20   Logarithms II

By studying this lesson you will be able to
- simplify expressions involving the product and quotient of numbers greater than 1, using the logarithms table
- identify the keys $\boxplus, \boxminus, \boxtimes, \boxdiv, \boxeq, \boxed{(}$ and $\boxed{)}$ on a calculator.

Logarithms Table

Let us recall several facts we have learnt earlier regarding logarithms.

Since $10^0 = 1$, we obtain $\log_{10} 1 = 0$. That is, the logarithm of 1 to base 10 is 0.

Since $10^1 = 10$, we obtain $\log_{10} 10 = 1$. That is, the logarithm of 10 to base 10 is 1.

Since $10^2 = 100$, we obtain $\log_{10} 100 = 2$. That is, the logarithm of 100 to base 10 is 2.

Since $10^3 = 1000$, we obtain $\log_{10} 1000 = 3$. That is, the logarithm of 1000 to base 10 is 3.

The following table has been prepared accordingly.

Number	1	10	100	1 000	10 000
Logarithm to base 10	0	1	2	3	4

This table provides the logarithm of the numbers 1, 10, 100, 1000 and 10 000 to base ten. Logarithm values exist for numbers between 0 and 1, between 1 and 10, between 10 and 100 etc also. These values are not whole numbers. The Scottish mathematician Henry Briggs who lived about four centuries ago prepared the logarithms table by calculating these values. He included only the logarithms of numbers from 1 up to 10. Below is given part of this logarithms table.

N	0	1	2	3	4	5	6	7	8	9	Mean difference								
											1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25

The numbers represented in the leftmost column under N by 10, 11, 12, …, 99 are 1.0, 1.1, 1.2, … , 9.9 which are from 1 up to 10. The decimal point (.) which should be placed in these numbers has not been indicated in the logarithms table. (This has been done to simplify the table). However, when using the table, the decimal point should be placed in the correct position. In the uppermost row, the numbers 0, 1, 2, 3, …, 9 have been written from left to right, and in the same row, on the right, the numbers 1, 2, 3, …, 9 have been written under the mean difference.

As an example the 29th row of the table is given below. In this row, the value in the 6th column is 4713. As mentioned earlier, the decimal point of the numbers is not written. But, when using the table, the decimal point should be placed in the correct position. That is, the value is 0.4713.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	11	12

This means that, the logarithm of 2.96 to base 10 is 0.4713. This also means that $10^{0.4713} = 2.96$. That is, when 2.96 is written as a power of ten it is $10^{0.4713}$. Using the logarithms table we can find the logarithm of numbers consisting of up to 4 digits. When writing logarithms to base 10, instead of writing $\log_{10}$ we write lg in short. Example: $\log_{10} 100 = 2$ is written as lg 100 = 2.

Note that to find the logarithm of 2.9, it must be written as 2.90. The value corresponding to the 29th row and the column indicated by 0 must be taken as the logarithm of 2.90; that is 0.4624.
This is written as, $\log_{10} 2.9 = 0.4624$ or lg 2.9 = 0.4624.
Writing this in index form we obtain $2.9 = 10^{0.4624}$.

Note: Here the logarithm of a number is an approximate value.

20.1 The logarithm of a number with up to two decimal places lying between 1 and 10

Let us identify how lg 4.58 is obtained using the logarithms table. The value that belongs to the row containing 45 which are the first two digits of 4.58 and the column containing 8, which is the remaining digit, is 0.6609. This value is the required logarithm.
That is,
the logarithm of 4.58 = lg 4.58 = 0.6609
Writing this in index form we obtain $4.58 = 10^{0.6609}$.

	0	1	2	3	4	5	6	7	8	9	1	2	3
45									→6609				

Example 1

Find the logarithm of each of the following numbers using the logarithms table. Indicate the relevant index form too.

(i) 6.85   (ii) 3.4   (iii) 8

(i) lg 6.85 = 0.8357, in index form $6.85 = 10^{0.8357}$

(ii) lg 3.4 = 0.5315, in index form $3.4 = 10^{0.5315}$ (Writing 3.4 as 3.40)

(iii) lg 8 = 0.9031, in index form $8 = 10^{0.9031}$

Exercise 20.1

1. Using the logarithms table, find the logarithm of each of the following numbers. Indicate the relevant index form too.

(i) 7.32	(ii) 1.05	(iii) 9.99	(iv) 5.8	(v) 9.2
(vi) 3.1	(vii) 4	(viii) 7	(ix) 1	(x) 1.01

20.2   The logarithm of a number with up to three decimal places lying between 1 and 10

We now know how to obtain the logarithm of a number with up to two decimal places which lies between 1 and 10. Next let us consider how the logarithm of a number with three decimal places which lies between 1 and 10 is found.

Let us identify how the logarithm of 5.075 which is a number with three decimal places is found using the table. The number 7050 is obtained from the table as the value which belongs to the row containing 50, which are the first two digits of 5.075, and the column containing 7 which is the third digit of this number. The mean difference in the same row, under the column containing 5, which is the fourth digit of 5.075 is 4.

			7	8	9	Mean difference 1	2	3	4	5	6	7	8	9
50			→7050							4				

Now add 4 to 7050. Then since 7050 + 4 = 7054, we obtain lg 5.075 = 0.7054. The index form of this is $5.075 = 10^{0.7054}$.

Example 2

Using the logarithms table, find the logarithm of each of the following numbers and then write each number in index form.

(i) 1.099   (ii) 5.875   (iii) 9.071

(i) lg 1.099 = 0.0411, in index form $1.099 = 10^{0.0411}$.

(ii) lg 5.875 = 0.7690, in index form $5.875 = 10^{0.7690}$.

(iii) lg 9.071 = 0.9576, in index form $9.071 = 10^{0.9576}$.

Exercise 20.2

Using the logarithms table, find the logarithm of each of the following numbers and then write each number in index form.

(i) 1.254 (ii) 3.752 (iii) 2.837 (iv) 8.032 (v) 9.998 (vi) 7.543

20.3 The logarithm of numbers which are greater than 10

Although only the logarithms of numbers from 1 up to 10 are included in the logarithms table, the logarithm of any other number (up to four digits or rounded off up to four digits) can also be found using the table. Now let us consider the strategy that is used in such a case.

Example 1

Find the logarithm of 54.37

Method (i): lg 54.37 = lg (5.437×10^1) (Representing in scientific notation)
       = lg 5.437 + lg 10^1 (Applying the laws of logarithms)
       = 0.7354 + 1 (From the logarithms table and since lg 10 = 1)
       = 1.7354

Method (ii): Using indices
      54.37 = 5.437×10^1
         = $10^{0.7354} \times 10^1$ (Writing the value that is obtained from the
            table 0.7354 as an index of 10)

         = $10^{1.7354}$
     $\therefore$ lg 54.37 = 1.7354

Example 2

Find the logarithm of each of the following numbers.

 (i) 8.583   (ii) 85.83   (iii) 858.3   (iv) 8583

(i) lg 8.583 = lg (8.583×10^0) = lg 8.583 + lg 10^0 = 0.9337 + 0 = 0.9337

(ii) lg 85.83 = lg (8.583×10^1) = lg 8.583 + lg 10^1 = 0.9337 + 1 = 1.9337

(iii) lg 858.3 = lg (8.583×10^2) = lg 8.583 + lg 10^2 = 0.9337 + 2 = 2.9337

(iv) lg 8583 = lg (8.583×10^3) = lg 8.583 + lg 10^3 = 0.9337 + 3 = 3.9337

(The decimal part of these logarithms remains the same since what is obtained from the table in each instance is the value in the 85th row, the 8th column and the 3rd mean difference column).

In the logarithm 1.9337 of 85.83 in the above example, the decimal part of the logarithm is called the **mantissa** and the whole number 1 which is on the left of the decimal point is called the **characteristic** of the logarithm.

Observe the following table.

Number	Number of digits in the integral part	Scientific Notation	Logarithm	Characteristic of the logarithm
8.583	1	8.583×10^0	0.9337	0
85.83	2	8.583×10^1	1.9337	1
858.3	3	8.583×10^2	2.9337	2

According to the table, the characteristic of the logarithm of a number is the index of the power of ten, in the expression obtained when the number is expressed in scientific notation.

For numbers which are larger than 1, the characteristic of the logarithm is one less than the number of digits in the integral part of the number. Accordingly, for numbers such as 5.673 which have just one digit in the integral part, the characteristic is 0.

Example 3

Find the logarithm of each number using the logarithms table. Write each number in index form too.

(i) 69.34   (ii) 957.1   (iii) 1248

 (i)   lg $69.34 = 1.8409$, in index form $69.34 = 10^{1.8409}$

(ii)   lg $957.1 = 2.9809$, in index form $957.1 = 10^{2.9809}$

(iii) lg $1248 = 3.0962$, in index form $1248 = 10^{3.0962}$

Exercise 20.3

1. Find the logarithm of each of the following numbers using the logarithms table, and write the numbers in index form too.

 (i) 59.1   (ii) 100.2   (iii) 95.41   (iv) 1412   (v) 592.1   (vi) 890

2. If $10^{0.8939} = 7.832$, find each of the following values.

 (i) lg 7.832   (ii) lg 78.32   (iii) lg 7832

20.4 Antilogarithm

According to the logarithms table, lg 59.3 = 1.7731. That is, the logarithm of 59.3 is 1.7731 . Another way of saying this is that 1.7731 is the logarithm of 59.3. Then we say that the antilogarithm of 1.7731 is 59.3. We write this as antilog 1.7731 = 59.3.

Now let us consider how the antilogarithm is obtained when the mean difference section is also involved.

Example 1

Find antilog 0.8436 using the logarithms table.

	0	1	2	3	4	5	6	⑦	8	9	1	2	3	4	5	⑥	7	8	9
											Mean difference								
㉖⑨	---	---	---	---	---	---	---	▷8432								4			

antilog 0.8436 = 6.976
We can describe how the antilogarithm of 0.8436 was found using the above table as follows. Since this value is not in the table, we consider the number in the table which is less than this and closest to it. This value is 8432 which lies in the 69th row and 7th column. The difference 4 (= 8436 – 8432) is in the 6th column of the mean difference section. Accordingly, the required antilogarithm is 6.976 .
(Since the characteristic of 0.8436 is 0 there is only one digit in the integral part of the antilog)

When the characteristic of the logarithm is 0, the antilogarithm can be written directly from the table as in the above example, as a number between 1 and 10. However when the characteristic is greater than 0, the antilog can be found as shown in the following example.

Example 2

Find the value of antilog 1.8436
Antilog $1.8436 = 6.976 \times 10^1$ (6.971 from the mantissa and 1 in 10^1 from the characteristic)
　　　　　 = 69.76

Example 3

Find using the logarithms table.
　(i) Antilog 1.5432　　　(ii) Antilog 2.5432　　　(iii) Antilog 3.5432
　(i) Antilog $1.5432 = 3.493 \times 10^1$　(ii) Antilog $2.5432 = 3.493 \times 10^2$　(iii) Antilog $3.5432 = 3.493 \times 10^3$
　　　　　　 = 34.93　　　　　　　　　　 = 349.3　　　　　　　　　 = 3493

1. Find the following values using the logarithms table.
  (i) Antilog 0.7350   (ii) Antilog 2.4337   (iii) Antilog 3.5419
 (iv) Antilog 1.0072   (v) Antilog 2.9114   (vi) Antilog 3.8413

2. If lg x = 0.7845,
   (i) find the value of x.
   (ii) find the value of x by expressing antilog 1.7854 in scientific notation.
   (iii) find the value of antilog 2.7854.
   (iv) If lg $10y$ = 0.7845, find the value of y.

20.5   Multiplying and dividing numbers greater than 1 using the logarithms table

We know the two laws of logarithms lg (MN) = lg M + lg N and lg $\left(\dfrac{M}{N}\right)$ = lg M − lg N.

Now let us see how numbers can be multiplied and divided using the knowledge gained on logarithms up to this point, and the above laws.

Example 1

Evaluate using the logarithms table.
(i) 4.975 × 10.31   (ii) 53.21 ÷ 4.97

Let us take P = 4.975 × 10.31
Then, lg P = lg (4.975 × 10.31)
                = lg 4.975 + lg 10.31   (Logarithm laws)
                = 0.6968 + 1.0132   (From the logarithms table)
                = 1.7100
          ∴ P = Antilog 1.7100
                = 51.28
4.975 × 10.31 = <u>51.28</u>

This product can also be obtained using the index laws
4.975 × 10.31 = $10^{0.6968}$ × $10^{1.0132}$ (From the logarithms table)
                = $10^{1.7100}$ (Sum of the indices)
                = $10^{0.7100}$ × 10^{1}
                = 5.128 × 10^{1} (Antilog of 0.7100 using the tables)
                = <u>51.28</u>

(ii) $53.21 \div 4.97$

Let us take $P = 53.21 \div 4.97$

Then $\lg P = \lg(53.21 \div 4.97)$

$\qquad = \lg 53.21 - \lg 4.97$

$\qquad = 1.7260 - 0.6964$

$\qquad = 1.0296$

$\qquad \therefore P = $ antilog 1.0296

$\qquad = \underline{\underline{10.71}}$

> **Simplifying using indices**
>
> $53.21 \div 4.97 = 10^{1.7260} \div 10^{0.6964}$
>
> $\qquad = 10^{1.7260\ -\ 0.6964}$
>
> $\qquad = 10^{1.0296}$
>
> $\qquad = 1.071 \times 10^{1}$
>
> $\qquad = 10.71$

The simplification of an expression which involves both a product and a quotient is given in the following example.

Example 2

Using the logarithms table, find the value of $\dfrac{594.2 \times 9.275}{84.21}$

Let us take $P = \dfrac{594.2 \times 9.275}{84.21}$

$\therefore \lg P = \lg\left(\dfrac{594.2 \times 9.275}{84.21}\right)$

$\qquad = \lg(594.2 \times 9.275) - \lg 84.21$

$\qquad = \lg 594.2 + \lg 9.275 - \lg 84.21$

$\qquad = 2.7739 + 0.9673 - 1.9254$

$\qquad = 1.8158$

$\qquad \therefore P = $ Antilog 1.8158

$\qquad = 6.543 \times 10^{1}$

$\qquad = \underline{\underline{65.43}}$

Exercise 20.5

1. Find the value using the logarithms table.

(i) 54.3×1.75 (ii) 323.8×2.832 (iii) $54.1 \times 27.15 \times 43$

(iv) $523.2 \div 93.75$ (v) $43.17 \div 8.931$ (vi) $\dfrac{73.1 \times 25.41}{18.32}$

(vii) $\dfrac{85.72 \times 58.1}{29.73}$ (viii) $\dfrac{112.8 \times 73.45}{82.11}$ (ix) $\dfrac{953.1 \times 457}{23.25 \times 99.8}$

2. The circumference of a circle is given by the formula $c = 2\pi r$. If it is given that $\pi = 3.142$ and that $r = 10.5$ cm, find the value of c using the logarithms table.

3. The area of the curved surface of a cylinder is given by $A = 2\pi rh$. If it is given that $\pi = 3.142$, $r = 5.31$ cm and $h = 20$ cm, find the value of A using the logarithms table.

20.6 The calculator

The calculator is an outstanding invention of the 19th century which is used to perform calculations quickly and accurately.
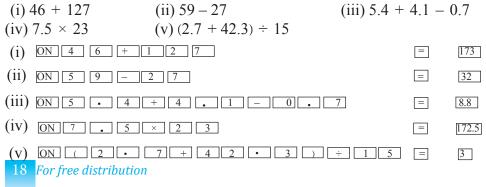
There are two types of calculators, namely the ordinary calculator and the scientific calculator. In an ordinary calculator, the calculations are done according to the order in which the operations are given. However, in a scientific calculator, the operations are performed according to mathematical principles (BODMAS). A calculator consists of a keyboard to perform operations and a screen to display the answers.

The following table provides information on the task performed by each of the given keys in a calculator.

Key	Result of operating the keys
ON	Switch on the calculator
OFF	Switch off the calculator
CE	Erase the last number or operation entered
AC	Clear the screen
+ − × ÷	Perform the relevant mathematical operation
3 4 5 6 8 7 9 2 1 0	Use the relevant digit
=	Show the result of the operations on the screen
.	Placing the period as required in a decimal number
(	Start including the required part within brackets
)	End the part within brackets

Example 1

Write down in order, the keys that should be operated to perform each of the following calculations using a scientific calculator. Write down the result that appears on the screen too.

(i) 46 + 127   (ii) 59 − 27   (iii) 5.4 + 4.1 − 0.7
(iv) 7.5 × 23   (v) (2.7 + 42.3) ÷ 15

(i) $\boxed{\text{ON}}$ $\boxed{4}$ $\boxed{6}$ $\boxed{+}$ $\boxed{1}$ $\boxed{2}$ $\boxed{7}$ $\boxed{=}$ $\boxed{173}$

(ii) $\boxed{\text{ON}}$ $\boxed{5}$ $\boxed{9}$ $\boxed{-}$ $\boxed{2}$ $\boxed{7}$ $\boxed{=}$ $\boxed{32}$

(iii) $\boxed{\text{ON}}$ $\boxed{5}$ $\boxed{.}$ $\boxed{4}$ $\boxed{+}$ $\boxed{4}$ $\boxed{.}$ $\boxed{1}$ $\boxed{-}$ $\boxed{0}$ $\boxed{.}$ $\boxed{7}$ $\boxed{=}$ $\boxed{8.8}$

(iv) $\boxed{\text{ON}}$ $\boxed{7}$ $\boxed{.}$ $\boxed{5}$ $\boxed{×}$ $\boxed{2}$ $\boxed{3}$ $\boxed{=}$ $\boxed{172.5}$

(v) $\boxed{\text{ON}}$ $\boxed{(}$ $\boxed{2}$ $\boxed{.}$ $\boxed{7}$ $\boxed{+}$ $\boxed{4}$ $\boxed{2}$ $\boxed{.}$ $\boxed{3}$ $\boxed{)}$ $\boxed{÷}$ $\boxed{1}$ $\boxed{5}$ $\boxed{=}$ $\boxed{3}$

1. Write down in order, the keys of the scientific calculator that need to be used to perform the following calculations. Write down the result that is obtained on the screen too.

(i) $543 + 275 + 17$ (ii) $2003 - 125$ (iii) $25.1 + 3.04 - 1.1$
(iv) 57.3×1.75 (v) $49.5 \div 15$ (vi) $(32.1 \times 4.3) + 1.5$

2. Simplify each of the following expressions using the logarithms table. Find the value of each expression using a scientific calculator too. Examine up to which decimal place the values obtained by the two methods are equal.

(i) 42.7×39.25 (ii) $514.1 \div 31.7$ (iii) $\dfrac{372.1 \times 4.3}{59.25}$
(iv) $\dfrac{753 \times 1.4}{101.5}$ (v) $(12.5 \times 62.4) \div 253.2$

Miscellaneous Exercise

1. Find the value of $\log_4 64 + \log_3 81 - \log_5 5 + 1$.

2. Find the value of each of the following expressions if $\lg 6.143 = 0.7884$
  (i) $10^{0.7884}$ (ii) $10^{1.7884}$ (iii) $10^{2.7884}$

3. Find the value of each of the following expressions if $10^{0.6582} = 4.552$
  (i) $\lg 4.552$ (ii) $\lg 45.52$ (iii) $\lg 455.2$

4. Find the value of each of the following expressions if antilog $1.6443 = 44.08$
  (i) Antilog 0.6443 (ii) Antilog 2.6443 (iii) Antilog 3.6443

5. (i) If $\lg a = x$ and $\lg b = 2x$, express $\lg (ab)$ in terms of x.

  (ii) If $\lg x = 0.9451$ and $\lg y = 0.8710$, find the value of $\lg \left(\dfrac{x}{y}\right)$

6. Simplify the following using the logarithms table. Examine the accuracy of your solution using a scientific calculator.

(i) $\dfrac{38.72 \times 1.003}{5.1}$ (ii) $\dfrac{5.432 \times 989.1}{379.1}$ (iii) $\dfrac{785.8}{27.2 \times 3.8}$

(iv) $\dfrac{75.23 \times 131.2}{5.74 \times 95.2}$ (v) $\dfrac{5.743 \times 83.21 \times 5.91}{12.75 \times 4.875}$ (vi) $\dfrac{573 \times 2.123 \times 6.1}{9.875 \times 54.21}$