18 Sets

By studying this lesson, you will be able to
- identify methods of describing sets
- identify the regions in a Venn diagram when at most two sets are represented in the Venn diagram and solve problems using the formula relating the number of elements in these sets.

Set Notation

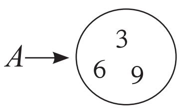
You have learnt three methods of describing sets before. They are
- the descriptive method
- the method of listing elements
- the Venn diagram method.

Let A be the set of all multiples of 3 between 1 and 10. Then, A can be denoted using the above three methods as follows:

• As a description,
A = {multiples of 3 between 1 and 10} or
A = set of all multiples of 3 between 1 and 10.

• Listing the elements
$A = \{3, 6, 9\}$

• In a Venn diagram,



18.1 Set Builder Method

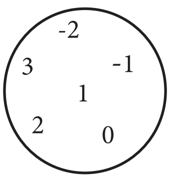
Set builder method is another method to describe sets. The set of all multiples of 3 between 1 and 10 can be denoted using the set builder method as follows:

$$A = \{x : x \text{ is a multiple of 3 and } 1 < x < 10\}.$$

The symbol x here is like a variable. You may use any symbol in place of x. The statement after the colon describes how the elements should be. There are several ways a set can be represented using the set builder method. For example, the following are three different ways of denoting the set $A = \{1, 2\}$ using the set builder method.

$A = \{ x : (x-1)(x-2) = 0 \}$
$A = \{ y : y \in \mathbb{Z} \text{ and } 1 \leq y \leq 2 \}$
$A = \{ n \in \mathbb{Z} \ : \ 0 < n \leq 2 \}$

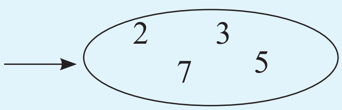
Look at the following table for more examples on the set builder method.

Set	Set builder form
A = {Positive integers less than 10}	$A = \{x : x \in \mathbb{Z}^+ \text{ and } 0 < x < 10\}$ or $A = \{x \in \mathbb{Z}^+ : 0 < x < 10\}$
$B = \{16, 25, 36, 49\}$	$B = \{ x : x \text{ is a perfect square. } 16 \leq x \leq 49\}$
C 	$C = \{ x : x \in \mathbb{Z}, -2 \leq x \leq 3 \}$ or $C = \{x \in \mathbb{Z} : -2 \leq x \leq 3 \}$

Exercise 18.1

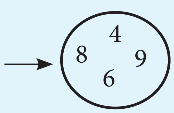
1. Describe the set of all positive integers from10 to 15 using
(i) the descriptive method
(ii) the listing method
(iii) a Venn diagram
(iv) the set builder method.

2. Describe each of the following sets using the descriptive method.
(i) A = {3, 6, 9, 12}

(ii) B


(iii) $C = \{ x : x \text{ is a perfect square and } 10 < x < 100 \}$

3. Describe each of the following sets using the descriptive method.
i. X = {All letters in the word "ANURADHAPURA"}
ii. $A = \{x : x \text{ is a prime number and } 10 < x < 20\}$

iii. B


4. Describe each of the following sets using a Venn diagram.

(i) $A = \{7, 14, 21, 28\}$

(ii) $B = \{\text{Vowels in the English alphabet}\}$

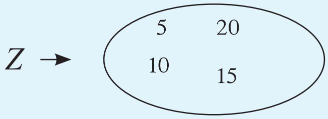
(iii) $Y = \{x \in \mathbb{Z} : x^2 = 4\}$

5. Describe each of the following sets using the set builder method.

(i) $X = \{\text{Odd numbers between 1 and 10}\}$

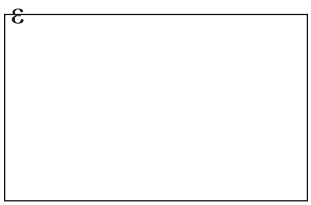
(ii) $Y = \{0, 1, 2, 3\}$

(iii)



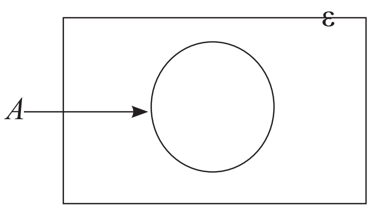
18. 2 Regions in a Venn Diagram

In Venn diagrams, the universal set is represented by a rectangular region and is denoted by ε.

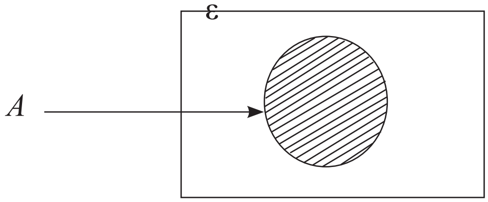


The subsets of the universal set are represented by circular (or elliptical) regions. These subsets give rise to several regions in the Venn diagram that represents the universal set. Now let us investigate these regions.

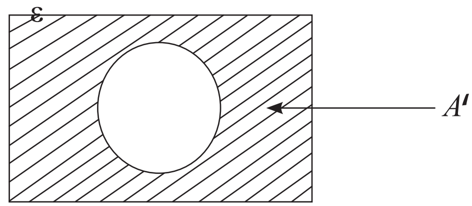
1. When there is one subset represented in the universal set.



The subset A divides the universal set into two regions which are shaded in the next figure.The subsets corresponding to these regions can be denoted by A and A' using set notation.



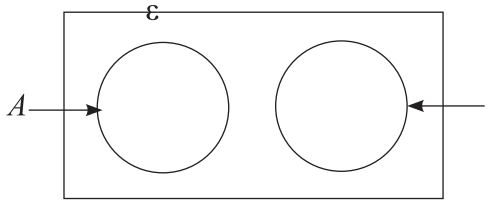
A



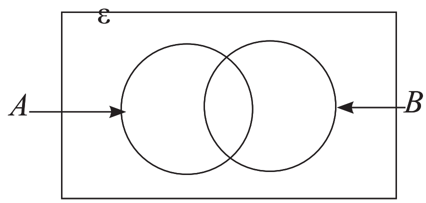
Complement of $A = A'$

2. When two subsets are represented in the Venn diagram

Let A and B be the two subsets. When there are no common elements in A and B, that is, when $A \cap B = \emptyset$, and when there are common elements in A and B, that is when $A \cap B \neq \emptyset$, the relevant Venn diagrams are shown below.



$A \cap B = \emptyset$



$A \cap B \neq \emptyset$

Before investigating the regions let us recall the following definitions.

A' = Set of elements not in A

$A \cap B$ = Set of elements belonging to both A and B

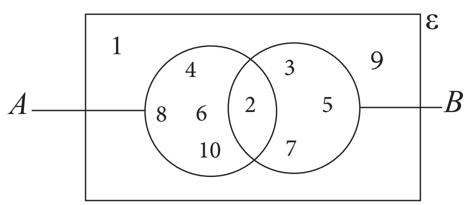
$A \cup B$ = Set of elements belonging to A or B (or both)

As an example, let us take $\varepsilon = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{2, 4, 6, 8, 10\}$ and

$B = \{2, 3, 5, 7\}$

Then, we can represent the above information in a Venn diagram as given below.



According to the given information, it is clear that

$A' = \{1, 3, 5, 7, 9\}$

$A \cap B = \{2\}$ and

$A \cup B = \{2, 3, 4, 5, 6, 7, 8, 10\}$

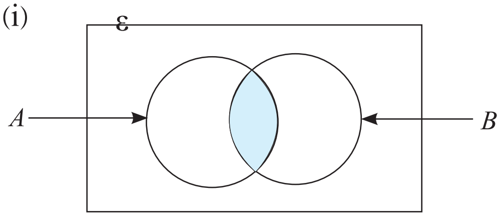
Also, when we observe the Venn diagram we can see that

$(A \cup B)' = \{1, 9\}$ and

$(A \cap B)' = \{1, 3, 4, 5, 6, 7, 8, 9, 10\}$

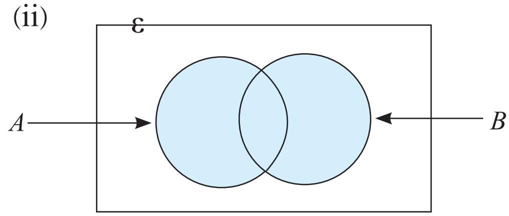
Two subsets of a universal set represented in a Venn diagram give rise to several regions. Some such regions and the way they can be written down using set complement, set intersection and set union are shown below.

(i)



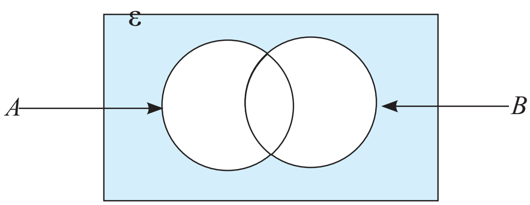
$A \cap B$

(ii)



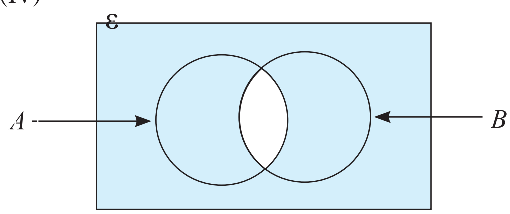
$A \cup B$

(iii)



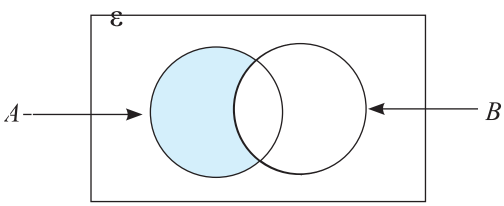
$(A \cup B)'$

(iv)



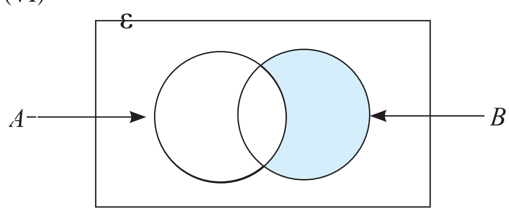
$(A \cap B)'$

(v)



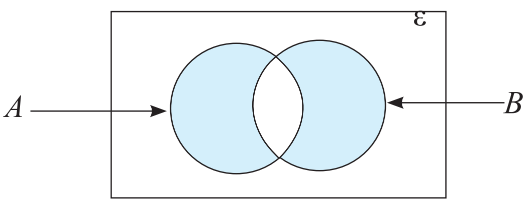
$A \cap B'$

(vi)



$A' \cap B$

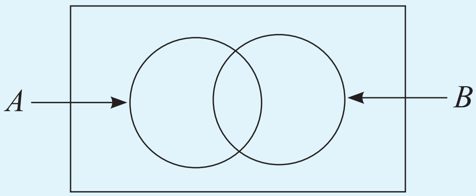
For the example we discussed above, we have that
$A \cap B' = \{4, 6, 8, 10\}$ and $A' \cap B = \{3, 5, 7\}$
Also, the Venn diagram given below is obtained by the Venn diagrams (v) and (vi)



$(A \cap B') \cup (A' \cap B)$

Exercise 18.2

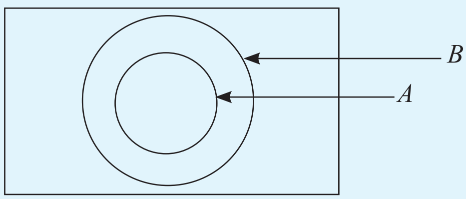
1. Shade the region denoted by each of the following sets in separate Venn diagrams.



(i) $A'\cap B'$ (ii) $A' \cup B'$
(iii) $(A\cap B)'$ (iv) $(A\cup B)'$
(v) $(A\cap \mathrm{B}) \cup (A\cup B)'$ (vi) $(A\cap B')'$
(vii) $(A'\cap B)'$ (viii) $(A \cup B')'$
(ix) $(A'\cup B)'$

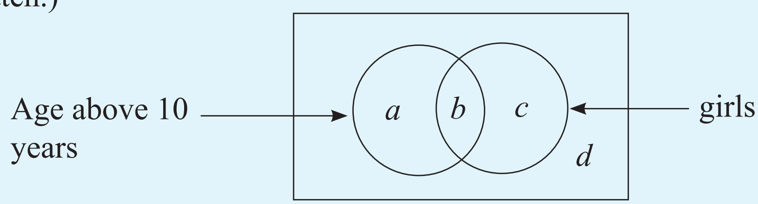
b. By investigating the regions you shades in part (a) above, find all pairs of equal sets.

2. Shown below is the Venn diagram of two sets A and B where $A\subset B$. In 6 copies of this Venn diagram, shade each of the given 6 regions.



(i) $A\cap B$ (ii) $A\cup B$
(iii) $A'\cap B$ (iv) $A'\cup B$
(v) $(A\cup B)'$ (vi) $(A'\cup B)'$

3. The information on the children in a society is shown in the following Venn diagram. (The letters *a, b, c* and *d* indicate the regions in which the letters are written.)



Describe in words the regions indicated by each of the letters *a, b, c* and *d*. For example, the boys whose ages are above 10 years are indicated by *a*.

4. Let $\varepsilon = \{1, 2, 3, 4, 5, 6, 7\}$

$A'\cap B = \{4, 5\}$

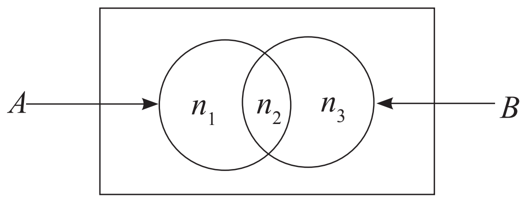
$A\cap B = \{3\}$

$(A\cup B)' = \{1\}$

Include the above information in a suitable Venn diagram and hence find A, $A\cup B$ and $B'\cap A$.

18.3 Relationship between the numbers of elements in two sets

Shown below is a Venn diagram with two subsets A and B of the universal set such that $A \cap B \neq \emptyset$. Here n_1, n_2 and n_3 denote the number of elements in the respective regions. (Though it is expected to write the elements inside the regions, we have written down the number of elements for ease.)



Let us the denote the number of elements belonging to the subset A by n(A) etc. Then, from the figure, we see that

$$n(A) = n_1 + n_2$$
$$n(B) = n_2 + n_3$$
$$n(A \cap B) = n_2$$
$$n(A \cup B) = n_1 + n_2 + n_3$$

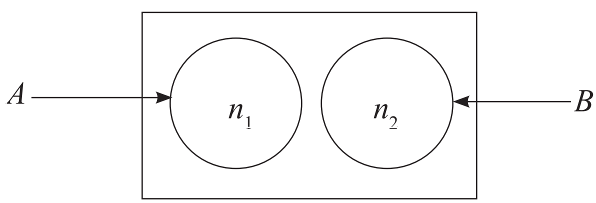
Now, we write, $n(A \cup B) = \underbrace{n_1 + n_2} + \underbrace{n_2 + n_3} - n_2$

$$= n(A) + n(B) - n(A \cap B)$$

Thus, we obtain the formula

$$\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$$

The Venn diagram when the two subsets A and B are disjoint (i.e.,when $A \cap B = \emptyset$) is shown below.



In this case, we see that

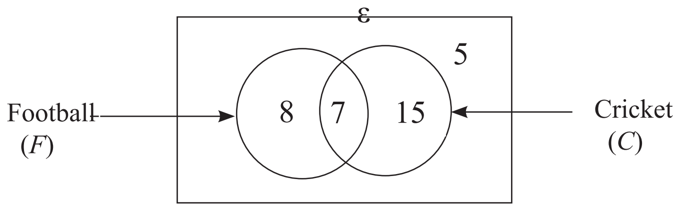
$$n(A) = n_1$$
$$n(B) = n_2$$
$$n(A \cup B) = n_1 + n_2$$

Thus, when $A \cap B = \emptyset$ we obtain the formula

$$\boxed{n\ (A \cup B) = n\ (A) + n\ (B)}$$

Example 1

Information on the numbers of students who participate in football and cricket are shown in the following diagram (In this case too, the numbers in the regions indicate the numbers of students in the regions).

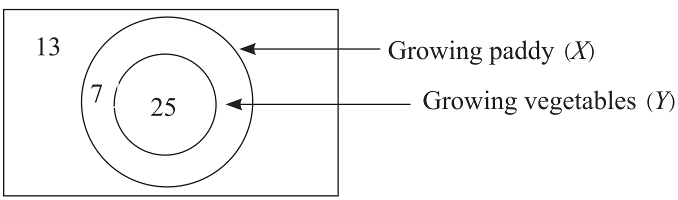


How many students are

1. participating in football? $n\ (F) = 8 + 7 = 15$
2. participating in cricket? $n\ (C) = 7 + 15 = 22$
3. participating in both sports (both football and cricket)? $n\ (F \cap C) = 7$
4. participating in cricket only? $n\ (C \cap F') = 15$
5. participating in football only? $n\ (F \cap C') = 8$
6. participating in football or cricket (or both)? $n\ (F \cup C) = 8 + 7 + 15 = 30$
7. not participating in football? $n\ (F') = 15 + 5 = 20$
8. not participating in cricket? $n\ (C') = 8 + 5 = 13$
9. participating in exactly in one of the two sports? $n\ \{(F \cap C') \cup (F' \cap C)\}$
$= 8 + 15 = 23$
10. Participating in neither of the two sports? $n\ (F \cup C)' = 5$

Example 2

The information obtained through a survey from a set of farmers in a certain village about the kind of crop they grow in their farms is shown in the following Venn diagram.



(a) How many farmers are growing

1. vegetables? $n\ (Y) = 25$
2. paddy? $n\ (X) = 7 + 25 = 32$
3. only paddy? $n\ (Y' \cap X) = 7$
4. only vegetables? $n(X' \cap Y) = 0$
5. both vegetables and paddy? $n\ (X \cap Y) = 25$
6. vegetables or paddy? $n\ (X \cup Y) = 7 + 25 = 32$
7. neither of the two crops? $n\ (X \cup Y)' = 13$

(b) How many farmers were surveyed?

$$n\ (\varepsilon) = 13 + 7 + 25 = 45$$

Exercise 18.3

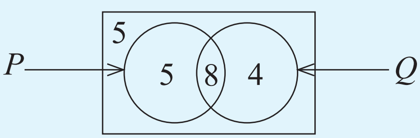
1. Find $n(A \cup B)$ if $n\ (A) = 35$, $n\ (B) = 24$, $n(A \cap B) = 11$.

2. Find $n(Y)$ if $n\ (X) = 16$, $n\ (X \cap Y) = 5$, $n\ (X \cup Y) = 29$.

3. Find $n(P \cap Q)$ if $n\ (P) = 70$, $n\ (Q) = 55$, $n(\ P \cup Q) = 110$.

4. Find $n(A \cap B)$ if $n(A) = 19$, $n(B) = 16$, $n\ (A \cup B) = 35$. What is special about the sets A and B?

5. The numbers belonging to each region is indicated in the following Venn diagram.



Find $n(P)$, $n(Q)$, $n(P \cap Q)$, $n(P \cup Q)$ and verify the formula

$$n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$$

6. In a sports club of 60 members, 30 play cricket, 25 play elle and 15 play both.
(i) Include this information in a Venn diagram.
(ii) How many members do not play either of the above two sports?
(iii) How many members play elle, but not cricket?

7. Out of 30 who attended a party, 12 ate Kavum, 20 ate kokis and 5 did not eat either of these. Represent this information in a Venn diagram.
(i) Find the number that ate both these food items.
(ii) How many of them ate only one of these two food items?

8. Out of 40 students in a class, 21 do not like to listen to the radio, 10 do not like to watch TV, and 8 do not like either of the two activities.
(i) Represent this information in a Venn diagram.
(ii) How many of the students like both activities?
(iii) How many of the students like only to watch TV?

9. From a group of 35 children who participated in a game, 19 were boys and 17 were above 15 years. 6 of the girls who participated were aged below 15 years.
   (i) Represent this information in a Venn diagram.
   (ii) How many boys were above 15 years?

10. From a group of 80 who went on an trip, 50% were wearing hats but were not wearing wrist watches. 40% of the group were wearing wrist watches, out of which 30 were also wearing hats.
   (i) Represent this information in a Venn diagram.
   (ii) How many of the group were not wearing either of the above mentioned two items?

11. In a certain village, 36 farmers grow potatoes and 18 farmers grow only chillies. Furthermore, the number of those who do not grow potatoes is 24 and the number of those who do not grow chillies is 26.
   (i) Represent this information in a Venn diagram.
   (ii) How many of the farmers grow neither of the two crops?
   (iii) How many of the farmers grow both crops?

12. A survey conducted in a certain village on 80 randomly selected households revealed the following information.
   * 5 households had neither water supply nor electricity.
   * 30 households had no electricity.

The number of households having only water supply was 7 more than the number that had both water supply and electricity.
   (i) Represent this information in a Venn diagram.
   (ii) How many households had both water supply and electricity?
   (iii) How many households had electricity but not water supply?
   (iv) How many households had no water supply?
   (v) How many households has exactly one of these facilities?