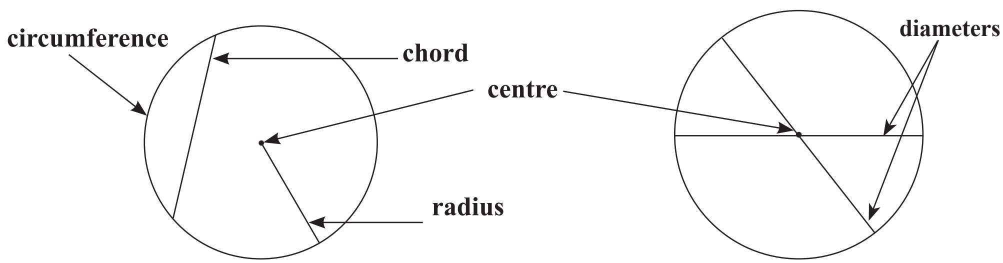
27 Chords of a Circle

By studying this lesson you will be able to

- solve problems by using the theorem that the straight line joining the midpoint of a chord of a circle to the centre is perpendicular to the chord, and
- solve problems by using the theorem that the perpendicular drawn from the centre of a circle to a chord bisects the chord.



A straight line segment drawn from the centre of a circle to a point on the circle is called a **radius** (Observe the above figure). The length of this line segment is the same irrespective of the point on the circumference that is selected. Therefore, such a line segment is called the radius of the circle. The length of the radius is also called the **radius**.

A straight line segment joining two points on a circle is defined as a **chord**.

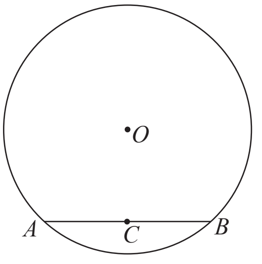
A chord which passes through the centre of the circle is called a **diameter**. All the diameters of a circle are equal in length. This length is also called the diameter. A diameter is the longest chord of a circle. The length of a diameter is twice the length of the radius.

27.1 The straight line segment drawn from the centre of a circle to the midpoint of a chord

Activity 1

- On a piece of paper, construct a circle of radius 3 cm using a pair of compasses and name its centre as O. Draw a chord AB which is not a diameter of the circle.
- By measuring the length using a ruler, mark the midpoint of the chord as C and join OC.

- Measure the magnitude of the angle $O\hat{C}A$ (or $O\hat{C}B$) with the aid of a protractor. Observe that this angle is 90º, that is, that OC and AB are perpendicular to each other.



- Draw several more chords of different lengths within this circle itself and observe that the straight line joining the midpoint of each chord to the centre is perpendicular to the respective chord.
- Draw several circles of different radii and do the above activity with these circles too.

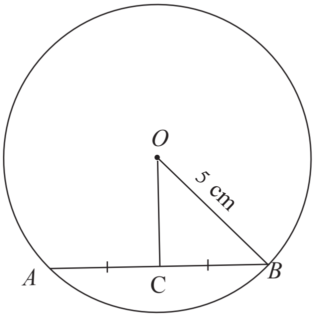
Discuss what you learnt through this activity with the other students in the class. The relationship that was identified is a theorem related to the chords of a circle.

Theorem
The straight line joining the centre of the circle to the midpoint of a chord is perpendicular to the chord.

Let us now consider how calculations are done using the above theorem.

Example 1

AB is a chord of a circle with centre O and radius 5 cm. The midpoint of AB is C. If AB = 8 cm, find the length of OC.
Let us represent this information in a figure.



$O\hat{C}B = 90°$ (Since the straight line joining the centre of the circle to the midpoint of a chord is perpendicular to the chord)

$\therefore$ OCB is a right angled triangle.

Let us find the length of OC by applying Pythagoras' theorem.

Now, $BC = \frac{8}{2} = 4$ cm (Since C is the midpoint of AB)

Also $OB = 5$ cm (OB is the radius of the circle)

$OB^2 = OC^2 + CB^2$ (Pythagoras' theorem)

$$\therefore \quad 5^2 = OC^2 + 4^2$$

$$25 = OC^2 + 16$$

$$25 - 16 = OC^2$$

$$OC^2 = 9$$

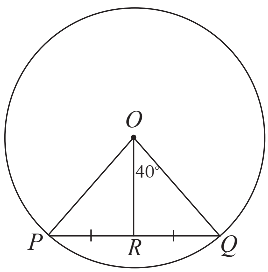
$$\therefore \quad OC = \sqrt{9}$$

$$= 3$$

$\therefore$ The length of OC is 3 cm.

Example 2

PQ is a chord of a circle with centre O. The midpoint of PQ *is* R. If $Q\hat{O}R = 40°$, find $O\hat{P}R$.



$O\hat{R}Q = 90^\circ$ (Since the straight line joining the centre of the circle to the midpoint of a chord is perpendicular to the chord)

Since the sum of the interior angles of a triangle is 180°,

$$O\hat{Q}R = 180^\circ - (40^\circ + 90^\circ)$$

$$\therefore O\hat{Q}R = 50^\circ$$

Now let us consider the triangle OPQ

$OQ = OP$ (Radii of the same circle)

$\therefore$ OPQ is an isosceles triangle.

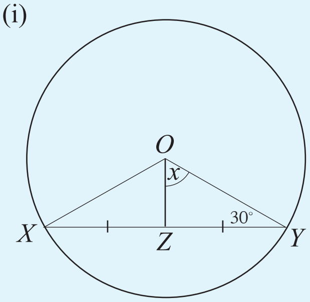
$$\therefore O\hat{P}R = O\hat{Q}R$$

$$\therefore O\hat{P}R = \underline{\underline{50^\circ}}$$

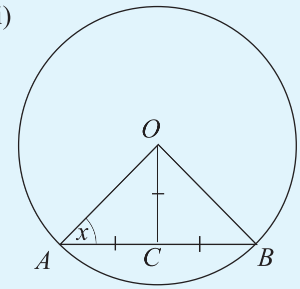
Exercise 27.1

1. Find the value of x in each of the following figures using the given data. In each figure, O denotes the centre of the circle.

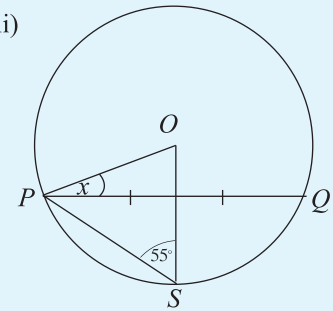
(i)



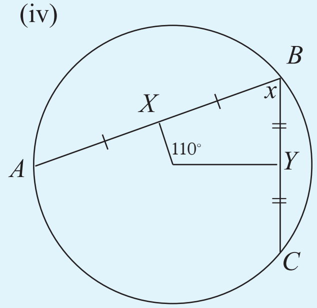
(ii)



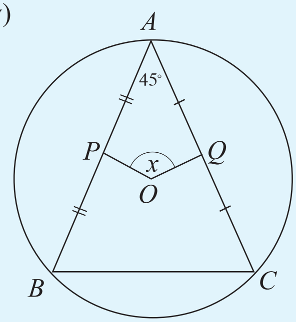
(iii)



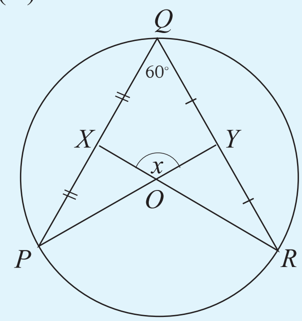
(iv)



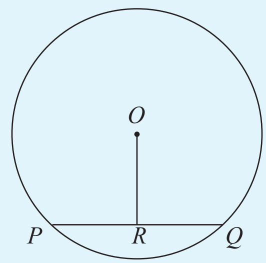
(v)



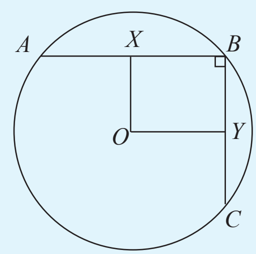
(vi)

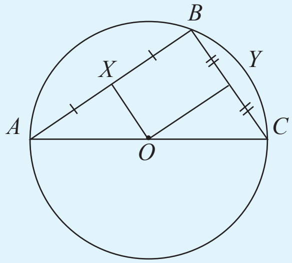


2. PQ is a chord of a circle with centre O. The midpoint of the chord is R. If PQ=12 cm and OR = 8 cm, find the radius of the circle.



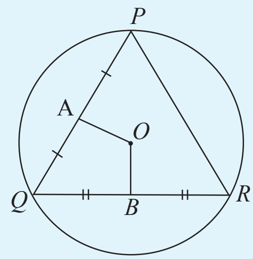
3. AB and BC are two chords of a circle with centre O which are perpendicular to each other. AB = 12 cm and BC = 8 cm. The mid-points of AB and BC are X and Y respectively. Find the perimeter of the quadrilateral $OXBY$.



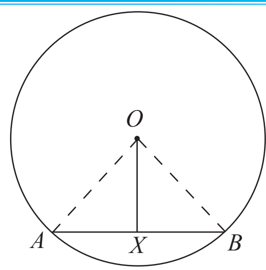
4. 

AB and BC are two chords of a circle with centre O which are perpendicular to each other. The mid-points of AB and BC are X and Y respectively. If AB = 8 cm and BC = 6 cm, find the perimeter of the quadrilateral $BXOY$.

5. The vertices P, Q and R of the triangle PQR lie on a circle with centre O. The midpoints of PQ and QR are A and B respectively. If PQ = 16 cm, OA = 6 cm and $OB = \sqrt{19}$ cm, calculate the length of QR.



27.2 The formal proof of the theorem that "the straight line joining the midpoint of a chord of a circle to the centre is perpendicular to the chord"



Data : AB is a chord of a circle with centre O. The midpoint of AB is X.

To be proved: OX is perpendicular to AB.

Construction: Join OA, OB.

Proof : In the triangles OXA and OXB,

$AO = BO$ (Radii of the same circle)

$AX = XB$ (Since X is the midpoint of AB)

OX is a common side.

$\therefore\ OXA\Delta \equiv OXB\Delta$ (SSS)

$\therefore\ O\hat{X}A = O\hat{X}B$

But, $O\hat{X}A + O\hat{X}B = 180^\circ$

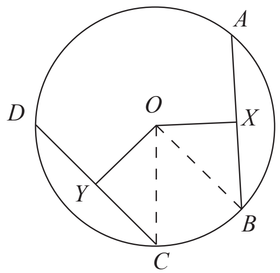
$\therefore\ 2O\hat{X}A = 180^\circ$

$O\hat{X}A = 90^\circ$

$\therefore\ OX$ is perpendicular to AB

Let us now consider how riders are proved using the above theorem.

Example 1



AB and CD are two equal chords of a circle of centre O. The midpoints of AB and CD are X and Y respectively. Show that $OX = OY$.

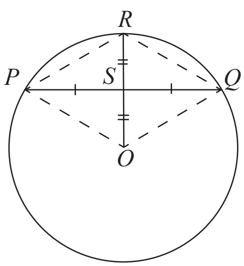
To show that $OX = OY$, let us first show that the two triangles OXB and OYC are congruent to each other under the conditions of hypotenuse-side.

In the triangles OXB and OYC,
since X is the midpoint of AB and Y is the midpoint of CD,
$O\hat{X}B = 90^\circ$ and $O\hat{Y}C = 90^\circ$.
Since they are radii of the same circle, $OB = OC$.
Also, since it is given that $AB = CD$, we obtain $\frac{1}{2}AB = \frac{1}{2}CD$.

Therefore, since X and Y are the midpoints of the chords AB and CD, we obtain $XB = YC$.

$\therefore\ \Delta OXB \equiv \Delta OYC$ (Hypotenuse-side)
$\therefore\ OX = OY$ (Corresponding elements of congruent triangles are equal)

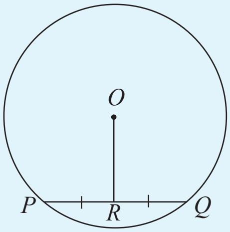
Example 2



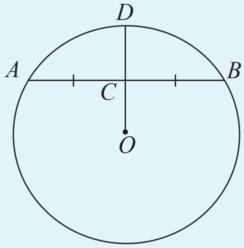
S is the midpoint of the chord PQ of the circle with centre O. OS produced meets the circle at R. If $RS = SO$, show that $OPRQ$ is a rhombus.

$PS = SQ$ (Since S is the midpoint of the chord PQ)
$RS = SO$ (Given)
$\therefore$ $OPRQ$ is a parallelogram.(Since the diagonals bisect each other)
$P\hat{S}O = 90^\circ$ (Since the straight line joining the midpoint of a chord of a circle to the centre is perpendicular to the chord)
That is PQ and RO bisect each other perpendicularly.
$\therefore$ $PRQO$ is a rhombus.

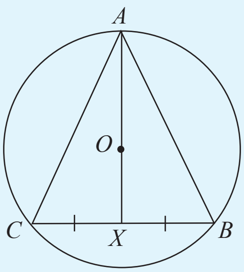
Exercise 27.2

1. 

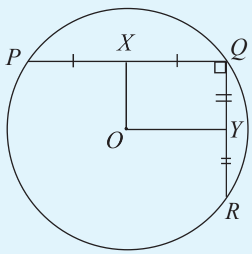
R is the midpoint of the chord PQ of the circle with centre O. If $R\hat{O}Q = 45^\circ$, show that $RQ = OR$.

2. 

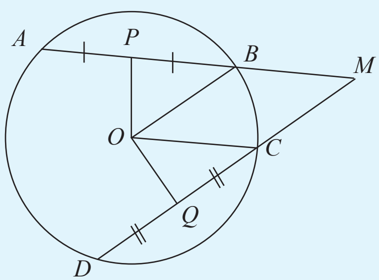
AB is a chord of the circle with centre O. Its midpoint is C. When OC is produced it meets the circle at D. Show that $AD = DB$.

3. 

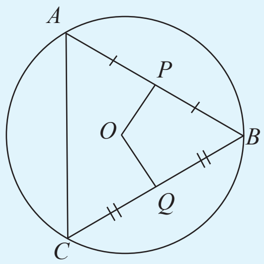
The vertices A,B and C of the triangle ABC lie on a circle with centre O. The midpoint of BC is X. If O lies on AX, show that $AB = AC$.

4. 

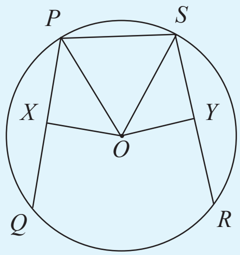
PQ and QR are two chords of a circle with centre O which are perpendicular to each other. The mid-points of PQ and QR are X and Y respectively. Show that $OXQY$ is a rectangle.

5. 

AB and CD are two chords of a circle with centre O. The midpoints of AB and CD are P and Q respectively. The chords AB and DC produced meet at M. Show that $P\hat{O}Q$ and $P\hat{M}Q$ are supplementary angles.

6. 

The midpoints of the chords AB and BC of the circle with centre O are P and Q respectively.
Show that $P\hat{O}Q = B\hat{A}C + A\hat{C}B$.

7. 

PQ and RS are two equal chords of a circle with centre O. The midpoints of PQ and RS are X and Y respectively.
Show that $X\hat{P}S = Y\hat{S}P$.

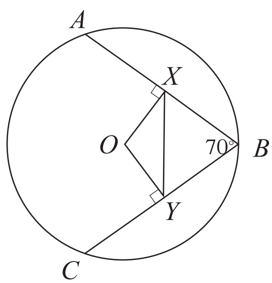
27.3 Converse of the theorem and its applications

The theorem studied above states that the straight line joining the midpoint of a chord of a circle to the centre is perpendicular to the chord. The converse of this theorem is also true. It is expressed as a theorem as follows.

Theorem: The perpendicular drawn from the centre of a circle to a chord bisects the chord.

Now let us consider a couple of examples of calculations done using the theorem that "the perpendicular drawn from the centre of a circle to a chord bisects the chord".

Example 1



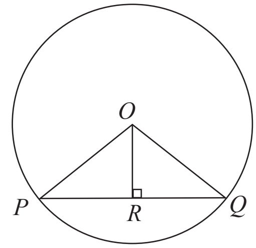
AB and BC are two equal chords of a circle with centre O. The perpendiculars drawn from O to the two chords are OX and OY respectively. If $X\hat{B}Y = 70^\circ$, then find the magnitude of $B\hat{X}Y$.
Since $OX \perp AB$ and $OY \perp BC$,
X is the midpoint of AB and Y is the midpoint of BC.

Also, since it has been given that $AB = BC$, we obtain that $XB = YB$.
Therefore, BXY is an isosceles triangle.

$\therefore B\hat{X}Y = B\hat{Y}X.$

$$\therefore B\hat{X}Y = \frac{180^\circ - 70^\circ}{2}$$
$$= \underline{\underline{55^\circ}}$$

Example 2



OR is the perpendicular drawn from a circle with centre *O* to the chord *PQ*. If *OR*= 3 cm and *PQ* = 8 cm, find the radius of the circle.

Since $OR \perp PQ$, *R* is the midpoint of *PQ*.

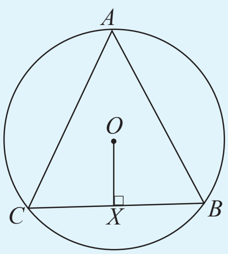
$$\therefore \; PR = \frac{8}{2} = 4 \text{ cm}.$$

Now, by applying Pythagoras' theorem to the triangle *ORP* ,

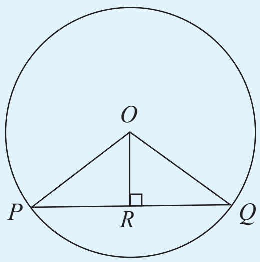
$$OP^2 = OR^2 + PR^2$$
$$= 3^2 + 4^2$$
$$= 25$$
$$\therefore OP = \sqrt{25}$$
$$= 5$$

$\therefore$ $\underline{\underline{\text{Therefore, the radius of the circle is 5 cm}}}$.

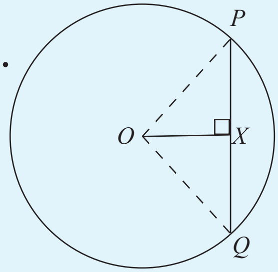
Exercise 27.3

1. 

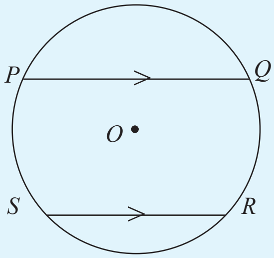
ABC is an equilateral triangle. The vertices *A,B* and *C* lie on the circle with centre *O*. The perpendicular drawn from *O* to *BC* is *OX*. If *XB* = 6 cm, find the perimeter of the triangle *ABC*.

2. 

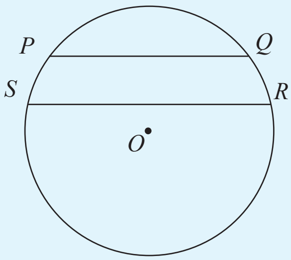
PQ is a chord of a circle with centre *O*. The perpendicular drawn from *O* to *PQ* is *OR*. If *PQ* = 12 cm and *OR* = 8 cm, find the perimeter of the triangle *OPQ*.

3. 

PQ is a chord of a circle with centre O. The perpendicular drawn from O to PQ is OX. If $PQ = 6$ cm and the radius of the circle is 5 cm, find the length of OX.

4. 

PQ and RS are two parallel chords of a circle which lie on opposite sides of the centre O. The radius of the circle is 10 cm. If $PQ = 16$ cm and $SR = 12$ cm, find the distance between the two chords.

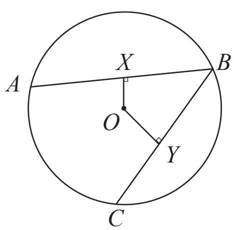
5. 

PQ and RS are two parallel chords of the circle with centre O as shown in the figure. The radius of the circle is 10 cm. If $PQ = 12$ cm and $SR = 16$ cm, find the distance between the two chords.

27.4 Proving riders using the theorem that "the perpendicular drawn from the centre of a circle to a chord bisects the chord"

Example 1

AB and BC are two equal chords of a circle with centre O. The perpendiculars drawn from O to the two chords are OX and OY respectively. Prove that $OX = OY$.



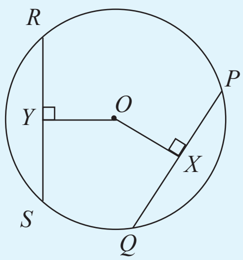
We will prove that $OX = OY$ by showing that the right triangles OXB and OYB are congruent under the conditions of hypotenuse-side.
OB is the common side of these two triangles.

Since $AB=BC$, by the above theorem we have that $BX = YB$.

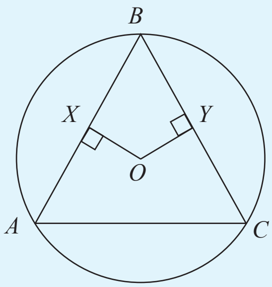
$\therefore \Delta\, OXB \equiv \Delta\, AYB$ (Hypotenuse-side)

$\therefore OX = OY.$ (The remaining elements of congruent triangels are equal to each other.)

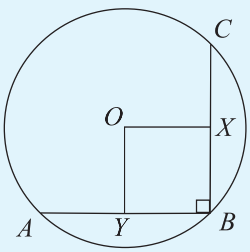
Exercise 27.4

1. 

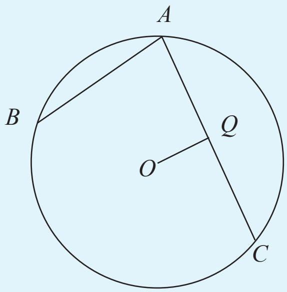
PQ and RS are two chords of a circle with centre O. The perpendiculars drawn from O to PQ and RS are OX and OY respectively. If $OX = OY$ prove that $PQ = RS$.

2. 

The vertices A, B and C of the triangle ABC lie on the circle with centre O. The perpendiculars drawn from O to AB and BC are OX and OY respectively. If $AX = CY$, then prove that $B\hat{A}C = B\hat{C}A$.

3. 

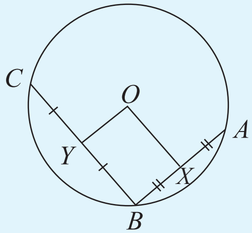
AB and BC are two equal chords of a circle with centre O which are perpendicular to each other. Prove that $OXBY$ is a square.

4. 

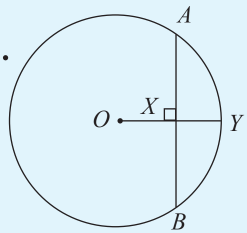
AB and AC are two chords of a circle with centre O. The perpendicular drawn from O to AC is OQ. If $2\,AB = AC$, prove that $AB = AQ$.

Miscellaneous Exercises

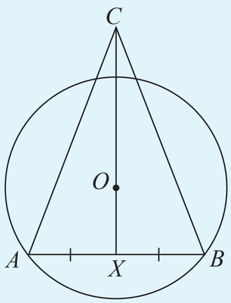
1. A chord of a circle lies 8 cm from the centre. If the length of the chord is 12 cm, find the radius of the circle.

2. 

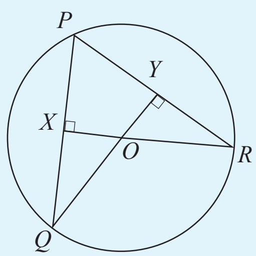
The radius of a circle with centre O is 5 cm. The lengths of the chords AB and BC are 6 cm and 8 cm respectively. The midpoints of the chords are X and Y respectively. Find the perimeter of the quadrilateral $OXBY$.

3. 

The length of the chord AB of the circle with centre O is 8 cm. The perpendicular drawn from O to the chord, intersects the chord at X, and meets the circle at Y. If $XY = 3$ cm, find the radius of the circle.

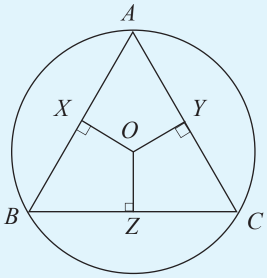
4. 

AB is a chord of a circle with centre O. The midpoint of the chord is X. The point C lies on the line drawn from X through the point O. Prove that $AC = BC$.

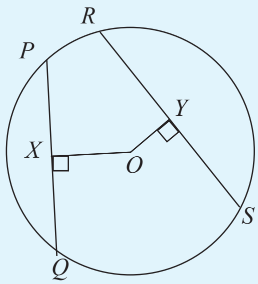
5. 

PQ and PR are two chords of a circle with centre O. The perpendiculars drawn from O to PQ and PR are OX and OY respectively. If XR and QY are straight lines, prove that $PQ = PR$.

6. A chord of length 24 cm lies 5 cm away from the centre of the circle. Another chord lies 12 cm from the centre. Find its length.

7. 

The vertices A,B and C of the equilateral triangle ABC lie on the circle with centre O. The perpendiculars drawn from O to AB, AC and BC are OX, OY and OZ respectively. Prove that $OX = OY = OZ$.

8. 

PQ and RS are two chords of a circle with centre O. The perpendiculars drawn from O to PQ and RS are OX and OY respectively. Show that $PQ^2 - RS^2 = 4OY^2 - 4OX^2$.