2 Square Root

By studying this lesson you will be able to
- approximate the square root of a positive number which is not a perfect square
- use the division method to find an approximate value for the square root of a positive number which is not a perfect square.

2.1 The square root of a positive number

You have earlier learnt some facts about the square of a number, and the square root of a positive number. Let us briefly recall what you have learnt.
The value of 3×3 is 9. We denote 3×3 in short by 3^2. This is read as 'three squared'. The '2' in 3^2 denotes the fact that three is multiplied 'twice' over. Accordingly, three squared is 9 and this can be written as $3^2 = 9$.

The squares of several numbers are given in the following table.

Number	How the square of the number is obtained	How the square of the number is denoted	Square of the number
1	1×1	1^2	1
2	2×2	2^2	4
3	3×3	3^2	9
4	4×4	4^2	16
5	5×5	5^2	25

Numbers such as 1, 4, 9, 16 are perfect squares.
Finding the square root is the inverse of squaring. For example, since $3^2 = 9$, we say that the square root of 9 is 3. It will be clear to you, according to the first and last columns of the above table that

the square root of 1 is 1,
the square root of 4 is 2,
the square root of 9 is 3,
the square root of 16 is 4
and the square root of 25 is 5.

The symbol $\sqrt{}$ is used to denote the square root. Accordingly we can write
$\sqrt{1}=1,\ \sqrt{4}=2,\ \sqrt{9}=3,\ \sqrt{16}=4,\ \sqrt{25}=5$ etc.

It is clear that every number has a square. However, does every positive number have a square root? Let us investigate this.

According to the above table, the square root of 4 is 2 and the square root of 9 is 3. The square root of a number between 4 and 9 is a value between 2 and 3. Accordingly, it is clear that the square root of a number between 4 and 9 is not an integer. It is a decimal number. In this lesson we will consider how an approximate value is found for this. We call such a value an **approximation**.

Let us for example consider how an approximate value is obtained for the square root of 5.

Consider the following table.

Number	How the square of the number is found	How the square of the number is written	The square of the number
2	2×2	2^2	4
2.1	2.1×2.1	2.1^2	4.41
2.2	2.2×2.2	2.2^2	4.84
2.3	2.3×2.3	2.3^2	5.29
2.4	2.4×2.4	2.4^2	5.76
2.5	2.5×2.5	2.5^2	6.25
2.6	2.6×2.6	2.6^2	6.76
2.7	2.7×2.7	2.7^2	7.29

From the values in the right hand side column, the two values that are closest to 5 are 4.84 and 5.29. They are the squares of 2.2 and 2.3 respectively. According to the above table, the square roots of 4.84 and 5.29 are respectively 2.2 and 2.3. This can be written symbolically as $\sqrt{4.84}=2.2$ and $\sqrt{5.29}=2.3$.

Now let us examine which value from 4.84 and 5.29 is closer to 5.

The difference between 4.84 and 5 = 5 – 4.84 = 0.16

The difference between 5.29 and 5 = 5.29 – 5 = 0.29

Accordingly, the value that is closer to 5 is 4.84. Therefore, 2.2 can be taken as an approximate value for the square root of 5. The value that is obtained for the square root of a positive integer which is correct to the first decimal place is called the "**approximation to the first decimal place**" of the square root of the given number (or more simply the "**first approximation**")

Accordingly, the approximation of square root of 5 to the first decimal place is 2.2. When an approximate value is given, the symbol $\approx$ is used. Accordingly, we can write $\sqrt{5} \approx 2.2$.

By providing reasons in a similar manner, we can conclude that the approximation of square root of 6 to the first decimal place is 2.4 and the approximation of square root of 7 to the first decimal place is 2.6

That is, $\sqrt{6} \approx 2.4$

$\sqrt{7} \approx 2.6$

By considering the following examples let us now learn a specific method of finding the first approximation of the square root of a positive number which is not a perfect square.

Example 1

Approximate $\sqrt{17}$ to the first decimal place.

- From the perfect square numbers which are less than 17, the one which is closest to it is 16, and from the perfect square numbers which are greater than 17, the one which is closest to it is 25.

  Accordingly, let us write

  $$16 < 17 < 25$$

- When we write the square root of each of these numbers we obtain.

  $$\sqrt{16} < \sqrt{17} < \sqrt{25}$$

  $$\therefore \; 4 < \sqrt{17} < 5$$

Accordingly, the square root of 17 is greater than 4 which is the square root of 16, and less than 5 which is the square root of 25.

i.e., $\sqrt{17}$ lies between 4 and 5.

- To find an approximate value close to $\sqrt{17}$, let us check whether 17 is closer to 16 or to 25.

The difference between 16 and 17 is 1.

The difference between 17 and 25 is 8.

$\therefore$ 17 is closer to 16 than to 25.

$\therefore \sqrt{17}$ is a value close to 4.

i.e., one of the values 4.1, 4.2, 4.3 and 4.4 is the first approximation of $\sqrt{17}$

- Let us now multiply each of these numbers by itself to identify the number which has a product which is closest to 17.

When the first two values are squared we obtain

$$4.1 \times 4.1 = 16.81$$
$$4.2 \times 4.2 = 17.64$$

Since the value of 4.2^2 exceeds 17, it is unnecessary to find 4.3^2 and 4.4^2.
16.81 is the closer value to 17 from these two.

$\therefore$ First approximation of $\sqrt{17}$ is 4.1

Example 2

Find the first approximation of $\sqrt{245}$.
Since $15^2 = 225$ and $16^2 = 256$,

write
$$225 < 245 < 256$$

Accordingly, $15 < \sqrt{245} < 16$

$\therefore \sqrt{245}$ is a value between 15 and 16.

Since 245 is closer to 256 than to 225, $\sqrt{245}$ is closer to 16 than to 15.
$\therefore$ The first approximation of $\sqrt{245}$ is one of 15.5, 15.6, 15.7, 15.8 and 15.9.
Let us now determine this value.

$$15.9 \times 15.9 = 252.81$$
$$15.8 \times 15.8 = 249.64$$
$$15.7 \times 15.7 = 246.49$$
$$15.6 \times 15.6 = 243.36$$

From the above values, 246.49 is closest to 245.

$\therefore$ First approximation of $\sqrt{245}$ is 15.7.

Exercise 2.1

Find the first approximation of each of the following numbers.

(i) $\sqrt{5}$ (ii) $\sqrt{20}$ (iii) $\sqrt{67}$ (iv) $\sqrt{115}$ (v) $\sqrt{1070}$

2.2 The Division Method

Let us now consider a method of finding the square root of any positive number. This method is called the division method. Let us study this method by considering several examples.

Example 1 Find the square root of 1764.

Step 1

Separate 1764 as shown below, by grouping the digits of 1764 in pairs, starting from the units position and proceeding towards the left.

17 64

Step 2

Find the perfect square number which is closest to the leftmost digit or pair of digits of the separated number, and as indicated below, write its square root above and to the left of the drawn lines.

$$\begin{array}{r|l} & 4 \\ 4 & \overline{17\;\;64} \end{array}$$

Step 3

Write down the product 4 x 4 of the number above and to the left of the lines, below the number 17 as indicated, and subtract it from 17.

$$\begin{array}{r|l} & \;\;4 \\ 4 & \overline{17\,64} \\ & \underline{16} \\ & \;\;1 \end{array}$$

Step 4

Now carry down the next two digits 64, as indicated below.

$$\begin{array}{r|l} & \;\;4 \\ 4 & \overline{17\,64} \\ & \underline{16} \\ & \;\;1\,64 \end{array}$$

Step 5

Next, write on the left as shown below, the digit 8, which is two times the number above the line, leaving space for another number to be written. (i.e., leave space for the digit in the units position)

$$\begin{array}{r|l} & \ \ 4 \\ \hline 4 & 17\ 64 \\ & 16 \\ \cline{2-2} 4\times 2=8 \rightarrow 8\ \square & \ \ 1\ 64 \end{array}$$

Step 6

The same digit should be written above the line to the right of 4 and in the space left in the units position on the left. This digit should be selected so that the product of this digit and the number obtained on the left when this digit is written in the units position (in this case 82), is equal to 164, or is the closest number less than 164 that can be obtained in this manner.

$$\begin{array}{r|r} & 4\ \boxed{2} \\ \hline 4 & 17\ 64 \\ & 16\ \ \ \ \\ \cline{2-2} 8\ \boxed{2} & 1\ 64 \\ & 1\ 64 \\ \cline{2-2} & 0 \end{array}$$

Then $\sqrt{1764} = \underline{\underline{42}}$

When finding the square root of a decimal number, separate the digits in pairs on both sides of the decimal point, starting at the decimal point as shown below.

$$\begin{array}{rcl} 3.61 & \longrightarrow & 3.\ 61 \\ 12.321 & \longrightarrow & 12.\ 32\ 10 \\ 143.456 & \longrightarrow & 1\ 43.\ 45\ 60 \end{array}$$

Example 2

Find the value of $\sqrt{3.61}$.

$$\begin{array}{r|l} & 1.\ \boxed{9} \\ \hline 1 & 3.\ 61 \\ & 1 \\ \cline{2-2} \boxed{1\times 2=2} \rightarrow 2\ \boxed{9} & 2\ \ 61 \\ & 2\ \ 61 \\ \cline{2-2} & \ \ \ \ 00 \end{array}$$

$\therefore \sqrt{3.61} = \underline{\underline{1.9}}$

Example 3

Find the value of $\sqrt{2737}$ accurate to two decimal places.

We must find the value to three decimal places and round off to two decimal places. To find to three decimal places we must write three pairs of zeros after the decimal point.

$$
\begin{array}{rr|l}
 & & 5\ \boxed{2}.\boxed{3}\ \boxed{1}\ \boxed{6} \\
\hline
 & 5 & 27\ 37.\ 00\ 00\ 00 \\
 & & 25 \\
\hline
\boxed{5 \times 2 = 10} \rightarrow & 10\boxed{2} & 2\ 37 \\
 & & 2\ 04 \\
\hline
\boxed{52 \times 2 = 104} \rightarrow & 104\boxed{3} & 33\ 00 \\
 & & 31\ 29 \\
\hline
\boxed{523 \times 2 = 1046} \rightarrow & 1046\boxed{1} & 1\ 71\ 00 \\
 & & 1\ 04\ 61 \\
\hline
\boxed{5231 \times 2 = 10462} \rightarrow & 10462\boxed{6} & 66\ 39\ 00 \\
 & & 62\ 77\ 56 \\
\hline
 & & 3\ 61\ 44
\end{array}
$$

$\therefore \sqrt{2737} \approx \underline{\underline{52.32}}$

Example 4 Find the value of $\sqrt{3.421}$ accurate to two decimal places.

As above let us find the value to three decimal places and round off to two decimal places. For this there must be three pairs of decimal places after the decimal point.

$$
\begin{array}{r|l}
 & 1.\ \boxed{8}\ \boxed{4}\ \boxed{9} \\
\hline
1 & 3.\ 42\ 10\ 00 \\
 & 1 \\
\hline
2\boxed{8} & 2\ 42 \\
 & 2\ 24 \\
\hline
36\boxed{4} & 18\ 10 \\
 & 14\ 56 \\
\hline
368\boxed{9} & 3\ 54\ 00 \\
 & 3\ 32\ 01 \\
\hline
 & 21\ 99
\end{array}
$$

$\therefore \sqrt{3.421} \approx \underline{\underline{1.85}}$

Exercise 2.2

1. Find the square root of each of the following numbers.

(i) 676 (ii) 1024 (iii) 2209 (iv) 2809 (v) 3721

2. Find the value accurate to one decimal place.

(a)

(i) $\sqrt{8}$ (ii) $\sqrt{19}$ (iii) $\sqrt{26}$

(iv) $\sqrt{263}$ (v) $\sqrt{2745}$ (vi) $\sqrt{3630}$

(b)

(i) $\sqrt{5.4}$ (ii) $\sqrt{3.45}$ (iii) $\sqrt{15.3}$ (iv) $\sqrt{243.2}$

(v) $\sqrt{4061.3}$ (vi) $\sqrt{85.124}$ (vii) $\sqrt{0.0064}$ (iv) $\sqrt{0.000144}$

2.3 Using the square roots of numbers to solve problems

Example 1

Find the length of a side of a square of area 441 cm^2.

$$\begin{aligned}
\text{Area of the square} &= (\text{side length})^2 \\
\therefore \text{ Length of a side of the square} &= \sqrt{\text{area of the square}} \\
\text{Area of the square} &= 441 \text{ cm}^2 \\
\therefore \text{ Length of a side of the square} &= \sqrt{441} \text{ cm} \\
&= \underline{\underline{21 \text{ cm}}}
\end{aligned}$$

$$\begin{array}{r|ll}
 & 2 & 1 \\ \hline
2 & 4 & 41 \\
 & 4 & \\ \hline
41 & 0 & 41 \\
 & & \underline{41} \\
 & & 00
\end{array}$$

Example 2

324 square shaped garden tiles of area 900 cm^2 each have been placed in a square shaped courtyard so that the courtyard is completely covered with the tiles. Find the length of a side of the courtyard.

$$\begin{aligned}
\text{The number of garden tiles in one row} &= \sqrt{324} \\
&= 18 \\
\text{The length of a garden tile} &= \sqrt{900} \text{ cm} \\
&= 30 \text{ cm} \\
\text{The length of a side of the courtyard} &= 18 \times 30 \text{ cm} \\
&= 540 \text{ cm} \\
&= \underline{\underline{5.4 \text{ m}}}
\end{aligned}$$

Exercise 2.3

1. What is the length of a side of a square shaped piece of cardboard of area 1225 cm^2?

2. What is the length of a side of a square of area the same as that of a rectangle of length 27 cm and breadth 12 cm?

3. 196 children participating in a drill display have been placed such that they form an equal number of rows and columns. How many children are there in a row?

4. The surface area of a cube is 1350 cm^2. Find the length of a side of the cube.

5. A rectangular walkway has been made by placing 10 flat square shaped concrete slabs along 200 rows. If the area of the flat surface of a concrete slab is 231.04 cm^2, what is the length and the breadth of the walkway?

Miscellaneous Exercise

1. Find the value accurate to the second decimal place.

(i) $\sqrt{3669}$ (ii) $\sqrt{4302}$ (iii) $\sqrt{22.79}$ (iv) $\sqrt{0.1296}$ (v) $\sqrt{5.344}$

2. The length and breadth of a rectangular shaped plot of land are respectively 25 m and 12 m. Find to the nearest metre, the least distance that a child standing at one corner of the plot should travel to reach the diagonally opposite corner of the plot.

3. If the length of the hypotenuse of an isosceles right angled triangle is 12 cm, find the length of a remaining side. (Give the answer to two decimal places).

4. 9, 16, 25, … is a number pattern. Which term of the pattern is the number 729?