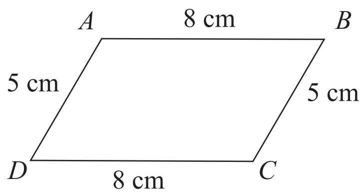
17 Parallelograms II

By studying this lesson you will be able to

identify the conditions that need to be satisfied for a quadrilateral to be a parallelogram.

Theorem: If the opposite sides of a quadrilateral are equal, then it is a parallelogram.

For example, in the given figure, $AB = DC$ and $AD = BC$.
Therefore, $ABCD$ is a parallelogram.

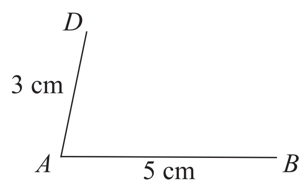


Let us engage in the following activity to establish the truth of the above theorem.

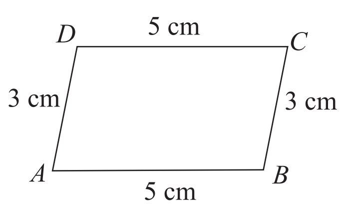
Activity 1

- Draw $D\hat{A}B$ as shown in the figure, such that the sides of the angle are of length 5 cm and 3 cm.

- As shown in the second figure, obtain the point C which is 3 cm from B and 5 cm from D.
- Now complete the quadrilateral $ABCD$.
- Then it can be seen that $AB = DC$ and $AD = BC$.



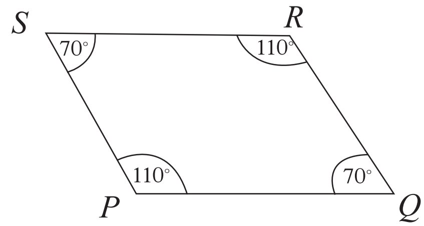
- By using a set square and a ruler or by measuring the angles and obtaining that the sum of a pair of allied angles is 180^{o}, observe that the opposite sides of the quadrilateral are parallel. That is, obtain that $AB//DC$ and $AD//BC$.



It can be observed that in a quadrilateral with opposite side equal, the opposite sides are parallel too.

Theorem: If the opposite angles of a quadrilateral are equal, then it is a parallelogram.

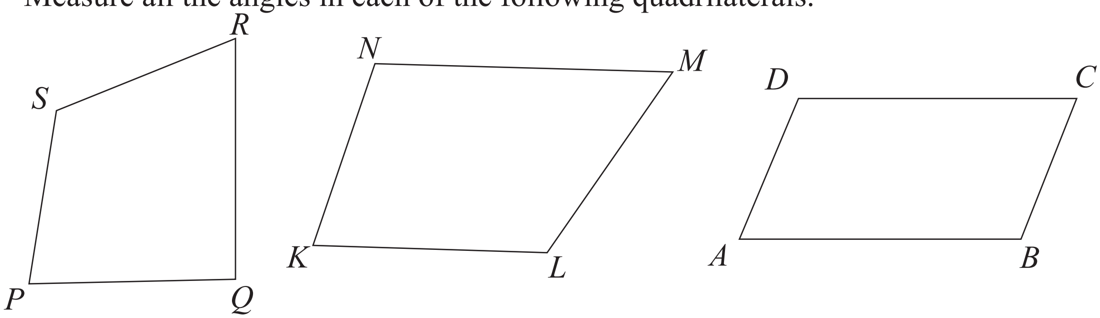
For example, in the given figure, since $P\hat{Q}R = P\hat{S}R$ and $Q\hat{R}S = Q\hat{P}S$, by the theorem, *PQRS* is a parallelogram.



Let us engage in the following activity to establish the truth of the above theorem.

Activity 2

- Measure all the angles in each of the following quadrilaterals.

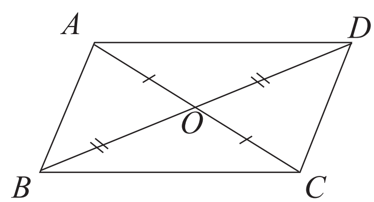


- Check whether the opposite pairs of angles of each quadrilateral are equal.
- Check whether the pairs of opposite sides are parallel in the quadrilateral with opposite angles equal. (See whether the sum of the allied angles is 180°.)

Accordingly, observe that in the quadrilateral with opposite angles equal, the opposite sides are parallel to each other.

Theorem: If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.

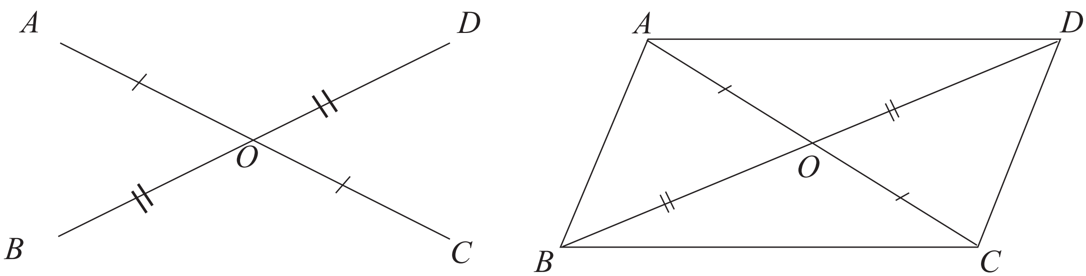
For example, in the quadrilateral *ABCD*, since $AO = OC$ and $BO = OD$, according to the theorem, *ABCD* is a parallelogram.



Engage in the following activity to establish the truth of the above theorem.

Activity 3

- To draw the quadrilateral *ABCD* with diagonals *AC* and *BD*, first draw the diagonal *AC* and name its midpoint *O*.
- Now draw another straight line segment such that it intersects the diagonal *AC* at *O*. Mark the points *B* and *D* on this line segment such that *OB* = *OD*.

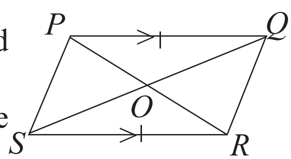


- Now complete the quadrilateral *ABCD* as shown above.
- Check whether the sides *AB* and *DC* as well as the sides *BC* and *AD* in the quadrilateral *ABCD* are parallel to each other by using a set square and a ruler or by measuring a pair of alternate angles.

Accordingly, it can be seen that if the diagonals of a quadrilateral bisect each other, then the opposite sides are parallel to each other.

Theorem: In a quadrilateral, if a pair of opposite sides is equal and parallel, then the quadrilateral is a parallelogram.

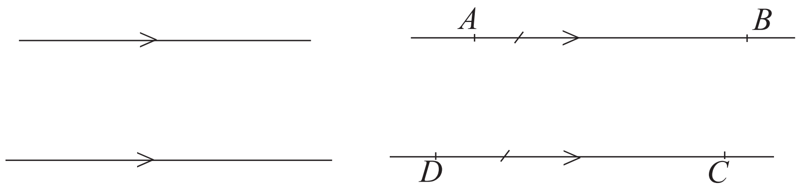
For example, in the quadrilateral *PQRS,* since *PQ* = *SR* and *PQ*//*SR,* it is a parallelogram.



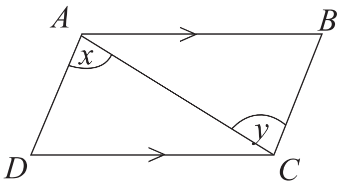
Engage in the following activity to establish the truth of the above theorem.

Activity 4

- Draw a pair of parallel lines using a set square and a ruler or by some other method.
- Mark two points *A* and *B* on one of these lines.
- Mark a length equal to *AB* on the other line as shown in the figure, and name it *DC*.



• Now complete the quadrilateral *ABCD* and draw the diagonal *AC* as shown in the figure.



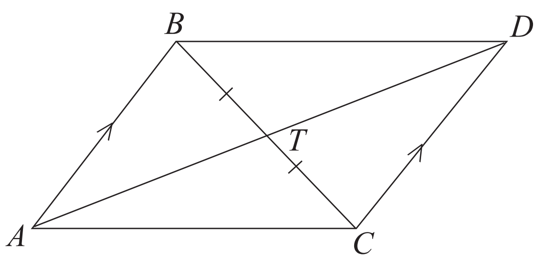
By measuring the pair of alternate angles *x* and *y* using a protractor, or by using a set square and ruler, observe that *AD* and *BC* are parallel to each other.

Accordingly, it can be seen that in a quadrilateral, if a pair of opposite sides is equal and parallel, then the quadrilateral is a parallelogram.

Now, by considering the following example, let us see how riders are proved using the above theorems.

Example 1

T is the midpoint of the side *BC* of the triangle *ABC*. The straight line drawn through *C*, parallel to *AB* meets *AT* produced at *D*. Prove that *ABDC* is a parallelogram.
First, let us draw the figure according to the given information.



We know that in a quadrilateral, if a pair of opposite sides is equal and parallel, then it is a parallelogram. Therefore, let us show that *ABDC* is a parallelogram by showing that a pair of opposite sides is equal and parallel. It is given that $AB//CD$. Let us show that $AB = CD$.

To obtain this, let us show that the triangles *ABT* and *CTD* are congruent.

In the triangles ABT and CTD

$BT = TC$ (Given)

$A\hat{T}B = C\hat{T}D$ (Vertically opposite angle)

$A\hat{B}T = T\hat{C}D$ $(AB//CD$, Alternate angles$)$

$\therefore \Delta\, ABT \equiv \Delta\, CTD$ (A.A.S.)

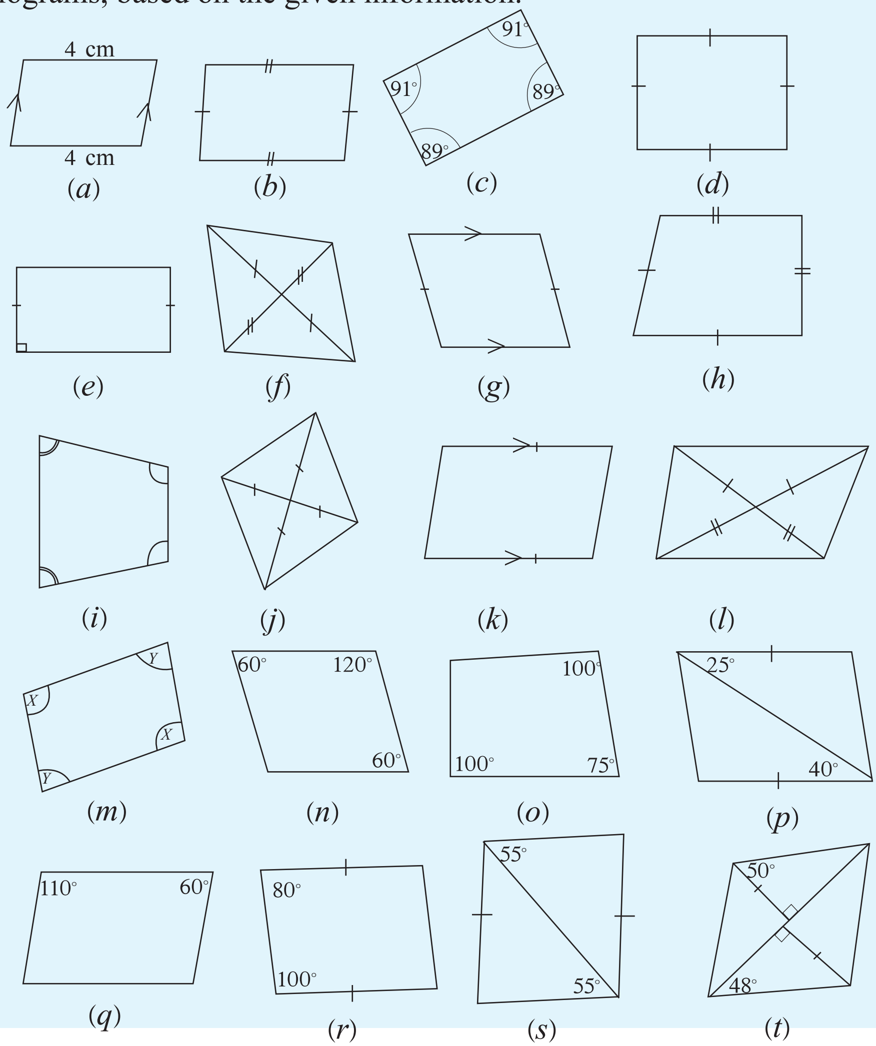
Since corresponding elements of congruent triangles are equal,

$AB = CD$.

Since $AB = CD$ and $AB\,//\,CD$, we obtain that $ABDC$ is a parallelogram.

Exercise 17.1

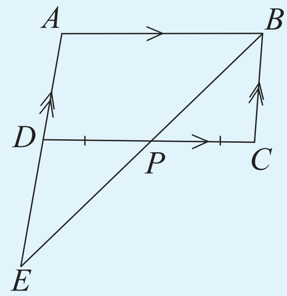
1. From the following quadrilaterals, select the ones which can be concluded to be parallelograms, based on the given information.



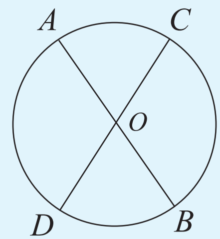
2. The midpoint of the side DC of the parallelogram $ABCD$ in the figure is P. AD and BP produced meet at E.

(i) Prove that $\Delta BCP \equiv \Delta DPE$

(ii) Prove that the quadrilateral $BCED$ is a parallelogram.



3. AB and CD are two diameters of the circle of centre O in the given figure. Prove that A, C, B and D are the vertices of a parallelogram.



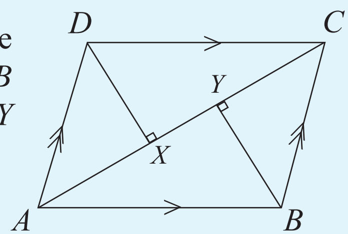
4. In the parallelogram $ABCD$ in the figure, the perpendiculars drawn from the points D and B to the diagonal AC meet AC at the points X and Y respectively.

Prove that,

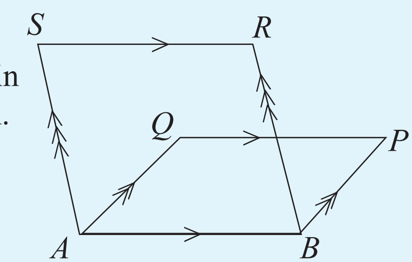
(i) $\Delta AXD \equiv \Delta BYC$,

(ii) $DX = BY$,

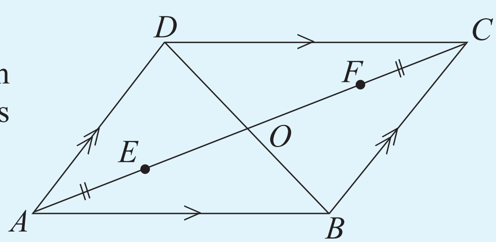
(iii) $BYDX$ is a parallelogram.



5. Two parallelograms $ABPQ$ and $ABRS$ are given in the figure. Prove that $QPRS$ is a parallelogram.



6. The figure illustrates a parallelogram $ABCD$. If $AE = FC$, prove that $EBFD$ is a parallelogram.

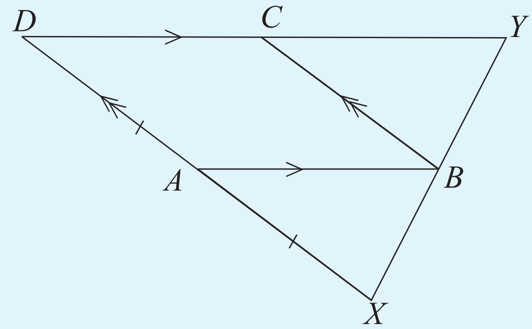


7. In the given figure, $ABCD$ is a parallelogram. The side DA has been produced up to X such that $DA = AX$. Also, DC produced and XB produced meet at Y. Prove that,

(i) $AXBC$ is a parallelogram,

(ii) $ABYC$ is a parallelogram,

(iii) $DC = CY$.



8. The diagonals of the parallelogram $PQRS$ intersect each other at O. The points M and T lie on PO and OR respectively and the points L and N lie on QO and OS respectively such that $PM = RT$ and $SN = QL$.

Prove that,

(i) $MO = OT$,

(ii) $LMNT$ is a parallelogram,

(iii) $MSTQ$ is a parallelogram.

Parallelograms with special properties

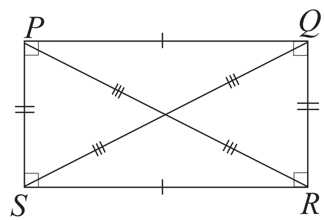
1. Rectangle

If one of the angles of a parallelogram is a right angle, then the other angles too are right angles. Such a parallelogram is a rectangle.

Apart from the properties of a parallelogram, a rectangle also has the following properties.

(i) All the vertex angles are right angles.

(ii) The diagonals are equal in length.



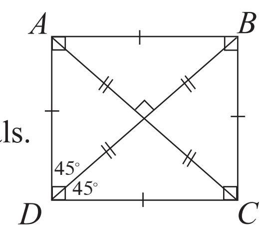
2. Square

A rectangle with two equal adjacent sides is a square. Apart from the properties of a rectangle, a square also has the following properties.

(i) All the sides are equal in length.

(ii) The diagonals bisect each other at right angles.

(iii) The angles at the vertices are bisected by the diagonals.



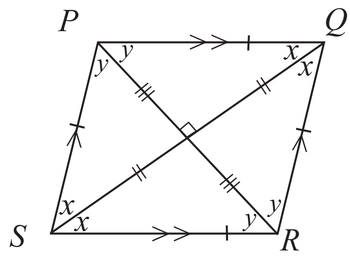
3. Rhombus

When two adjacent sides of a parallelogram are equal in length, then all four sides are equal in length. Such a parallelogram is called a rhombus. Apart from the properties of a parallelogram, a rhombus also has the following properties.

(i) All the sides are equal to each other.

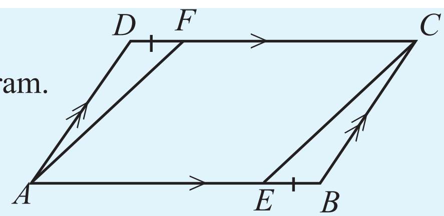
(ii) The diagonals bisect each other at right angles.

(iii) The angles at the vertices are bisected by the diagonals.



Miscellaneous Exercise

1. *ABCD* in the figure is a parallelogram.
If $DF = EB$, prove that *AECF* is a parallelogram.



2. In the triangle *ABC*, the bisector of $A\hat{B}C$ meets the side *AC* at the point *P*. The straight line through *A* drawn parallel to *BC*, meets *BP* produced at *D* such that $BP = PD$.

(i) Prove that $\Delta BCP \equiv \Delta ADP$

(ii) Show that *ABCD* is a rhombus.

(iii) If $AC = 18$ cm and $BD = 24$ cm, find the length of *AB*.

3. In the triangle *ABC*, the midpoints of the sides *AB* and *AC* are respectively *X* and *Y*. The straight line drawn through *C*, parallel to *AB*, meets *XY* produced at *Z*. Prove that,

(i) $\Delta AXY \equiv \Delta CYZ$

(ii) *BCZX* is a parallelogram.

4. In the parallelogram *ABCD*, the midpoints of the sides *AB, BC, CD* and *AD* are *P, Q, R* and *S* respectively. Prove that ,

(i) $\Delta ASP \equiv \Delta CQR$,

(ii) *PQRS* is a parallelogram.