10 Inverse Proportions

By studying this lesson you will be able to

solve problems related to inverse proportions.

Ratios

Do the following exercise to recall the facts you have learnt earlier about ratios and direct proportions.

Review Exercise

1. Find the number suitable for the box, for each of the following to be a direct proportion.

(i) 5 : 2 = 20 : ☐ (iii) 4 : ☐ = 20 : 25

(ii) 2 : 3 = ☐ : 15 (iv) ☐ : 4 = 60 : 80

2. The daily income earned by a vehicle used in a taxi service is Rs 8000, while the daily cost incurred by it is Rs 4500. Write the ratio of the daily income to the daily cost in its simplest form.

3. In a certain scale diagram, an actual distance of 1000 m is represented by 2 cm. Express this scale as a ratio.

4. The gravitational force on earth is six times the gravitational force on the moon. Therefore, the ratio of the weight of an object on the moon to the weight of the object on earth is 1 : 6. How much would the weight of an astronaut be on the moon, if his weight on earth is 540 N.

5. To make a cement mixture, cement and sand are mixed together in the ratio 1 : 6.

(i) What would be the fraction of cement in such a mixture?

(ii) How many pans of cement are required for 18 pans of sand?

(iii) A certain bag of cement contains 5 pans of cement. If it is necessary to make a cement mixture using the whole bag, how many pans of sand are required?

(iv) Find separately the number of pans of cement and the number of pans of sand that are required to make 70 pans of the cement mixture.

10.1 Inverse Proportions

We know that, when we consider two quantities, if when one quantity increases according to a certain ratio, the other quantity also increases in the same ratio, or when one quantity decreases according to a certain ratio, the other quantity also decreases in the same ratio, then the two quantities are said to be **directly proportional** to each other.

In an **inverse proportion**, when one quantity increases according to a certain ratio, the other quantity decreases in the same ratio, or when one quantity decreases according to a certain ratio, the other quantity increases in the same ratio.

Let us establish this fact further by considering the following example.

In a certain hostel, there is sufficient food stored for 12 hostellers for 4 days. Keeping this amount of food in mind, let us consider the following questions.

(i) If the number of hostellers is 15, will the food be sufficient for four days?

(ii) If the number of hostellers is 6, for how many days will the food be sufficient?

(iii) When the number of hostellers decreases, does the number of days for which the food is sufficient decrease? Or does it increase?

(iv) For how many days will this food, which is sufficient for 12 hostellers for 4 days, be sufficient for one person?

It is clear that the food which is sufficient for 12 people for 4 days will be sufficient for 6 people for 8 days, and for one person for 48 days. The following relationships between the number of hostellers and the number of days for which the food is sufficient can easily be identified.

Number of hostellers	Number of days
12	4
⑧	⑥
6	8
4	12
②	㉔
1	48

Let us see how the two quantities, namely the number of hostellers and the number of days for which the food is sufficient change proportionally. According to the above chart, when the number of hostellers decreases from 8 to 2, the number of days for which the food is sufficient increases from 6 to 24.

In this case, the ratio of the number of hostellers = 8 : 2 = 4 : 1
The number of days for which the food is sufficient in this case has increased from 6 to 24.
The ratio of the number of days = 6 : 24 = 1 : 4

Although the ratio 1 : 4 is not equal to the ratio 4 : 1, the ratio that is obtained by interchanging the two numbers of one ratio is the same as the other ratio.

Then,

the ratio of the number of hostellers = 8 : 2 = 4 : 1
the ratio of the corresponding number of days interchanged = 24 : 6 = 4 : 1

A relationship such as the above one between the number of hostellers and the number of days for which the food is sufficient is called an **inverse proportion**.

Let us consider two more instances of the relationship between the number of hostellers and the number of days for which the food is sufficient.

Number of hostellers	**Number of days**
12	4
1	48

Ratio of the number of hostellers = 12 : 1
Ratio of the corresponding number of days interchanged = 48 : 4 = 12 : 1

The inverse proportion relationship should hold for any two cases that are considered, in the same manner that was observed in the above cases.

Two more examples of inverse proportions are given below.

(i) The number of people required to complete a particular task and the amount of time taken to complete the task.

(ii) When a vehicle travels a constant distance with uniform speed, the speed of the car and the time taken to complete the journey.

Now, let us consider the following example.

Example 1

It takes 5 men 8 days to complete a certain task. Find the number of days required by 10 men to complete the same task.

Let us look at two methods that can be used to solve this problem.
Note that this is a problem on inverse proportions.

Method 1

Let us take the number of days required by 10 men to complete the task as x. Then,

Number of men	Number of days
5	8
10	x

Since this is an inverse proportion,

$$5 : 10 = x : 8$$
$$\frac{5}{10} = \frac{x}{8}$$
$$10x = 8 \times 5$$
$$= 40$$
$$\therefore x = \frac{40}{10}$$
$$= 4$$

$\therefore$ The number of days required by 10 men to complete the task = 4

Method 2

Time taken for 5 men to complete the task = 8 days

Time taken for one man to complete the task = 8 days $\times$ 5

= 40 days

$\therefore$ Time taken for 10 men to complete the task = 40 days $\div$ 10

= 4 days

Note: In the above example, the number of days taken by one man to complete the task, which is 40, can be taken as a measurement of the magnitude of the task. This value is called the **number of man days**.

Magnitude of the task = Amount of time required by one man to complete the task

= Number of men $\times$ Number of days

Accordingly, the magnitude of the above task can be considered as 40 man days. The magnitude of a task can be measured in terms of man days as well as in terms of man hours.

Example 2

It takes 5 men 8 days to complete a certain task. How many men are required to complete the same task in 2 days?

Let us use method (ii) of example 1.

The number of days required by 5 men to complete the task = 8

$\therefore$ The number of days required by one man to complete the task = 8 $\times$ 5

$\therefore$ The magnitude of the task = 8 $\times$ 5 man days

= 40 man days

$\therefore$ The number of men required to complete the task in 2 days = 40 $\div$ 2

= 20

Example 3

Food sufficient for 12 days for a group of 40 men employed at a worksite has been stored at the site. If 8 more men join the workforce after 6 days, for how many more days will the food that is remaining be sufficient?
Let us now consider how this problem is solved by two different methods.

Method 1

Amount of food sufficient for 40 men for 12 days $= 40 \times 12$
$= 480$
Amount of food sufficient for 40 men for 6 days $= 40 \times 6$
$= 240$
Amount of food remaining $= 480 - 240$
$= 240$
Number of days this food is sufficient for 48 men $= 240 \div 48$
$= 5$ days

Let us now see how this sum can be solved algebraically.

Method 2

The food which is sufficient for the group of 40 men for 12 days will be sufficient for the 40 of them for 6 days and for a few more days for the 48 men, which includes the 8 who joined later.

Let us take the number of days for which the food will be sufficient for the 48 men after the 6^{th} day as x. We can then equate the amount of food sufficient for the 40 men for 12 days to the sum of the amount of food sufficient for the 40 men for 6 days and the 48 men for x days.

$$\therefore 40 \times 12 = (40 \times 6) + (48 \times x)$$
$$480 = 240 + 48x$$
$$48x = 480 - 240$$
$$= 240$$
$$\therefore x = \frac{240}{48}$$
$$= 5$$

$\therefore$ Therefore, the remaining food will be sufficient for 5 days.

Exercise 10.1

1. For each of the situations described below, select the suitable answer from (*a*), (*b*) and (*c*) and write it within the brackets next to it.

(*a*) Is not a proportion (*b*) Is a direct proportion (*c*) Is an inverse proportion

(i) The number of soldiers in a camp and the amount of food stored for their consumption (…….)
(ii) The radius of a circle and its area (…….)
(iii) The distance travelled by a vehicle travelling at a uniform speed and the time taken for the journey (…….)
(iv) The length and breadth of a rectangle of constant area (…….)
(v) The amount of sugar bought at a store by a person and the amount paid for it (…….)

2. It takes 8 men 9 days to complete a certain task.
(i) How many days will it take one man to complete the same task?
(ii) What is the magnitude of the task in man days?
(iii) If 12 men are assigned the task, how many days will it take them to complete the task?

3. A land owner estimates that it would take 10 men 8 days to clear his land. He hires 12 men for the initial two days.
(i) What is the magnitude of the task in man days?
(ii) How much of the task will be completed during the first two days?
(iii) If the landowner wishes to get the task done in 6 days, how many more men should he employ for the next four days?

4. In a certain farm there was sufficient food for 12 cattle for 10 days. After two days another four cattle were bought and brought to the farm.
(i) For how many days is the food in the farm sufficient for one of the cattle?
(ii) What is the reduction in the number of days for which the food is sufficient, due to the increase in the number of cattle?

5. Food sufficient for 24 trainees for 8 days was stored at a certain training camp. However, two days after the camp commenced, 6 of the trainees had to leave the camp due to illness. Show that the remaining food is sufficient for 2 extra days than was initially planned.

6. A particular tank of water can be emptied in four hours using three equal pumps. The three pumps were used to empty the tank. However one pump stopped working after an hour. Thereafter, the tank was emptied using the other two pumps. Find how much more time was required to empty the tank due to breakdown of one pump.

7. It takes half an hour for a certain vehicle travelling at a speed of 40 kmh^{-1} to complete a certain journey. Find the time in minutes that it would take the vehicle to complete the same journey, if it travels at a speed of 50 kmh^{-1}.

8. Four men who were given the responsibility of completing a task were able to finish only $\frac{2}{3}$ of it by working 6 hours a day for 3 days.

(Hint: Man hours = no.of men × no.of days × no.of working hours per day)

(i) What is the magnitude of the task in man hours?

(ii) The four men decide to complete the task on the next day. How many hours will they have to work to achieve this?

10.2 Representing inverse proportions algebraically

If it takes eight men one day to complete a certain task, then

- four men would require two days to complete the same task
- if only two men are engaged, four days would be required to complete the task
- it would take one man eight days to complete the task

Observe that in all four of the above situations, the product of the number of men and the number of days is a constant. That is,

number of men × number of days = a constant

This constant value is the magnitude of the task. The unit used to measure the magnitude of the task can be taken as man days.

Accordingly, if x is the number of men and y is the number of days, then

$$xy = k \quad (k \text{ is a constant})$$

$$\therefore \quad x = \frac{k}{y} \quad \text{or} \quad y = \frac{k}{x}$$

According to the definition of direct proportion, this can be expressed as $x \propto \frac{1}{y}$. That is, x and $\frac{1}{y}$ are directly proportional to each other. In this case we say that x and y are inversely proportional to each other.

Example 1

8 men can complete a certain task in 9 days. However, it was possible to employ only 6 men. How many days will it take these 6 men to complete the task?

Let us denote the number of men by x and the number of days by y. Then, by using the equation $xy = k$ and substituting the data, we obtain the equations,

$8 \times 9 = k$ and

$6y = k$

Since it is the same task, the constant k is the same.

Substituting for k, we obtain the equation

$$8 \times 9 = 6y$$

$$\text{i.e., } y = \frac{8 \times 9}{6}$$

$$= 12$$

Therefore, it will take these 6 men 12 days to complete the task.

Example 2

A group of men, who completed a certain task in 9 days, recruited 3 more men to work on a similar task. If together they completed the task in 6 days, find how many men there were in the initial group.

Suppose the number of men who were in the first group is x.

Then, according to the data, we obtain the equations,

$$x \times 9 = k \text{ and}$$

$$(x + 3) \times 6 = k$$

From these we obtain

$$9x = 6(x + 3)$$

$$9x = 6x + 18$$

$$3x = 18$$

$$x = 6$$

Therefore, the number of men in the initial group was 6.

Solve the sums in the exercise given below algebraically.

Exercise 10.2

1. It took 5 men 4 days to complete a certain task. How many days will 4 men require to complete the task?

2. It was necessary to engage 9 men for 4 days, at 5 hours per day to clear a certain land. How many men working 6 hours per day can complete the same task in 10 days?

3. A certain task can be completed in 6 days by 18 men. It is expected to complete another task which is twice the size of the initial task in 9 days.
Find how many men are required to complete the second task in 9 days.