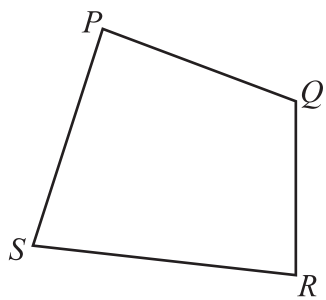
# 16 Parallelograms I

**By studying this lesson you will be able to**

solve problems and prove riders using the properties of parallelograms.

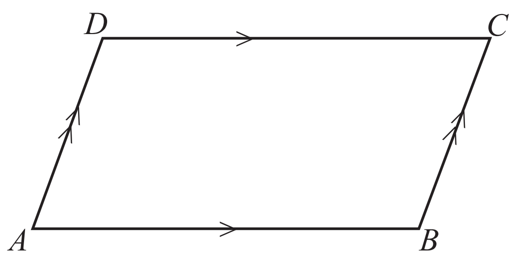
## Parallelograms

A closed plane figure bounded by four straight line segments is called a quadrilateral. Let us consider the opposite sides and the opposite angles of a quadrilateral.




In the quadrilateral $PQRS$, $PQ$ and $SR$ are a pair of opposite sides, and $PS$ and $QR$ are the other pair of opposite sides. While $S\hat{P}Q$ and $S\hat{R}Q$ form a pair of opposite angles, the other pair of opposite angles is $P\hat{Q}R$ and $P\hat{S}R$.

A quadrilateral with both pairs of opposite sides parallel is defined as a **parallelogram.**



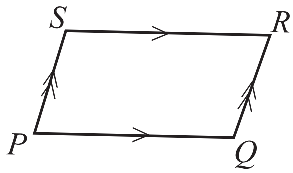

A pair of arrowheads have been used to indicate that the sides $AB$ and $DC$ of the above parallelogram are parallel to each other. Also, two arrowheads each have been used to indicate that the sides $BC$ and $AD$ are parallel.

## 16.1 Properties of Parallelograms

Do the following activities first to identify the properties of parallelograms.

**Activity 1**

Draw a parallelogram using a set square and a ruler. Name it $PQRS$ as indicated in the figure.




**1.** In the parallelogram $PQRS$ that you drew,

- measure the lengths of the sides $PQ$, $QR$, $SR$ and $PS$.
- What can you say about the lengths of the pair of opposite sides $PQ$ and $SR$, as well as the lengths of $PS$ and $QR$?

It should be clear to you that $PQ = SR$ and $PS = QR$.

**2.** In the parallelogram that you drew above,

- measure the magnitudes of $P\hat{Q}R$, $Q\hat{P}S$, $P\hat{S}R$ and $Q\hat{R}S$.
- What can you say about the magnitudes of the pair of opposite angles $Q\hat{P}S$ and $Q\hat{R}S$, as well as the magnitudes of the pair $P\hat{S}R$ and $P\hat{Q}R$?

It should be clear to you that $Q\hat{P}S = Q\hat{R}S$ and that $P\hat{S}R = P\hat{Q}R$.

**3.** Now copy the parallelogram $PQRS$ onto a piece of tissue paper, and using it draw two copies of $PQRS$ and cut them out.

- In one parallelogram, draw the diagonal $PR$.
- Now cut this parallelogram along the diagonal and see whether the two triangles that you obtain coincide.

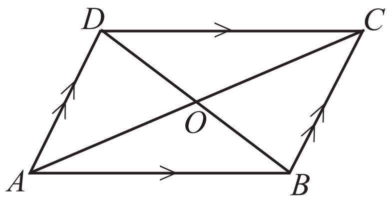
It should be clear to you that the two triangles coincide. Therefore, the areas of the two triangles are equal. Similarly, use the other parallelogram you cut out to verify that the areas of the two triangles obtained by cutting the parallelogram along the other diagonal are also equal.

According to the above activity,

> The opposite sides of a parallelogram are equal, the opposite angles of a parallelogram are equal, and the area of the parallelogram is bisected by each diagonal.

**Activity 2**

As in activity 1, draw a parallelogram using a set square and a ruler. Name it $ABCD$ as in the following figure.

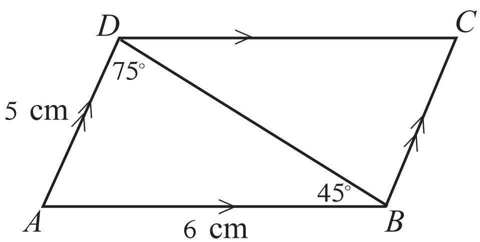



- Now draw the diagonals $AC$ and $BD$. Name their intersection point as $O$.
- Measure the lengths of $OA, OB, OC$ and $OD$.
- What can you say about the lengths of $OA$ and $OC$?
- What can you say about the lengths of $OB$ and $OD$?
- It should be clear to you that $OA = OC$ and that $OB = OD$.

Accordingly, it is clear that the diagonals of a parallelogram bisect each other.

Now let us consider how various elements of a parallelogram are found based on the data that is given.

Find the lengths of the sides and the magnitudes of the angles given below based on the data given in the parallelogram $ABCD$.




(i) Length of $BC$
(ii) Length of $DC$
(iii) Magnitude of $B\hat{A}D$
(iv) Magnitude of $B\hat{C}D$
(v) Magnitude of $A\hat{B}C$
(vi) Magnitude of $A\hat{D}C$

(i) Since the opposite sides of a parallelogram are equal, $AD = BC$ and $AB = CD$.

$\therefore BC = 5$ cm

(ii) $DC = 6$ cm

(iii) Since the sum of the interior angles of a triangle is $180°$,

$$B\hat{A}D = 180° - 75° - 45°$$
$$= \underline{\underline{60°}}$$

(iv) Since the opposite angles of a parallelogram are equal,

$$B\hat{C}D = B\hat{A}D$$
$$\underline{\therefore B\hat{C}D = 60°}$$

(v) $A\hat{D}B = C\hat{B}D$ ($AD \,//\, BC$, Alternate angles)

$\therefore C\hat{B}D = 75°$

$A\hat{B}C = A\hat{B}D + C\hat{B}D$

$\therefore A\hat{B}C = 45° + 75°$

$\underline{\underline{= 120°}}$

(vi) Since the opposite angles of a parallelogram are equal to each other,

$A\hat{B}C = A\hat{D}C$

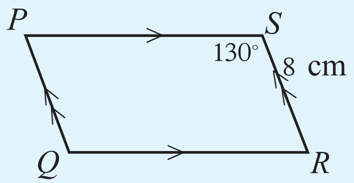
$\therefore \underline{\underline{A\hat{D}C = 120°}}$

**Exercise 16.1**

**1.** According to the information in the parallelogram $PQRS$,

(i) find the length of the side $PQ$.

(ii) find the magnitude of each of the angles $Q\hat{P}S$, $P\hat{Q}R$ and $Q\hat{R}S$.

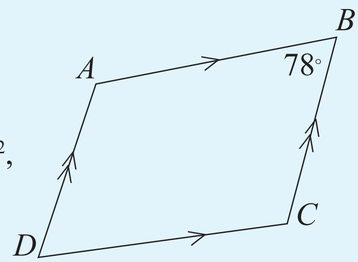



**2.** Based on the data in the figure,

(i) find the magnitude of $B\hat{C}D$.

(ii) If the area of the parallelogram $ABCD$ is 24 cm$^2$, what is the area of the triangle $BCD$?

(iii) What is the area of the triangle $ACD$?



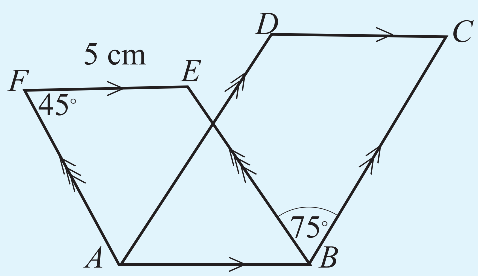

**3.** Based on the information in the figure,

(i) find the length of $DC$.

(ii) find the magnitude of $A\hat{B}E$.

(iii) find the magnitude of $A\hat{D}C$.

(iv) find the magnitude of $B\hat{C}D$.

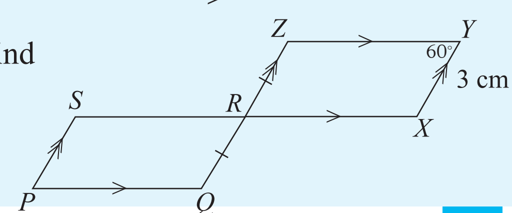



**4.** Based on the information in the figure, find

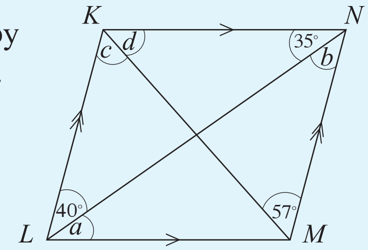
(i) the length of $PS$.

(ii) the magnitude of $Q\hat{P}S$.

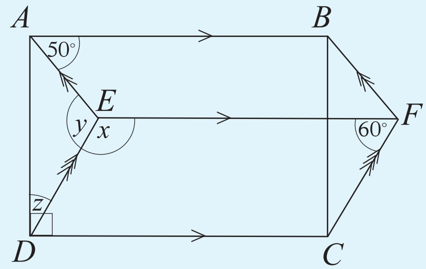
(iii) the magnitude of $P\hat{Q}R$.



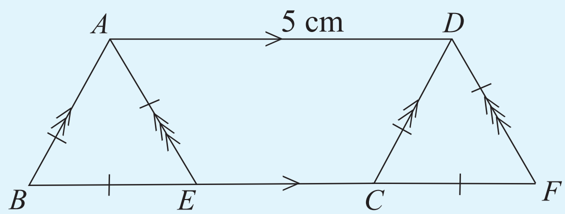
**5.** Find the magnitude of each of the angles denoted by *a, b, c* and *d*, based on the information in the figure.




**6.** Based on the information in the figure,
(i) write down two sides which are equal in length to *DC*.
(ii) find the magnitudes of the angles denoted by *x, y* and *z*.




**7.** The figure depicts two parallelograms *ABCD* and *ADFE*. According to the information given in the figure,
(i) find the length of *BC*.
(ii) find the magnitude of each of the angles, $C\hat{F}D, A\hat{D}C$ and $E\hat{C}D$.

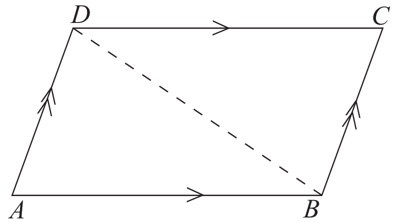



## 16.2 Properties of a parallelogram

Since the properties that we observed above in parallelograms are common to all parallelograms, we can present them in the form of a theorem as follows.

**Theorem: In a parallelogram,**
(i) opposite sides are equal.
(ii) opposite angles are equal.
(iii) the area of the parallelogram is bisected by each diagonal.
(iv) the diagonals bisect each other.

Now let us consider how the first three parts of this theorem are proved formally.




Data: $ABCD$ is a parallelogram.

To be proved: (i) $AB = DC$ and $AD = BC$

(ii) $B\hat{A}D = B\hat{C}D$ and $A\hat{D}C = A\hat{B}C$

(iii) Area of $\Delta\, ABD$ =Area of $\Delta\, BCD$

Area of $\Delta\, ACD$ = Area of $\Delta\, ABC$

Construction: Join $BD$

We can obtain the three results by showing that the triangles $ABD$ and $BCD$ are congruent. Let us prove that the two triangles are congruent under the case AAS as follows.

Proof: In the triangles $ABD$ and $BCD$,

$A\hat{D}B = C\hat{B}D$ (Alternate angles, $AD\,//\,BC$)

$A\hat{B}D = B\hat{D}C$ (Alternate angles, $AB\,//\,DC$)

$BD$ is the common side.

$\therefore \Delta\, ABD \equiv \Delta\, BCD$ (AAS)

Since the corresponding elements of congruent triangles are equal,

$AB = DC$ and $AD = BC$.

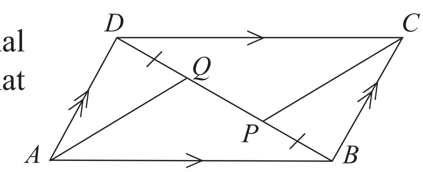
Also $B\hat{A}D = B\hat{C}D$.

Area of $\Delta\, ABD$ = Area of $\Delta\, BCD$ (Since $\Delta\, ABD \equiv \Delta\, BCD$)

$\therefore$ The area of the parallelogram $ABCD$ is bisected by the diagonal $BD$.

The above facts can also be proved by using the diagonal $AC$.

**Example 1**




The points $P$ and $Q$ are marked on the diagonal $BD$ of the parallelogram $ABCD$ such that $BP = DQ$. Prove that,

(i) $\Delta\, ADQ \equiv \Delta\, BPC$

(ii) $AQ//PC$

Proof: (i) In the triangles *ADQ* and *PBC,*

$DQ = BP$ (Given)

$AD = BC$ (The opposite sides of a parallelogram are equal)

$A\hat{D}Q = P\hat{B}C$ (Alternate angles, $AD // BC$)

$\therefore \underline{\underline{\Delta ADQ \equiv \Delta BPC}}$ (SAS)

(i) Since the triangles *ADQ* and *PBC* are congruent, the corresponding elements of the two triangles are equal.

$\therefore A\hat{Q}D = B\hat{P}C.$

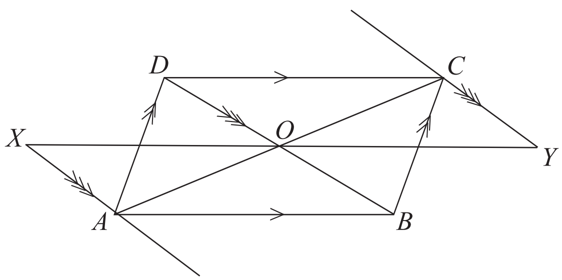
$\therefore A\hat{Q}P = Q\hat{P}C. \quad \left(A\hat{Q}D + A\hat{Q}P = B\hat{P}C + C\hat{P}Q = 180^0\right)$

But $A\hat{Q}P$ and $Q\hat{P}C$ are alternate angles.

Since a pair of alternate angles are equal, $AQ // PC.$

**Example 2**

According to the information given in the diagram, prove that *O* is the mid point of *XY*.




We need to prove that $XO = OY$. To do this, let us first show that the triangles *AOX* and *COY* are congruent.

Proof:

In the triangles *AOX* and *COY*,

$A\hat{X}O = C\hat{Y}O$ ($AX//CY$, Alternate angles)

$A\hat{O}X = C\hat{O}Y$ (Vertically opposite angles)

$AO = OC$ (Diagonals of a parallelogram bisect each other)

$\therefore \Delta AOX \equiv \Delta COY$ (A A S)

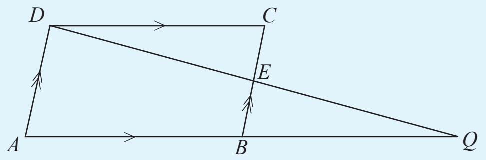
Corresponding elements of congruent triangles are equal.

$\therefore OX = OY$

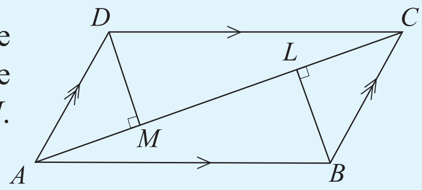
i.e., *O* is the mid point of *XY*.

## Exercise 16.2

**1.** In the parallelogram $ABCD$, the midpoint of $BC$ is $E$. $DE$ and $AB$ produced meet at $Q$. Prove that $AB = BQ$.



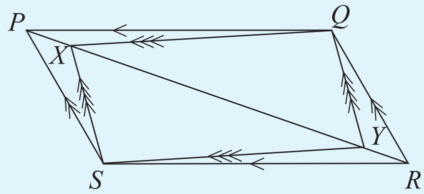

**2.** In the parallelogram $ABCD$ in the figure, the perpendiculars drawn from $B$ and $D$ to $AC$ are $BL$ and $DM$ respectively. Show that $BL = DM$.




**3.** The figure illustrates two parallelograms $PQRS$ and $QYSX$. Prove that,

(i) $PX = RY$.

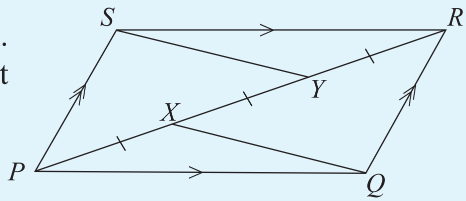
(ii) Area of $PSXQ$ = Area of $SRQY$




**4.** $PQRS$ in the figure is a parallelogram. The points $X$ and $Y$ lie on $PR$ such that $PX = XY = YR$. Prove that,

(i) $QX = SY$,

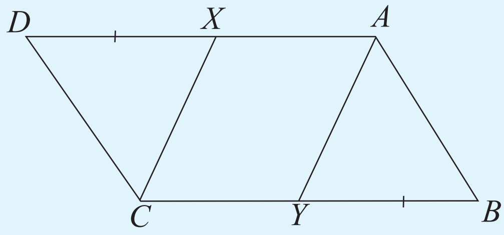
(ii) $QX // SY$.



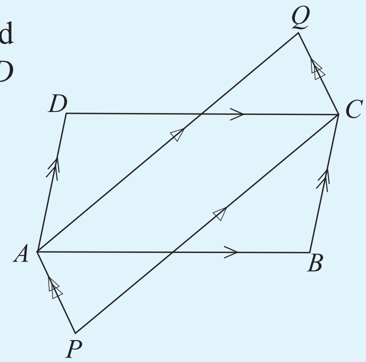

**5.** The figure illustrates a parallelogram $ABCD$. The points $X$ and $Y$ lie on the sides $AD$ and $BC$ respectively, such that $DX = BY$.

(i) Prove that $\Delta ABY \equiv \Delta DCX$

(ii) Show that $AY // XC$.



**6.** The figure depicts two parallelograms named *ABCD* and *APCQ*. Prove that, the lines *AC, BD* and *PQ* are concurrent.




**7.** In the parallelogram *PQRS,* the bisectors of $P\hat{S}R$ and $Q\hat{R}S$ meet at the point *X* on the side *PQ*.

(i) Draw a figure with the above information included in it.

(ii) Prove that $PX = PS$.

(iii) Prove that X is the mid-point of *PQ*.

(iv) Prove that $PQ = 2\,PS$.