12 Least Common Multiple of Algebraic Expressions

By studying this lesson you will be able to

find the least common multiple of several algebraic expressions.

Finding the least common multiple of numbers

The least common multiple (LCM) of several whole numbers is the smallest number which is divisible by all these numbers. You have learnt earlier how this is found. Let us recall what you have learnt.

Let us find the least common multiple of the numbers 6, 8 and 12 by writing these numbers as a product of their prime factors.

$$6 = 2 \times 3 = 2^1 \times 3^1$$

$$8 = 2 \times 2 \times 2 = 2^3$$

$$12 = 2 \times 2 \times 3 = 2^2 \times 3$$

The distinct prime factors of the above three numbers are 2 and 3. When all three numbers are considered,

the greatest power of 2 = 2^3

the greatest power of 3 = 3^1

$\therefore$ The least common multiple $= 2^3 \times 3$

$= \underline{\underline{24}}$

Accordingly, the least common multiple of several numbers can be found in the following manner.

1. Write each number as a product of its prime factors.
2. From the factors of all the numbers, select the greatest power of each prime number.
3. By multiplying all these powers together, obtain the required LCM.

Review Exercise

1. Find the least common multiple of each of the following triples, by writing each number as a product of its prime factors.

(i) 12, 18, 24 (ii) 6, 10, 15 (iii) 20, 30, 60

(iv) 8, 12, 24 (v) 24, 36, 48

2. An ice-cream producing company has three ice-cream vans. The three vans arrive at the housing complex "Isuruvimana" once every 3 days, once every 6 days and once every 8 days respectively. If all three vans arrived at "Isuruvimana" on a certain day, after how many days will they all arrive at this housing scheme again on the same day?

3. Mr. Gunatillake visits Galle Face grounds every Sunday evening to observe the setting of the sun. Mr. Mohamed and Mr. Perera too visit the same location once every 6 days and once every 8 days respectively for the same reason. If the three of them met each other for the first time in Galle Face grounds on Sunday the 8th of December 2013, after how many days did they all meet each other again at the same location? On which date did they meet again?

4. When a number is divided by each of the numbers 5, 6 and 7, a remainder of 1 is obtained. Find the smallest such number.

12.1 Finding the least common multiple of algebraic terms

Now let us consider what is meant by the least common multiple of several algebraic terms and how it is found.
Let us find the least common multiple of $4a^2$, $6ab$ and $8b$.
Let us write each term as a product of its factors.

$$4a^2 = 2\times2\times a\times a = 2^2\times a^2$$
$$6ab = 2\times3\times a\times b = 2^1\times3^1\times a^1\times b^1$$
$$8b = 2\times2\times2\times b^1 = 2^3\times b^1$$

The distinct factors of these algebraic terms are 2, 3, a and b.

The largest power of 2 is 2^3
The largest power of 3 is 3^1
The largest power of a is a^2
The largest power of b is b^1

$$\therefore\ \text{LCM} = 2^3\times3\times a^2\times b$$
$$= \underline{\underline{24a^2b}}$$

The least common multiple is obtained by taking the product of the largest powers of all the distinct factors of the given algebraic terms.

Exercise 12.1

1. For each of the following parts, find the LCM of the given terms.

(i) $xy,\ xy^2$ (ii) $a^2b,\ ab^2$

(iii) $6,\ 3a,\ 8b$ (iv) $24,\ 8x,\ 10x^2$

(v) $4m,\ 8mn,\ 12m^2$ (vi) $6p,\ 4pq,\ 12pq^2$

(vii) $4,\ 6x^2y,\ 8y$ (viii) $m^2n,\ nm,\ nm^2$

(ix) $ab,\ 4a^2b,\ 8a^2b^2$ (x) $5xy,\ 10x^2y,\ 2xy^2$

12.2 Finding the least common multiple of algebraic expressions which include binomial expressions

Let us find the least common multiple of $2x + 4$ and $3x - 9$.

To find the LCM of such expressions, their factors need to be found first.

$$2x + 4 = 2\,(x + 2)$$
$$3x - 9 = 3\,(x - 3)$$

The distinct factors are 2, 3, $(x + 2)$ and $(x - 3)$. The index of each of these factors is 1.

The product of the largest powers of these factors $= 2 \times 3 \times (x+2) \times (x-3)$

$\therefore$ LCM $= \underline{\underline{6\,(x+2)\,(x-3)}}$

Example 1

Find the least common multiple of $15x^2,\ 20\,(x + 1),\ 10\,(x + 1)^2$

$$15x^2 = 3 \times 5 \times x^2$$
$$20\,(x + 1) = 2 \times 2 \times 5 \times (x+1) = 2^2 \times 5 \times (x + 1)$$
$$10\,(x + 1)^2 = 2 \times 5 \times (x+1)^2$$

The distinct factors are 2, 3, 5, x and $(x + 1)$.

$\therefore$ LCM $= 2^2 \times 3 \times 5 \times x^2 (x+1)^2$

$= \underline{\underline{60x^2 (x+1)^2}}$

Example 2

Find the least common multiple of the algebraic expressions $(b - a)$, $2(a - b)$ and $4a^2\,(a - b)^2$

$$(b - a) = (-1) \times (a - b)$$
$$2(a - b) = 2 \times (a - b)$$
$$4a^2 (a - b)^2 = 2 \times 2 \times a^2 \times (a - b)^2$$
$$= 2^2 \times a^2 \times (a - b)^2$$

Since $(a - b)$ is a factor of two of these expressions, it is necessary to express $(b - a)$ as $-(a - b)$.

The distinct factors are 2, (-1), a and $(a-b)$.

$\therefore$ The product of the largest powers $= 2^2 \times (-1) \times a^2 \times (a-b)^2$

$\therefore$ LCM $= -4a^2(a-b)^2$

Note: Knowing that although $a-b = -(b-a)$, we have $(a-b)^2 = (b-a)^2$, facilitates problem solving.

Exercise 12.2

1. Find the LCM of the algebraic expressions in each of the following parts.

a. $3x+6,\ 2x-4$
b. $2a+8,\ 3a+12$
c. $p-4,\ 8-2p$
d. $8(x+5),\ 20(x+5)^2$
e. $3x,\ 15(x+1),\ 9(x-1)$
f. $a^2,\ 2(a-b),\ (b-a)$
g. $3(x-2),\ 5(3-x),\ (x-2)(x-3)$
h. $3x,\ 15(x-3),\ 6(x-3)^2$
i. $(t-1),\ (1-t)^2$
j. $2a-4,\ 12(a-2)^2,\ 8(a+2)(2-a)^2$

12.3 Finding the least common multiple of algebraic expressions, described further

(*a*) When there is a difference of two squares

Example 1

Find the least common multiple of the algebraic expressions $2x-6$, $4x(x-3)^2$ and $6(x^2-9)$

$2x-6 = 2(x-3)$

$4x(x-3)^2 = 2 \times 2 \times x \times (x-3)^2$

$6(x^2-9) = 2 \times 3 \times (x-3)(x+3)$

The distinct factors are $2, 3, x, (x-3)$ and $(x+3)$

$\therefore$ LCM $= 2^2 \times 3 \times x \times (x+3) \times (x-3)^2$

$= 12x(x+3)(x-3)^2$

(*b*) When there are trinomial quadratic expressions

Example 2

Find the least common multiple of the algebraic expressions $3(x+2)^2$, x^2+5x+6, and $2x^2+7x+3$

$3(x+2)^2 = 3 \times (x+2)^2$

$x^2+5x+6 = (x+2)(x+3)$

$2x^2+7x+3 = (x+3)(2x+1)$

The distinct factors are $3, (x+2), (x+3)$ and $(2x+1)$

$\therefore$ LCM $= 3(x+3)(2x+1)(x+2)^2$

Exercise 12.3

1. Find the LCM of the following algebraic expressions.

a. $3(x-2)$, (x^2-4)

b. $6(x-1)$, $2x(x^2-1)$

c. $3x-9$, $4x(x-3)$, (x^2-9)

d. $(a-b)$, (a^2-b^2)

e. $p(p-q)$, $pq(p^2-q^2)$

f. x^2+2x+1, $2(x+1)$

g. $x^2-8x+15$, $2x^2-x-15$

h. x^2-4, $3x^2-5x-2$, $3x^2-9x-12$

i. m^2-5m+6, m^2-2m-3

j. x^2-a^2, x^2-ax, $x^2-2ax+a^2$