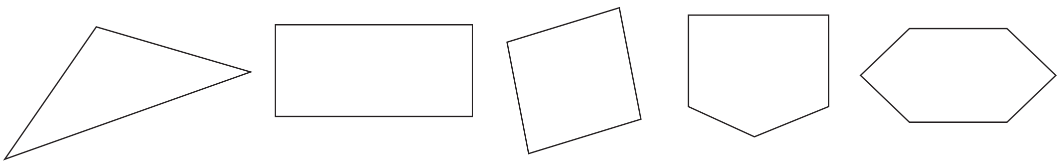
# 5 Congruence of Triangles

**By studying this lesson you will be able to**

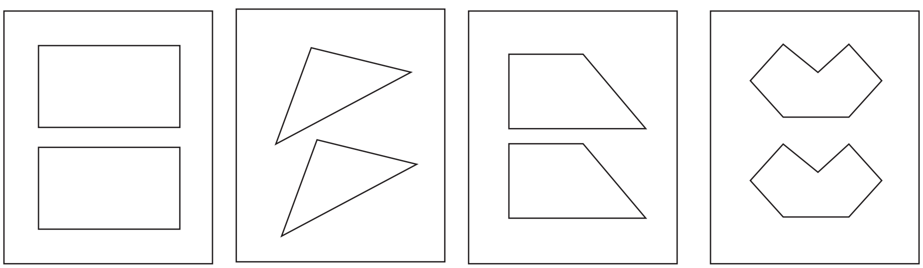
- recognize the congruence of two plane figures
- identify the necessary conditions for two triangles to be congruent
- prove riders by using the congruence of triangles

## Congruence of two plane figures



If we examine the above figures we see that they are all closed plane figures consisting of straight line segments. Such figures are called rectilinear plane figures. The sides and the angles are called the elements (parts) of these figures.

The pairs of rectilinear plane figures presented below in figures (i) to (iv), which are identical in shape and size can be placed on each other such that they coincide.



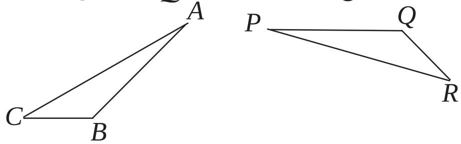
(i) (ii) (iii) (iv)

A pair of plane figures which may be made to coincide is called a pair of **congruent plane figures**. In this section we concentrate on the congruence of pairs of triangles.

## 5.1 Congruence of two triangles

A triangle has six elements. They are the three sides and the three angles of the triangle.

Let us assume that the two triangles *ABC* and *PQR* given below are congruent. Let us also assume that, when the two triangles are placed one on top of the other such that they coincide, then $AB$ coincides with $PQ$, $AC$ coincides with $PR$ and $BC$ coincides with $QR$. Then we say that, in the two triangles, the side corresponding to $AB$ is $PQ$, the side corresponding to $AC$ is $PR$ and the side corresponding to $BC$ is $QR$. Similarly, we say that the angle corresponding to $B\hat{A}C$ is $Q\hat{P}R$, the angle corresponding to $A\hat{B}C$ is $P\hat{Q}R$ and the angle corresponding to $A\hat{C}B$ is $P\hat{R}Q$.




Accordingly, in congruent triangles, the corresponding elements are equal to each other.

We indicate the fact that two triangles are congruent by using the symbol "$\equiv$". For example, if the two triangles $ABC$ and $PQR$ are congruent, we indicate this fact by writing $\Delta ABC \equiv \Delta PQR$

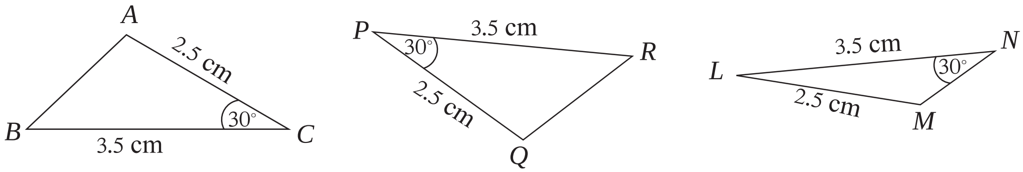
It is not necessary to show that the six elements of one triangle are equal to the six elements of the other triangle as indicated above to show that a pair of triangles is congruent. It is sufficient to show that three of the elements are equal. However, this does not mean that if any three elements of one triangle are equal to three elements of another triangle, then the two triangles are congruent. In certain cases, when three elements of one triangle are equal to three elements of another triangle, then the remaining elements too are equal and the triangles are congruent. There are four such cases. Let us now consider these four cases.

**(*a*) First Case**

The case in which two sides and the included angle of one triangle are equal to two sides and the included angle of another triangle

**Activity**

Three triangles with two sides of length 2.5 cm and 3.5 cm and an angle of magnitude 30° are given below.




- Copy the triangle $ABC$ onto a tissue paper and cut it out.
- Examine whether the cut out triangle can be made to coincide with the triangles $PQR$ and $LMN$.
- Accordingly, select the triangle which is congruent to the triangle $ABC$.

It must be clear to you through the above activity, that only the triangle *PQR* is congruent to the triangle *ABC*. Both the triangles *PQR* and *LMN* have three elements which are equal to the given three elements of the triangle *ABC*. However, the triangle *ABC* is only congruent to the triangle *PQR*. It should be clear to you from this that, just because two triangles have three elements which are equal to each other, it does not mean that the triangles are congruent.

Let us consider another method by which we can identify that the triangle *ABC* is congruent to the triangle *PQR* but not to the triangle *LMN*. The 30$^{o}$ angle of the triangle *ABC* is the included angle of the sides of length 2.5 cm and 3.5 cm. It is the same for the triangle *PQR*. However the 30$^{o}$ angle is not the included angle of the sides of length 2.5 cm and 3.5 cm of the triangle *LMN*. Two sides and the included angle of triangle *ABC* are equal to two sides and the included angle of triangle *PQR*. But this cannot be said about the triangles *ABC* and *LMN*. $\therefore$ There is insufficient data to state that and $\Delta ABC$ and $\Delta\ LMN$ are congruent.

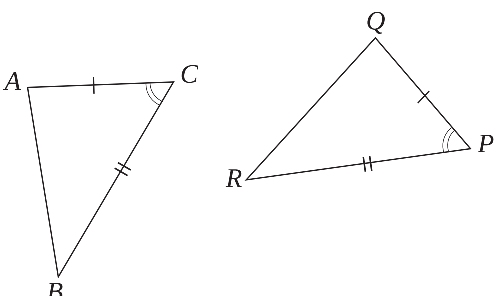
Note: Here the angle $A\hat{C}B$ which is 30$^{o}$ is called the included angle of the sides *AC* and *BC*. Similarly, the angle $R\hat{P}Q$ is the included angle of the sides *PR* and *PQ* of the triangle *PQR*.

This result which you discovered through the above activity has been used as an axiom of geometry from ancient times.

If two sides and the included angle of one triangle are equal to two sides and the included angle of another triangle, then the two triangles are congruent.

Showing that two triangles are congruent in the above manner is mentioned in short as being congruent under the case SAS.

The two triangles *ABC* and *PQR* given below can be shown to be congruent according to the above mentioned case using the given data as follows.




In the triangles *ABC* and *PQR*,

$AC = PQ$ (Given)

$A\hat{C}B = R\hat{P}Q$ (Given)

$BC = PR$ (Given)

$\therefore \underline{\underline{\Delta ABC \equiv \Delta PQR}}$ (SAS)

Since the above two triangles are congruent, the remaining corresponding elements are also equal.

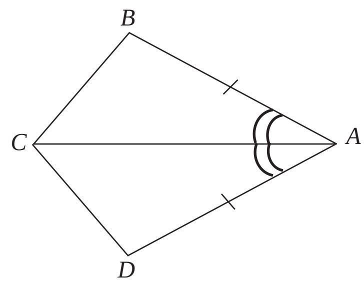
That is,

the sides $AB$ and $QR$ which are opposite the equal angles $A\hat{C}B$ and $Q\hat{P}R$ are equal to each other, the angles $A\hat{B}C$ and $Q\hat{R}P$ which are opposite the equal sides $AC$ and $PQ$ are equal to each other,
and the angles $C\hat{A}B$ and $P\hat{Q}R$ which are opposite the equal sides $BC$ and $PR$ are equal to each other.
Now let us consider an example.

**Example 1**

According to the data marked on the figure, prove that the triangles $ABC$ and $ADC$ are congruent, and write all the remaining equal corresponding elements.

**Proof:**




(i) In the triangles $ABC$ and $ACD$,

$AB = AD$ (Given)

$B\hat{A}C = C\hat{A}D$ (Given)

$AC$ is $a$ common side

$\therefore \underline{\underline{\Delta ABC \equiv \Delta ADC}}$ (S A S)

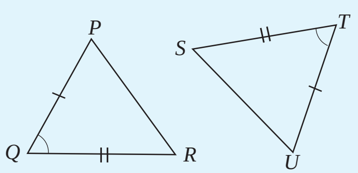
(ii) The corresponding elements of congruent triangles are equal.

$\therefore BC = CD,\ A\hat{B}C = A\hat{D}C$ and $A\hat{C}B = A\hat{C}D$.

**Exercise 5.1**

**1.** Determine for which pairs of triangles the SAS case can be applied to prove congruence based on the given information. For these pairs, prove that the two triangles are congruent and write down the remaining pairs of corresponding elements which are equal to each other.

(i)

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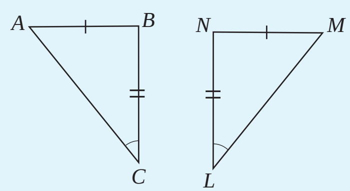


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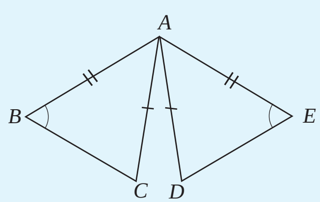


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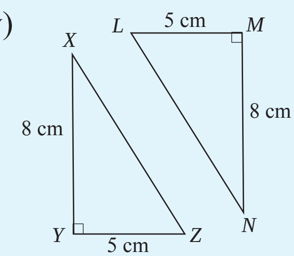

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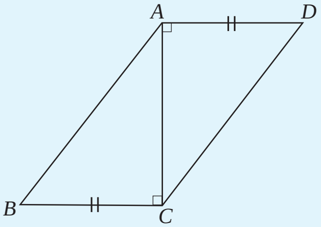


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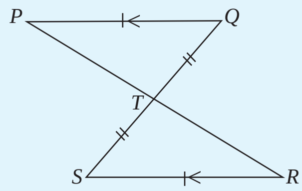


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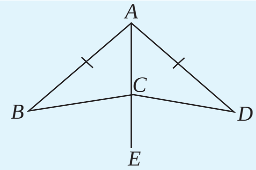


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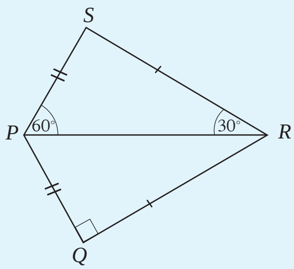


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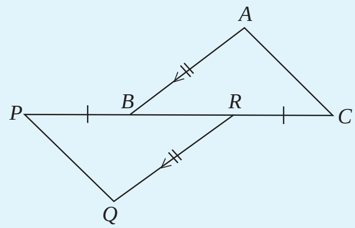


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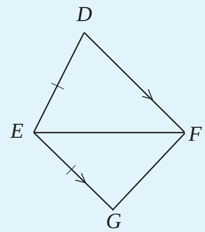



**2.** For each of the following parts, draw a sketch of the relevant pairs of triangles based on the information that is given. From these pairs of triangles, select the ones which are congruent and write down the remaining pairs of corresponding elements which are equal to each other.

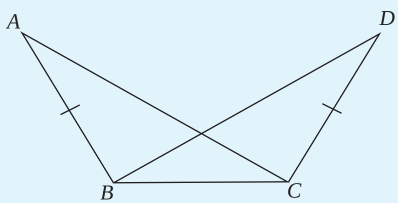
(i) In the triangles $PQR$ and $XYZ$, $PQ = XZ$, $QR = XY$, $P\hat{Q}R = Y\hat{X}Z$.

(ii) In the triangles $ABC$ and $LMN$, $AC = LN$, $BC = LM$, $A\hat{B}C = L\hat{M}N = 50°$.

(iii) In the triangles $DEF$ and $STU$, $EF = TU$, $DF = SU$, $E\hat{F}D = T\hat{U}S$.

(iv) In the triangles $ABC$ and $PQR$, $BC = PQ$, $C\hat{B}A = Q\hat{P}R$, $AC = PR$.

**3.**

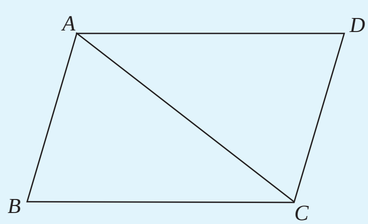



In the given figure, $AB = DC$ and $A\hat{B}C = B\hat{C}D$. Prove that,

(i) $\Delta ABC \equiv \Delta DCB$,

(ii) $AC = BD$.

**4.**

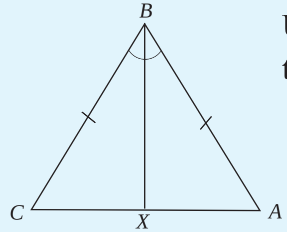



In the quadrilateral $ABCD$, the sides $AD$ and $BC$ are parallel and equal in length. Mark the given data in the diagram and prove that,

(i) $\Delta ABC \equiv \Delta ADC$,

(ii) $AB = DC$,

(iii) $AB$ and $DC$ are parallel to each other.

5. 


Using the information marked on the triangle $ABC$, prove that,

(i) $\Delta\, ABX \equiv \Delta\, CBX$

(ii) $A\hat{X}B = 90^\circ$.

6. In the quadrilateral $ABCD$, the diagonals $AC$ and $BD$ bisect each other at $O$. Prove that,

(i) $\Delta\, AOD \equiv \Delta\, BOC$

(ii) the lines $AD$ and $BC$ are parallel to each other.

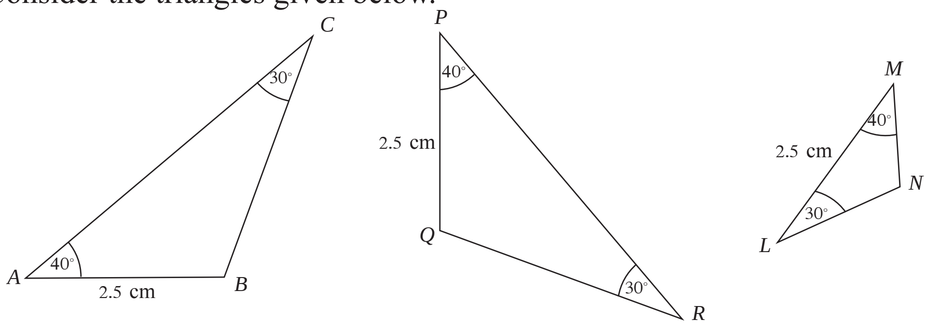
Now let us consider the second case by which the congruence of two triangles can be identified.

### (*b*) Second Case

The case in which the magnitudes of two angles and the length of a side of a triangle are equal to the magnitudes of two angles and the length of a corresponding side of another triangle

**Activity**

Consider the triangles given below.




- Copy the triangle $ABC$ onto a tissue paper and cut it out.
- Place it on the triangles $PQR$ and $LMN$ and examine which triangle it coincides with.
- Accordingly, which triangle is congruent to the triangle $ABC$?

It must be clear to you according to this activity that the trianlge $ABC$ is congruent only to the triangle $PQR$.

In this case too, as in the case (*a*), the two triangles $PQR$ and $LMN$ have three elements which are equal to three elements of the triangle $ABC$. However, although the triangle $ABC$ is congruent to the triangle $PQR$, it is not congruent to the triangle $LMN$. As before, it should be clear to you from this that, just because two triangles have three elements which are equal to each other, it does not mean that the triangles are congruent.

Therefore, let us consider another method by which we can identify that the triangle *ABC* is congruent to the triangle *PQR*. The given side of length 2.5 cm is opposite the given 30° angle of the triangle *ABC*. It is the same for the triangle *PQR*. However it is not the same in the triangle *LMN*. Accordingly, two angles of triangle *ABC* is equal to two angles of triangle *PQR*, and one side of triangle *ABC* is equal to the **corresponding** side of triangle *PQR*. However, the corresponding side of triangle *LMN* is not equal to that of triangle *ABC*.

**Note: Here, corresponding sides are defined as those which are opposite equal angles of the two triangles.**

If two angles and a side of one triangle are equal to two angles and the corresponding side of another triangle, then the two triangles are congruent.

Showing that two triangles are congruent under these conditions is stated concisely as being congruent according to the AAS case.

Using the given data, we can show as done below, that the triangles *STU* and *LMN* in the following figure are congruent according to the case mentioned above.

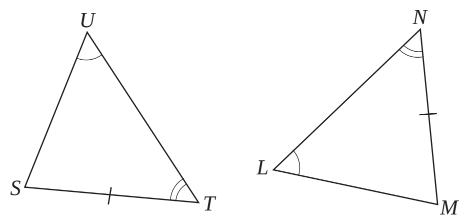
In the triangles *STU* and *LMN*

$S\hat{T}U = M\hat{N}L$ (Given)

$T\hat{U}S = N\hat{L}M$ (Given)

$ST = MN$ (Given)

$\therefore \Delta\, STU \equiv \Delta\, MNL$ (AAS)




Note: In the above two triangles *ST* and *MN* are a pair of corresponding sides which are also equal. Observe carefully that they are corresponding sides because they are opposite the angles $S\hat{U}T$ and $M\hat{L}N$ which are equal to each other.

**Example 1**

Based on the data marked on the figure, prove that, $\Delta\, ABX \equiv \Delta\, ACX$ and write all the remaining equal corresponding elements.

Proof:

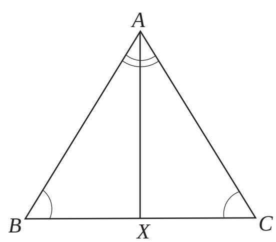
(i) In the triangles *ABX* and *ACX*,

$A\hat{B}X = A\hat{C}X$ (Given)

$B\hat{A}X = C\hat{A}X$ (Given)

$AX$ is a common side

$\therefore \Delta\, ABX \equiv \Delta\, ACX$ (A A S)

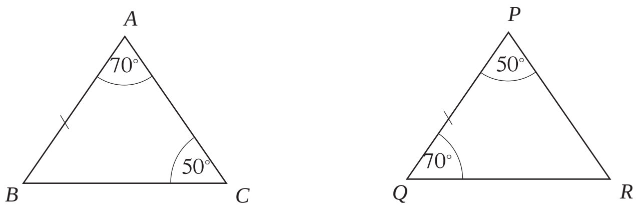


The corresponding elements of congruent triangles are equal.

Therefore $BX = CX$, $A\hat{X}B = A\hat{X}C$, $AB = AC$

**Example 2**

Determine whether the following pair of triangles is congruent under the case AAS.

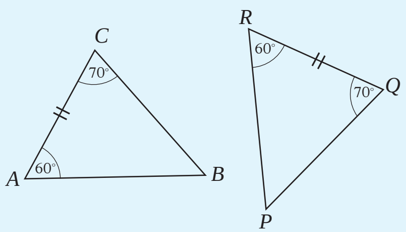



Two angles of the triangle $ABC$ are equal to two angles of the triangle $PQR$. Also, $AB = PQ$. However, they are not corresponding sides. The reason for this is that the angles $A\hat{C}B$ and $P\hat{R}Q$ which are opposite these two sides are not equal to each other. ( $A\hat{C}B = 50^\circ$, $P\hat{R}Q = 180^\circ - 50^\circ - 70^\circ = 60^\circ$ ) Therefore, there are insufficient reasons to say that the two triangles $ABC$ and $PQR$ are congruent to each other by the case AAS.

**Exercise 5.2**

Mention for which pairs of triangles the AAS conditions can be applied to prove congruence based on the given information. For these pairs, prove that the two triangles are congruent and write down the other pairs of corresponding elements which are equal to each other.

(i)

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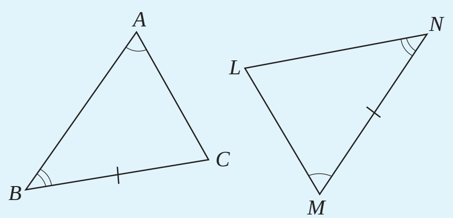


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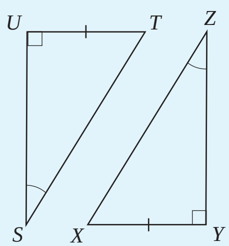


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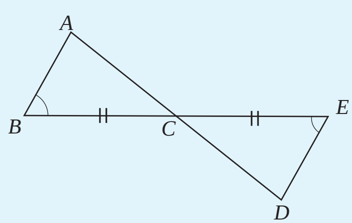

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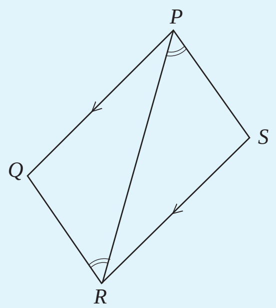


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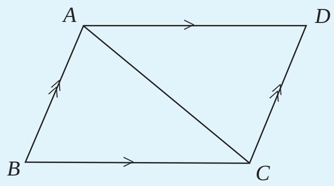


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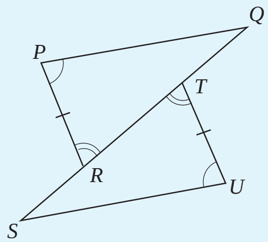


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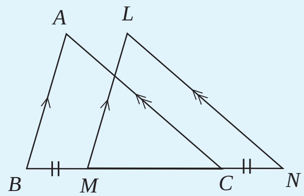



**2.** For each of the following parts, draw a sketch of the relevant pair of triangles based on the information that is given. From these pairs of triangles, select the ones that are congruent and write down the remaining pairs of corresponding elements which are equal to each other.

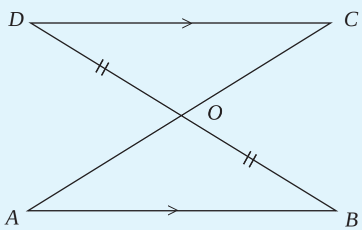
i. In the triangles $ABC$ and $PQR$, $A\hat{B}C = P\hat{Q}R$, $A\hat{C}B = P\hat{R}Q$, $BC = QR$

ii. In the triangles $XYZ$ and $LMN$, $X\hat{Y}Z = L\hat{M}N = 90^\circ$, $Y\hat{X}Z = 30^\circ$, $M\hat{N}L = 60^\circ$, $YZ = MN$

iii. In the triangles $STU$ and $PQR$, $T\hat{S}U = Q\hat{R}P$, $TU = PR$, $T\hat{U}S = P\hat{Q}R$

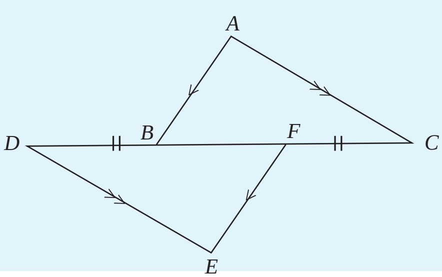
iv. In the triangles $DEF$ and $ABC$, $E\hat{D}F = B\hat{A}C = 40^\circ$, $D\hat{F}E = A\hat{C}B = 60^\circ$, $DE = BA$

**3.**



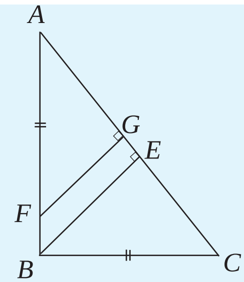

In the given figure, $AB$ and $CD$ are parallel to each other and $BO = OD$. Show that $\Delta AOB \equiv \Delta DOC$

**4.**




The pairs of sides $AB$ and $FE$, and $AC$ and $DE$ are parallel to each other. Show that $\Delta ABC \equiv \Delta EFD$

**5.** In the triangle $ABC$, $A\hat{B}C = 90^\circ$. If $AF = BC$, prove that $\Delta AFG \equiv \Delta BCE$.




**6.** In the quadrilateral $ABCD$, $\hat{A} = \hat{C} = 90^\circ$. The angles $A\hat{D}C$ and $A\hat{B}C$ are bisected by $BD$. Prove that $\Delta ABD \equiv \Delta CBD$.

Let us consider the third case by which the congruence of two triangles can be identified.

**(*c*) Third Case**

**The case of three sides of a triangle being equal to three sides of another triangle**

Can a unique triangle be constructed when the lengths of the three sides of the triangle are given? To determine this, engage in the following activity.

**Activity**

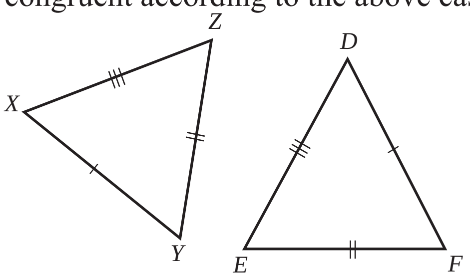
Break off pairs of ekel sticks of lengths 5 cm, 6 cm and 7 cm respectively. Make two triangles of side lengths 5 cm, 6 cm and 7 cm with these pieces. Do you see that the two triangles have to be congruent? By changing the positions of the pieces of ekel in one triangle, can you create a triangle which is not congruent to the other triangle? It must be clear to you that this is not possible.

This result which you established through the above activity can also be used as an axiom.

If the three sides of a triangle are equal to the three sides of another triangle, then the two triangles are congruent.

Showing that two triangles are congruent under these conditions is stated concisely as being congruent according to the SSS case.

We can show in the following manner that the pair of triangles $XYZ$ and $DEF$ are congruent according to the above case.




In the triangles $XYZ$ and $DEF$,

$XY = DF$ (Given)

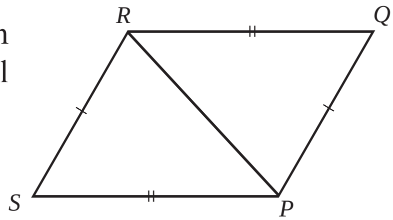
$YZ = EF$ (Given)

$ZX = DE$ (Given)

$\therefore \Delta XYZ \equiv \Delta DEF$ (S S S)

**Example 1**

Prove that $\Delta PQR \equiv \Delta PSR$ based on the information in the figure and write all the remaining equal corresponding elements.




Proof: In the triangles $PQR$ and $PSR$,

$PQ = RS$ (Given)

$QR = PS$ (Given)

$PR$ is a common side

$\therefore \Delta PQR \equiv \Delta RSP$ (S S S)

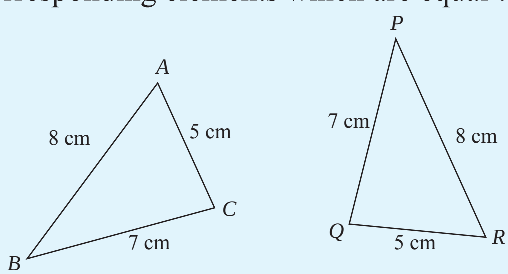
The corresponding elements of congruent triangle are equal

$\therefore \; R\hat{S}P = P\hat{Q}R,\, S\hat{R}P = Q\hat{P}R,\, S\hat{P}R = Q\hat{R}P.$

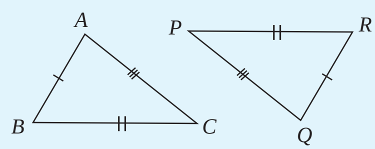
**Exercise 5.3**

**1.** Determine for which of the following pairs of the triangles the SSS conditions can be used to show congruence based on the given information. For these pairs, prove that the two triangles are congruent and write down the remaining pairs of corresponding elements which are equal to each other.

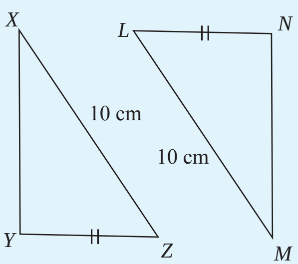
(i)



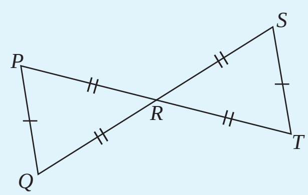

(ii)



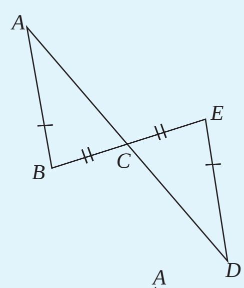

(iii)



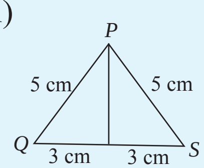

(iv)



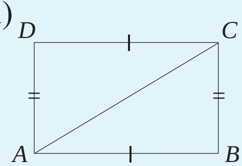

(v)



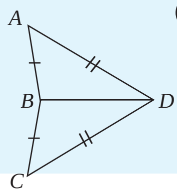

(vi)



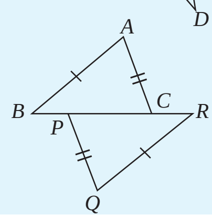

(vii)




(viii)




(ix)



**2.** Draw a sketch of the following triangles based on the information that is given. Select the triangles (if there are any) which are congruent according to the SSS case and write down the remaining pairs of corresponding elements which are equal to each other.

In triangle $PQR$, $PQ = 4$ cm, $QR = 6$ cm, $RP = 5$ cm

In triangle $XYZ$, $XY = 6$ cm, $YZ = 8$ cm, $ZX = 10$ cm

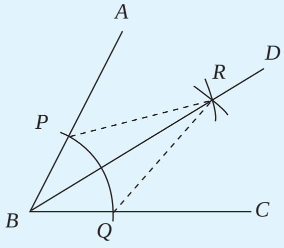
In triangle $LMN$, $LM = 5$ cm, $NM = 4$ cm, $NL = 6$ cm

In triangle $DEF$, $DE = 8$ cm, $EF = 10$ cm, $FD = 6$ cm

In triangle $ABC$, $BC = 8$ cm, $CA = 7$ cm, $AB = 9$ cm

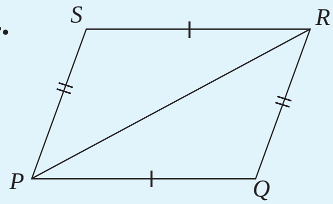
In triangle $STU$, $ST = 9$ cm, $TU = 7$ cm, $SU = 5$ cm

**3.**




To bisect the angle $A\hat{B}C$, a student selects the point $B$ as the centre and draws the arc $PQ$. The arc intersects $AB$ and $BC$ at the points $P$ and $Q$ respectively. Two equal arcs drawn from the points $P$ and $Q$ intersect at $R$. Prove that $P\hat{B}R = Q\hat{B}R$.

**4.**




The opposite sides of the quadrilateral $PQRS$ are equal in length. Prove that,

(i) $\Delta PSR \equiv \Delta PQR$

(ii) $P\hat{S}R = P\hat{Q}R$

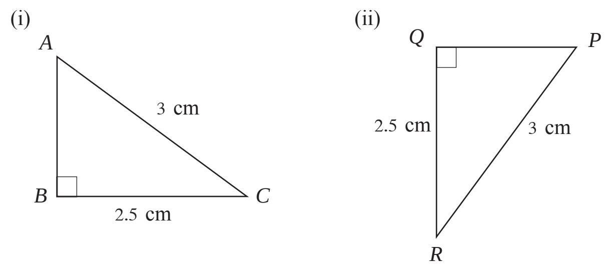
(iii) the opposite sides of the quadrilateral are parallel.

**5.** Prove that the straight line joining one vertex of an equilateral triangle to the mid-point of the opposite side is perpendicular to that side.

### (*d*) Fourth Case

The case of the hypotenuse and a side of a right angled triangle being equal to the hypotenuse and a side of another right angled triangle.

A pair of right-angled triangles drawn such that the hypotenuse is 3 cm and another side is 2.5 cm is shown below.



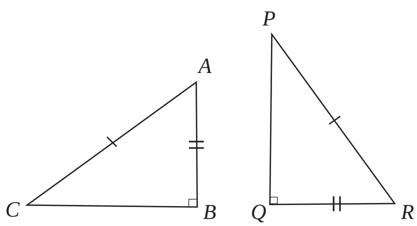
**Activity**

Copy the triangle in figure (i) onto a tissue paper and examine whether it can be made to coincide with the triangle in figure (ii).

Accordingly, the congruence of a pair of right-angled triangles can be expressed in terms of the equality of two elements as follows.

If the lengths of the hypotenuse and a side of a right-angled triangle are equal to the lengths of the hypotenuse and a side of another right-angled triangle, then the two triangles are congruent.
Showing that two triangles are congruent under these conditions is stated concisely as being congruent according to the RHS (right-angle-hypotenuse-side) case.

Let us prove that the two triangles given below are congruent based on the information that is given.



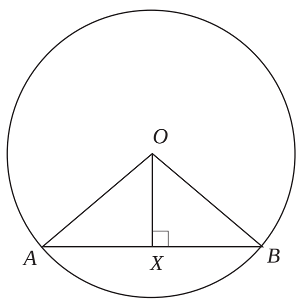

In the right-angled triangles $ABC$ and $PQR$

$$AC = PR \quad \text{(Given)}$$
$$AB = QR \quad \text{(Given)}$$
$$\therefore \Delta ABC \equiv \Delta PQR \quad \text{(RHS)}$$

Since the above pair of triangles is congruent, the remaining pairs of corresponding elements are also equal.
That is, $BC = PQ,\ B\hat{A}C = P\hat{R}Q,\ A\hat{C}B = Q\hat{P}R$.

**Example 1**

Based on the information in the figure, prove that, $\Delta OXA \equiv \Delta OXB$ and write all the remaining equal corresponding elements.




Proof:
In the right-angled triangles $OXA$ and $OXB$,

$$OA = OB \quad \text{(radii of same circle)}$$
$$OX \quad \text{is a common side}$$
$$\therefore \Delta OXA \equiv \Delta OXB \quad \text{(R H S)}$$

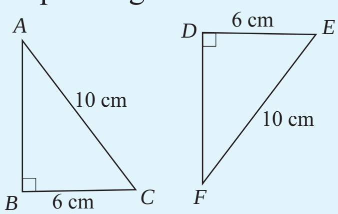
Corresponding elements of congruent triangles are equal.

$$\therefore \quad O\hat{A}X = O\hat{B}X,\ AX = BX,\ A\hat{O}X = B\hat{O}X.$$

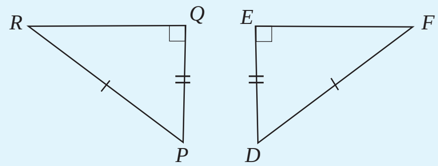
## Exercise 5.4

**1.** Determine for which of the following pairs of the triangles the RHS conditions can be used to show congruence based on the given information. For these pairs, prove that the two triangles are congruent and write down the remaining pairs of corresponding elements which are equal to each other.

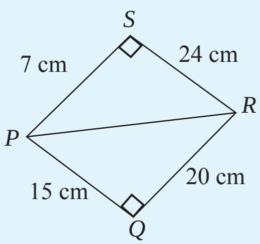
(i)



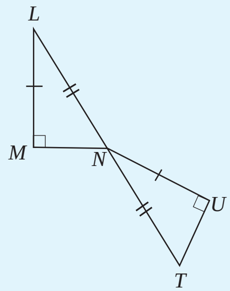

(ii)



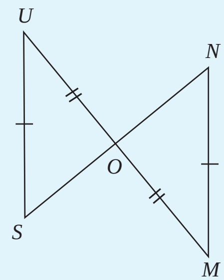

(iii)



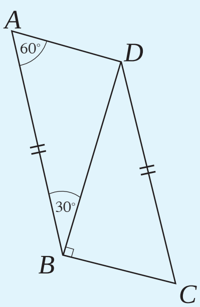

(iv)



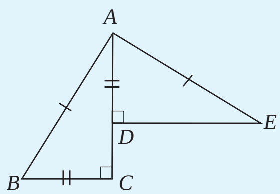

(v)



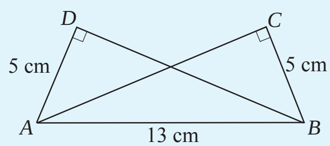

(vi)



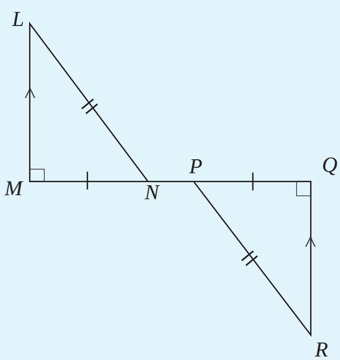

(vii)



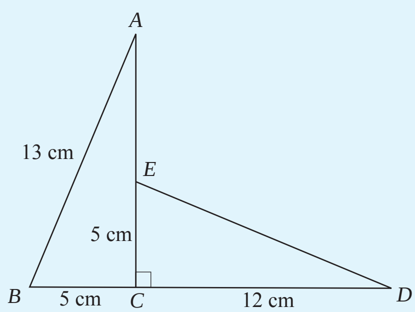

(viii)




(ix)




(x)



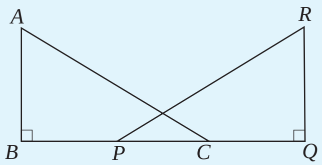
**2.** For each of the following parts, draw a sketch of the relevant pairs of triangles based on the information that is given. From these pairs, select the ones that are congruent (if there are any) and write down the remaining pairs of corresponding elements which are equal to each other.

(i) In the triangles $ABC$ and $PQR$, $A\hat{B}C = P\hat{Q}R = 90^\circ$, $AC = PR = 5$ cm, $BC = 3$ cm, $QP = 4$ cm.

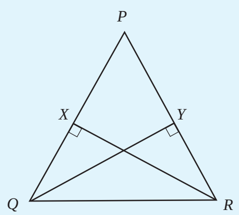
(ii) In the triangles $LMN$ and $XYZ$, $L\hat{M}N = X\hat{Y}Z = 90^\circ$, $LM = XY$, $MN = YZ$ .

(iii) In the triangles $DEF$ and $PQR$, $D\hat{E}F = P\hat{Q}R = 90^\circ$, $DF = PR$, $F\hat{D}E = 20^\circ$, $P\hat{R}Q = 70^\circ$, $EF = PQ$.

(iv) In the triangles $ABD$ and $ABC$, $A\hat{D}B = A\hat{C}B = 90^\circ$, $AD = CB$

**3.** 


If $AC = PR$ and $AB = RQ$ in the figure, show that $BP = CQ$.

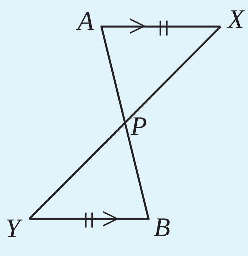
**4.** 


In the triangle $PQR$, the perpendiculars $QY$ and $RX$ are drawn from the points $Q$ and $R$ to the sides $RP$ and $QP$ respectively such that $QY=RX$. Prove that,

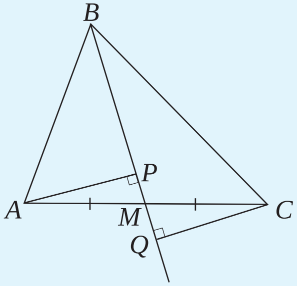
(i) $\Delta\, XQR \equiv \Delta\, YRQ$

(ii) $X\hat{R}Q = Y\hat{Q}R$

## Miscellaneous Exercise

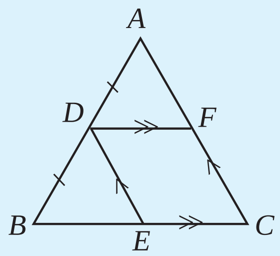
**1.** 


In the figure $AX // YB$ and $AX = YB$. Show that the straight lines $AB$ and $YX$ bisect each other at $P$.

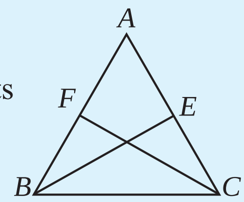
**2.** 


In the triangle $ABC$, the mid point of $AC$ is $M$. The perpendiculars drawn from $A$ and $C$ meet $BM$ and $BM$ produced at $P$ and $Q$ respectively. Show that $\Delta\, AMP \equiv \Delta\, MQC$.

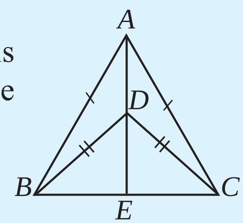
3. Using the information in the figure, show that
$\Delta ADF \equiv \Delta DBE$.



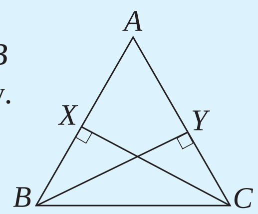

4. *ABC* in the figure is an equilateral triangle. The mid points of *AC* and *AB* are *E* and *F* respectively. Show that
   (i) *AB* and *FC* are perpendicular
   (ii) *AC* and *BE* are perpendicular
   (iii) $CF = BE$.



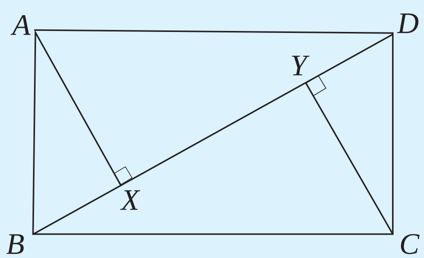

5. In the triangle *ABC* in the figure, $AB = AC$. The point *D* is such that $BD = DC$. *AD* produced meets *BC* at *E*. Prove that
   (i) $\Delta ABD \equiv \Delta CAD$
   (ii) $\Delta BAE \equiv \Delta CAE$
   (iii) *AE* and *BC* are perpendicular to each other.




6. In the given triangle *ABC*, the perpendicular drawn from *B* and *C* to the sides *AC* and *AB* are *BY* and *CX* respectively. If $BY = CX$ then show that
   (i) $AB = AC$
   (ii) $X\hat{B}C = Y\hat{C}B$ .



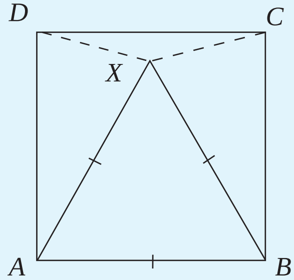

7. The perpendiculars drawn from *A* and *C* to the diagonal *BD* of the rectangle *ABCD* meet *BD* at *X* and *Y* respectively. Prove that
   (i) $\Delta AXD \equiv \Delta BCY$
   (ii) $AX = YC$
   (iii) $BX = YD$
   (iv) $\Delta YDC \equiv \Delta\ ABX$



8. The point $X$ lies in the interior of the square $ABCD$ such that $XAB$ is an equilateral triangle. Show that

(i) $\Delta\, AXD \equiv \Delta\, CBX$

(ii) $DXC$ is an isosceles triangle.




9. The equilateral triangles $BCF$ and $DCE$ are drawn on the sides $BC$ and $DC$ of the square $ABCD$ such that the triangles lie outside the square.

(i) Draw a rough sketch illustrating the above information.

Show that,

(ii) $\Delta\, EDA \equiv \Delta\, ABF$

(iii) $EAF$ is an equilateral triangle.

10. The perpendicular bisector of the side $BC$ of the triangle $ABC$ is $AE$. The point $D$ lies on $AE$. Prove that,

(i) $\Delta\, ABE \equiv \Delta\, AEC$

(ii) $\Delta\, BDE \equiv \Delta\, DEC$

(iii) $\Delta\, ABD \equiv \Delta\, ACD$

11. $ABCDE$ is a regular pentagon.

(i) Show that $\Delta\, ABC \equiv \Delta\, AED$.

(ii) If the foot of the perpendicular drawn from $A$ to the side $CD$ is $X$, prove that $CX = XD$.