.

3. (i) Using values of x such that $-3 \leq x \leq 3$, prepare a suitable table of values to draw the graphs of the functions $y = 2x^2$, $y = 4x^2$, $y = -\frac{1}{3}x^2$ and $y = -3x^2$.

(ii) Draw the graphs on a suitable coordinate plane.

(iii) For each of the graphs write down

(*a*) the equation of the axis of symmetry

(*b*) the coordinates of the turning point

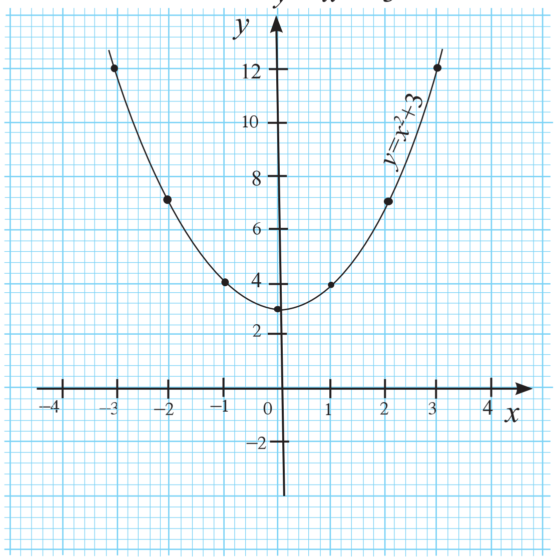
(*c*) the maximum or minimum value of the function.

21.5 Graph of a function of the form $y = ax^2 + b$

Let us draw the graph of the function $y = x^2 + 3$ to identify several basic properties of the graph of a function of the form $y = ax^2 + b$ (Here $a \neq 0$).

x	– 3	– 2	– 1	0	1	2	3
x^2	9	4	1	0	1	4	9
+3	+3	+3	+3	+3	+3	+3	+3
y	12	7	4	3	4	7	12

$y = x^2 + 3$



The graph of the function $y = x^2 + 3$ is a parabola with a minimum point.
For the function $y = x^2 + 3$,

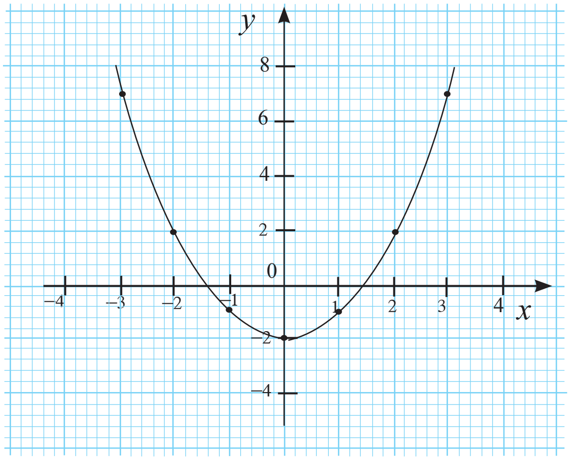
- the equation of the axis of symmetry of the graph is $x = 0$.
- the graph has a minimum point, with coordinates (0, 3).
- the minimum value of the y coordinates of the points on the graph is 3. Therefore, the minimum value of the function is 3.

Let us draw the graph of the function $y = x^2 – 2$ to identify the properties of the graph of a function of the form $y = ax^2 + b$ when the value of b is negative.

$y = x^2 – 2$

x	–3	–2	–1	0	1	2	3
x^2	9	4	1	0	1	4	9
–2	–2	–2	–2	–2	–2	–2	–2
y	7	2	–1	–2	–1	2	7

(–3, 7) (–2, 2) (1, –1) (0, –2) (1, –1) (2, 2) (3, 7)



The graph of the function $y = x^2 - 2$ is a parabola with a minimum value.
For the function $y = x^2 - 2$,

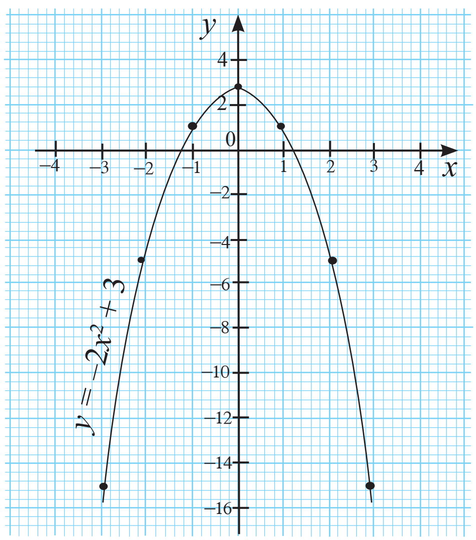
- the equation of the axis of symmetry is $x = 0$.
- the coordinates of the turning point are $(0, -2)$.
- the minimum value of the y coordinates of the points on the graph of the function is -2. Therefore, the minimum value of the function is -2.

Let us draw the graph of the function $y = -2x^2 + 3$ to identify the properties of the graph of a function of the form $y = ax^2 + b$. When the value of a is negative.

$y = -2x^2 + 3$

x	-3	-2	-1	0	1	2	3
x^2	9	4	1	0	1	4	9
$-2x^2$	-18	-8	-2	0	-2	-8	-18
$+3$	$+3$	$+3$	$+3$	$+3$	$+3$	$+3$	$+3$
y	-15	-5	$+1$	$+3$	$+1$	-5	-15

$(-3, -15)$ $(-2, -5)$ $(-1, 1)$ $(0, 3)$ $(1, 1)$ $(2, -5)$ $(3, -15)$



The graph of the function $y = -2x^2 + 3$ is a parabola with a maximum point.
For the function $y = -2x^2 + 3$,

- the equation of the axis of symmetry of the graph is $x = 0$.
- the coordinates of the turning point of the graph are (0, 3).
- the maximum value of the points on the graph of the function is 3.

Therefore, the maximum value of the function is 3.

Let us identify several common properties of the graphs of functions of the form $y = ax^2 + b$, by examining the graphs that were drawn of functions of this form.

The graph of a function of the form $y = ax^2 + b$,
- is a parabola with a minimum point when a is a positive value.
- is a parabola with a maximum point when a is a negative value.
- the equation of the axis of symmetry of the graph is $x = 0$.
- the coordinates of the maximum or minimum point (turning point) is $(0, b)$.
- the maximum or minimum value of the function is b.

Example 1

Write down for the graph of the function $y = 3x^2 - 5$,
(i) the equation of the axis of symmetry,
(ii) the coordinates of the turning point,
(iii) the maximum or minimum value of the function.

(i) Since the graphs of functions of the form $y = ax^2 + b$ are parabolas which are symmetric about the y - axis, the equation of the axis of symmetry of the graph of the function $y = 3x^2 - 5$ is $x = 0$.

(ii) Since the coordinates of the turning point of the graphs of functions of the form $y = ax^2 + b$ are $(0, b)$, the coordinates of the turning point of the graph of the function $y = 3x^2 - 5$ are $(0, -5)$.

(iii) Since the coefficient of x^2 in $y = 3x^2 - 5$ is positive, the graph has a minimum value. The minimum value of the function is -5.

Example 2

Write down for the function $y = 4 - 2x^2$,
(i) the equation of the axis of symmetry of the graph,
(ii) the coordinates of the turning point of the graph,
(iii) the maximum or minimum value of the function.

(i) The equation of the axis of symmetry of the graph of $y = 4 - 2x^2$, is $x = 0$.

(ii) The coordinates of the turning point are (0, 4).

(iii) Since the coefficient of x^2 in $y = 4 - 2x^2$ is negative, the graph has a maximum value. The maximum value of the function is 4.

Exercise 21.5

1. Without drawing the graphs of the functions of the form $y = ax^2 + b$ given below, complete the following table.

Function	**Equation of the axis of symmetry**	**Coordinates of the turning point of the graph**	**Whether the graph has a maximum or minimum value**	**The maximum or minimum value of the function**
$y = 3x^2 + 4$				
$y = 3 - 4x^2$				
$y = \frac{3}{2}x^2 + 4$				
$y = \frac{3}{2}x^2 - 5$				
$y = 2x^2 - \frac{1}{3}$				

2. Incomplete tables of values prepared to draw the graphs of the functions $y = 2x^2 - 4$ and $y = -x^2 + 5$ are given below.

$y = 2x^2 - 4$

x	-2	-1	0	1	2
y	4	___	___	-2	4

$y = -x^2 + 5$

x	-3	-2	-1	0	1	2	3
y	-4	___	$+4$	$+5$	___	$+1$	-4

(i) Complete each table and draw the corresponding graph. For each graph write down

(*a*) the equation of the axis of symmetry of the graph,
(*b*) the coordinates of the turning point,
(*c*) the maximum or minimum value of the function.

3. For each of the functions given in (a) to (d) below, prepare a table of values to draw the graph of the function, considering the integral values of x in the range $-3 \leq x \leq 3$.

For each function,
(i) draw the corresponding graph
(ii) write down the equation of the axis of symmetry of the graph
(iii) indicate the turning point on the graph and write down whether it is a maximum or a minimum
(iv) write down the maximum or minimum value of the function.

(*a*) $y = x^2 + 4$
(*b*) $y = 4 - x^2$
(*c*) $y = -(2x^2 + 3)$
(*d*) $y = 4x^2 - 5$

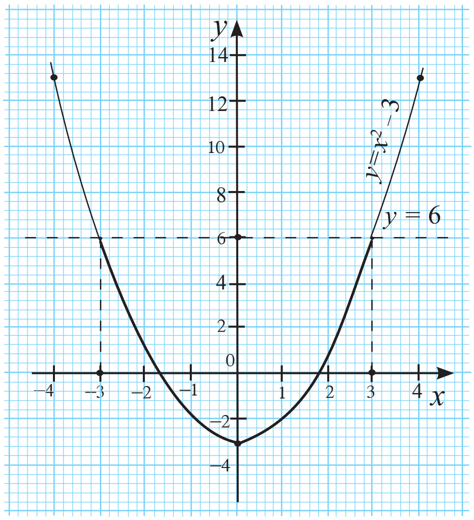
21.6 Finding the interval of values of x corresponding to an interval of values of y, for a function of the form $y = ax^2 + b$

Let us identify how the interval of values of x corresponding to an interval of values of y is found for a function with a minimum value, by considering the graph of the function $y = x^2 - 3$. Let us find the interval of values of x for which the value of the function is less than 6; that is $y < 6$. Let us first draw the graph of $y = x^2 - 3$.

$y = x^2 - 3$

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
-3	-3	-3	-3	-3	-3	-3	-3	-3	-3
y	13	6	1	-2	-3	-2	1	6	13

$(-4, 13)$ $(-3, 6)$ $(-2, 1)$ $(-1, -2)$ $(0, -3)$ $(1, -2)$ $(2, 1)$ $(3, 6)$ $(4, 13)$



To identify the region of the graph, belonging to the interval of values $y < 6$, let us draw the straight line $y = 6$. In the region of the graph below the line $y = 6$, the y coordinate takes values less than 6. This region of the graph has been indicated by a dark line.

Let us draw two lines parallel to the y - axis, from the points of intersection of the graph and the line $y = 6$, up to the x - axis. Let us mark the two points at which these two lines meet the x - axis (-3 and $+3$).

The interval of values of x between these two points is the interval of values of x for which $y < 6$. That is, when the value of x is greater than -3 and less than $+3$, the value of $y < 6$. Therefore, the interval of values of x for which the function $y = x^2 - 3$ satisfies the condition $y < 6$ is $-3 < x < 3$.

Example 1

For the function $y = x^2 - 4$,

(i) find the values of x for which $y \geq 0$.

(ii) what is the interval of values of x for which the function is increasing positively?

(iii) what is the interval of values of x for which the function is decreasing positively?

(iv) what is the interval of values of x for which the function is increasing negatively?

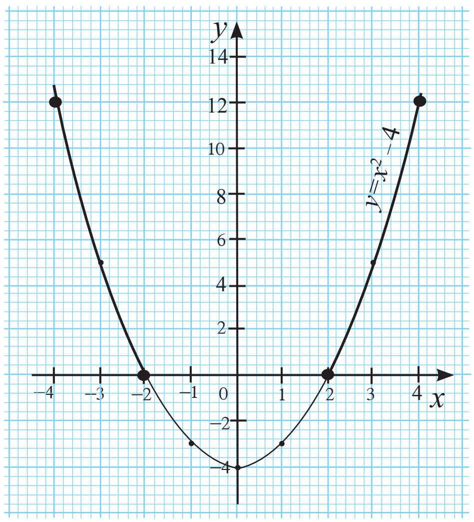
(v) what is the interval of values of x for which the function is decreasing negatively?

First let us draw the graph.

$$y = x^2 - 4$$

x	−4	−3	−2	−1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
−4	−4	−4	−4	−4	−4	−4	−4	−4	−4
y	12	5	0	−3	−4	−3	0	5	12

(−4, 12) (−3, 5) (−2, 0) (−1, −3) (0, −4) (1, −3) (2, 0) (3, 5) (4, 12)



(i) The portion of the graph for which $y \geq 0$, is the portion on and above the straight line $y = 0$.

The corresponding values of x are those which are less than or equal to –2 or greater than or equal to +2. That is, $x \leq -2$ or $x \geq 2$

(ii) $x > 2$.

(iii) $x < -2$.

(iv) $0 < x < 2$.

(v) $-2 < x < 0$.

Exercise 21.6

1. Draw the graph of $y = 3 - 2x^2$ and find the interval of values of x for which $y \geq 1$.

2. Draw the graph of $y = 2x^2 - 4$ and find,
(i) the interval of values of x for which $y < -3$.
(ii) the interval of values of x for which the function increases negatively.
(iii) the interval of values of x for which the function increases positively.
(iii) the interval of values of x for which the function decreases positively.
(iv) the interval of values of x for which the function decreases negatively.

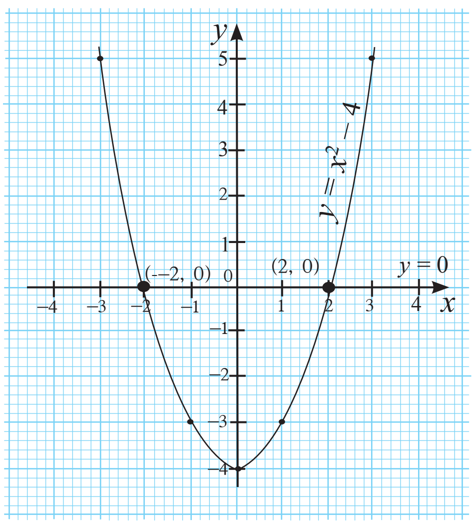
21.7 Finding the roots of an equation of the form $ax^2 + b = 0$ by considering the graph of a function of the form $y = ax^2 + b$

Let us consider for example how the roots of the equation $x^2 - 4 = 0$ are found. To do this, we need to first draw the graph of the function $y = x^2 - 4$.

$y = x^2 - 4$

x	-3	-2	-1	0	1	2	3
x^2	9	4	1	0	1	4	9
-4	-4	-4	-4	-4	-4	-4	-4
y	5	0	-3	-4	-3	0	5

$(-3,\ 5)$ $(-2,\ 0)$ $(-1,\ -3)$ $(0,\ -4)$ $(1,\ -3)$ $(2,\ 0)$ $(3,\ 5)$



The two points at which the graph of the function $y = x^2 - 4$ cuts the x - axis are $x = -2$ and $x = +2$. That is, when $x = -2$ and $x = +2$, the y coordinate of the graph is 0. Therefore, when $x = -2$ and $x = +2$, we have $x^2 - 4 = 0$. That is, $x = -2$ and $x = +2$ satisfy the equation $x^2 - 4 = 0$. To put this in another way, the roots of the equation $x^2 - 4 = 0$ are 2 and -2.

Exercise 21.7

1. Complete the following table of values to draw the graph of the function $y = 9 - 4x^2$.

x	-2	-1	$-\frac{1}{2}$	0	$+\frac{1}{2}$	1	2
y	-7	5	8	9	____	5	-7

(i) Using the table, draw the graph of the function $y = 9 - 4x^2$
(ii) Using the graph, find the roots of the equation $9 - 4x^2 = 0$

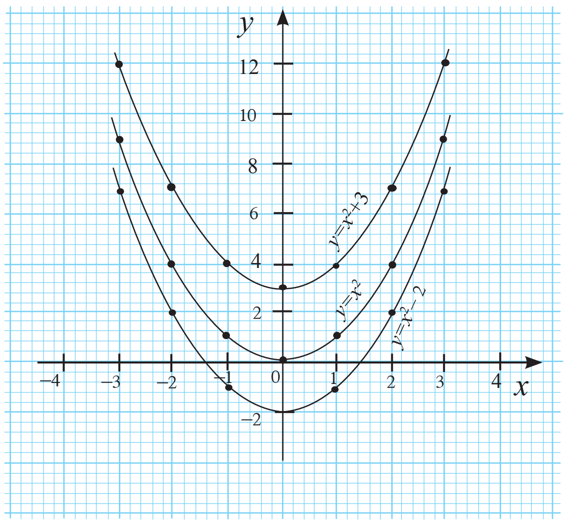
2. Prepare a table of values with $-3 \leq x \leq 3$ to draw the graph of the function $y = x^2 - 1$.
(i) Draw the graph of $y = x^2 - 1$.
(ii) Using the graph, find the roots of $x^2 - 1 = 0$.

3. Prepare a table of values with $-3 \leq x \leq 3$ to draw the graph of the function $y = 4 - x^2$.
(i) Draw the graph of $y = 4 - x^2$.
(ii) Using the graph, find the roots of $4 - x^2 = 0$.

4. Prepare a suitable table of values and draw the graph of $y = x^2 - 9$.
(i) Draw the graph of $y = x^2 - 9$.
(ii) Using the graph, find the roots of $9 - x^2 = 0$.

21.8 Verticle displacement of graphs of functions of the form $y = ax^2 + b$

Consider the graphs given below which you have studied earlier.



• Observe that,
by translating the graph of $y = x^2$ by 3 units vertically upwards, the graph corresponding to the function with the equation $y = x^2 + 3$ is obtained and also by translating the graph of $y = x^2$ by 2 units vertically downwards, the graph corresponding to the function with the equation $y = x^2 - 2$ is obtained.

Observe the following table.

Equation of the graph	Minimum point	Axis of symmetry
$y = x^2$	(0, 0)	$x = 0$
$y = x^2 + 3$	(0, 3)	$x = 0$
$y = x^2 - 2$	(0, –2)	$x = 0$

Accordingly,

- if we translate the graph of $y = x^2$ by 6 units vertically upwards, the equation of the graph of the corresponding function will be $y = x^2 + 6$.
- if we translate the graph of $y = x^2$ by 4 units vertically downwards, the equation of the corresponding function of the graph will be $y = x^2 - 4$.
- in general, the equation of the graph obtained by translating the graph of a function of the form $y = ax^2 + b$ vertically upwards or downwards by c units is respectively $y = ax^2 + b + c$ or $y = ax^2 + b - c$.

Exercise 21.8

1. If the graph of the function $y = x^2 + 2$,
(i) moves upwards along the y axis by 2 units
(ii) moves downwards along the y axis by 2 units
write the equation of the graph.

2. If the graph of the function $y = -x^2$,
(i) moves upwards along the y axis by 3 units
(ii) moves downwards along the y axis by 3 units
write the equation of the graph.

3. If the graph of the function $y = 2x^2 + 5$,
(i) moves upwards along the y axis by 6 units
(ii) moves downwards along the y axis by 6 units
write the equation of the graph.

Miscellaneous Exercise

1. The coordinates of two points on a straight line graph are (0,3) and (3, 1).
(i) Calculate the gradient of the graph.
(ii) Find the intercept of the graph.
(iii) Write down the function of the graph.

2. Without drawing the graph, providing reasons show that the points (–1, –3), (2, 4) and (4, 6) line on the same straight line graph.

3. Without drawing the graph, providing reasons show that the points (–2, –8), (0, –2), (3, 7) and (2, 4) lie on the same straight line graph.

4. Draw the graph of the function $y = \frac{x^2}{2} - 3$.
(i) Using the graph, find the interval of values of x for which $y \geq 1\frac{1}{2}$.
(ii) Using the graph, find the interval of values of x for which the value of the function is less than –1.

5. Construct the table of values to draw the graph of the function $y = 3 - 2x^2$ in the interval $-2 \leq x \leq 2$.
(i) Using the table of values, draw the graph of $y = 3 - 2x^2$.
(ii) Using the graph, find the roots of the equation $3 - 2x^2 = 0$.
(iii) Write down the equation of the graphs which is obtained when the above graph is shifted upwards by 2 units.