# 13 Algebraic Fractions

**By studying this lesson you will be able to**

simplify algebraic fractions with unequal denominators.

## Algebraic Fractions

Given below are several examples of algebraic fractions.

$$\frac{x}{4}, \quad \frac{2x+1}{x+3}, \quad \frac{3}{1+6y}, \quad \frac{x^2+x+1}{x^3-3x}$$

There are algebraic expressions in the numerator, the denominator or both the numerator and the denominator of each of these algebraic fractions.

Do the following exercise by applying what you have learnt earlier about adding and subtracting algebraic fractions.

**Review Exercise**

Simplify the following algebraic fractions.

(i) $\frac{x}{3} + \frac{x}{3}$ (ii) $\frac{x+1}{5} + \frac{2x+3}{3}$ (iii) $\frac{x}{3} + \frac{x}{2} + \frac{x}{4}$

(iv) $\frac{x+1}{3} + \frac{x+3}{6}$ (v) $\frac{2}{a} + \frac{3}{a} - \frac{1}{a}$ (vi) $\frac{5}{x+2} - \frac{3x+1}{x+2}$

## 13.1 Simplifying fractions with unequal algebraic terms in the denominator

**Simplify**

$$\frac{2}{x} + \frac{3}{2x}$$

$x$ and $2x$ are the two terms in the denominators of $\frac{2}{x}$ and $\frac{3}{2x}$ respectively. Since they are unequal, the two fractions cannot be added directly. Therefore, let us first write equivalent fractions that have a common denominator and then do the simplification.

That is,

$$\frac{2}{x} + \frac{3}{2x} = \frac{2 \times 2}{x \times 2} + \frac{3}{2x}$$

$$= \frac{4}{2x} + \frac{3}{2x}$$

$$= \frac{7}{2x}$$

Here $2x$ is the denominator of the equivalent fractions. Observe that $2x$ is the least common multiple of the denominators ($x$ and $2x$) of the given two fractions. Now consider the following algebraic fractions which have been simplified in a similar manner.

**Example 1**

$$\frac{5}{3a}-\frac{3}{4a}$$
$$=\frac{5\times 4}{3a\times 4}-\frac{3\times 3}{4a\times 3}$$
$$=\frac{20}{12a}-\frac{9}{12a}$$
$$=\underline{\underline{\frac{11}{12a}}}$$

**Example 2**

$$\frac{2}{3x}+\frac{5}{4y^2}$$
$$=\frac{2\times 4y^2}{3x\times 4y^2}+\frac{5\times 3x}{4y^2\times 3x}$$
$$=\frac{8y^2}{12xy^2}+\frac{15x}{12xy^2}$$
$$=\underline{\underline{\frac{8y^2+15x}{12xy^2}}}$$

**Example 3**

$$\frac{3b}{4a}+\frac{2a}{3b^2}+\frac{a}{2b}$$
$$=\frac{3b\times 3b^2}{4a\times 3b^2}+\frac{2a\times 4a}{3b^2\times 4a}+\frac{a\times 6ab}{2b\times 6ab}$$
$$=\frac{9b^3}{12ab^2}+\frac{8a^2}{12ab^2}+\frac{6a^2b}{12ab^2}$$
$$=\underline{\underline{\frac{9b^3+8a^2+6a^2b}{12ab^2}}}$$

**Exercise 13.1**

**1.** Simplify the following algebraic fractions.

**a.** $\frac{3}{x}+\frac{1}{3x}$ **b.** $\frac{7}{4a}-\frac{1}{2a}$ **c.** $\frac{3}{5m}+\frac{5}{4m^2}$ **d.** $\frac{1}{p}+\frac{1}{q}$

**e.** $\frac{7}{3x}-\frac{5}{4x}$ **f.** $\frac{3}{2a}+\frac{2}{a}-\frac{1}{3a}$ **g.** $\frac{3}{4x}-\frac{2}{3x}+\frac{4}{2x}$ **h.** $\frac{5}{m}+\frac{n}{3m}$

**i.** $\frac{a}{b}-\frac{b}{a}$ **j.** $\frac{1}{4a^2}+\frac{3}{5a}$ **k.** $\frac{3n}{m^2}-\frac{4}{5m}$ **l.** $\frac{3}{2a^2}-\frac{5}{4b}+\frac{4b}{3}$

## 13.2 Simplifying algebraic fractions with unequal binomial expressions in the denominators

As was done in 13.1 above, here too the LCM of the algebraic expressions in the denominators is first found and the simplification is done after the equivalent fractions with a common denominator are found.

**Example 1**

Simplify $\frac{1}{(p+1)}+\frac{1}{(p+5)}$

Since the LCM of $(p+1)$ and $(p+5)$ is $(p+1)\,(p+5)$,

$$\frac{1}{(p+1)}+\frac{1}{(p+5)}=\frac{(p+5)}{(p+1)\,(p+5)}+\frac{(p+1)}{(p+1)\,(p+5)}$$

$$= \frac{(p+5)+(p+1)}{(p+1)(p+5)}$$

$$= \frac{2p+6}{(p+1)(p+5)}$$

$$= \frac{2(p+3)}{(p+1)(p+5)}$$

**Example 2**

$$\frac{4}{x+3} - \frac{3}{x+4}$$

$$= \frac{4\ (x+4)}{(x+3)(x+4)} - \frac{3(x+3)}{(x+3)(x+4)}$$

Since the LCM of $(x+3)$ and $(x+4)$ is $(x+3)(x+4)$

$$= \frac{4\,(x+4)-3(x+3)}{(x+3)(x+4)}$$

$$= \frac{4x+16-3x-9}{(x+3)(x+4)}$$

$$= \frac{x+7}{(x+3)(x+4)}$$

When there are quadratic expressions in the denominators, they have to be first written in terms of their factors, and then the simplification needs to be done as above by finding the LCM of the denominators.

**Example 3**

$$\frac{1}{(x+2)} + \frac{1}{(x^2-3x-10)}$$

$$= \frac{1}{(x+2)} + \frac{1}{(x+2)(x-5)}$$

$$= \frac{(x-5)+1}{(x+2)(x-5)}$$

$$= \frac{(x-4)}{(x+2)(x-5)}$$

**Example 4**

$$\frac{1}{(x-1)} + \frac{3}{(x+1)} - \frac{2}{(x^2-1)}$$

$$= \frac{(x+1)}{(x-1)(x+1)} + \frac{3(x-1)}{(x-1)(x+1)} - \frac{2}{(x-1)(x+1)}$$

$$= \frac{x+1+3x-3-2}{(x-1)(x+1)}$$

$$= \frac{4x-4}{(x-1)(x+1)}$$

$$= \frac{4\cancel{(x-1)}}{\cancel{(x-1)}\,(x+1)}$$

$$= \frac{4}{(x+1)}$$

**Exercise 13.2**

Simplify the following algebraic fractions.

(*A*)

a. $\dfrac{1}{a}+\dfrac{2}{a+2}$

b. $\dfrac{5}{x}+\dfrac{3}{x+1}$

c. $\dfrac{1}{x+1}+\dfrac{2}{x+3}$

d. $5+\dfrac{2}{x+3}$

e. $\dfrac{5}{4x+1}-\dfrac{1}{3(2x+1)}$

f. $\dfrac{8}{x+5}-\dfrac{3}{5-x}$

g. $\dfrac{2}{x+5}+\dfrac{3}{x-2}+\dfrac{1}{x}$

h. $\dfrac{2}{1-x}-\dfrac{3}{5-x}$

i. $\dfrac{3}{2(y-2)}+\dfrac{2}{3(y-2)}$

j. $\dfrac{1}{m-3}-\dfrac{2}{2m-1}$

k. $\dfrac{3}{x-6}-\dfrac{2}{2x-5}$

l. $\dfrac{4}{3(x+1)}-\dfrac{2}{5(x-1)}$

(*B*)

a. $\dfrac{x+3}{x^2-1}+\dfrac{1}{x+1}$

b. $\dfrac{t-1}{t+1}+\dfrac{1}{t^2-1}$

c. $\dfrac{1}{x+1}+\dfrac{1}{(x+1)^2}+\dfrac{1}{x^2-1}$

d. $\dfrac{1}{a-3}+\dfrac{1}{a^2-a-6}$

e. $\dfrac{1}{x+3}+\dfrac{1}{x^2+x-6}$

f. $\dfrac{3}{x^2+x-2}-\dfrac{1}{x^2-x-6}$

g. $\dfrac{4}{p^2+p-6}-\dfrac{2}{p^2+5p+6}$

h. $\dfrac{1}{x^2+4x+4}-\dfrac{1}{(x-2)(x+2)}$

i. $\dfrac{3}{a^2+5a+6}+\dfrac{1}{a^2+4a+3}$

j. $\dfrac{1}{2a+1}+\dfrac{1}{a^2+3a+2}$