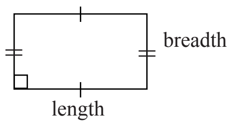
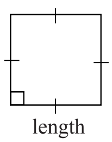
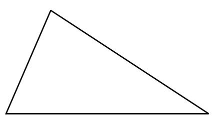
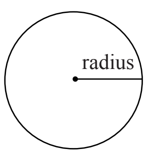
1 Perimeter

By studying this lesson you will be able to

- find the perimeter of a sector of a circle and
- solve problems related to the perimeter of plane figures containing sectors of circles.

Perimeter of Plane Figures

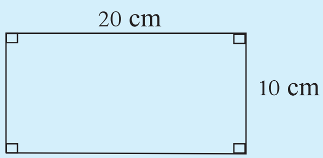
In previous grades you have learnt how to find the perimeter of plane figures such as a rectangle, a square, a triangle and a circle. Facts relating to these can be summarized as follows.

Plane Figure		Perimeter
Rectangle		2 (length + breadth)
Square		$4 \times$ length of a side
Triangle		Sum of the lengths of the three sides
Circle		$2\pi \times$ radius

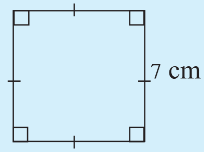
Review Exercise

1. Find the perimeter of each of the following plane figures.

(i)



(ii)



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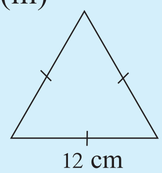
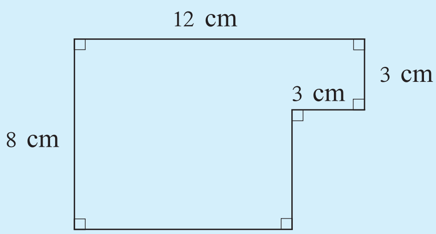
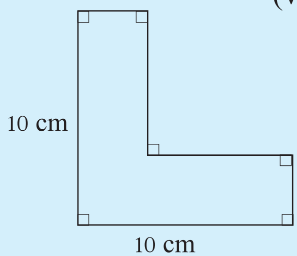


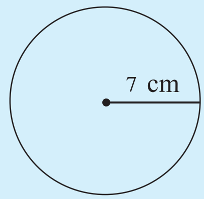
(iv)



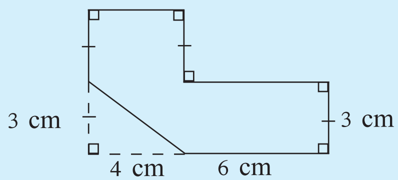
(v)



(vi)

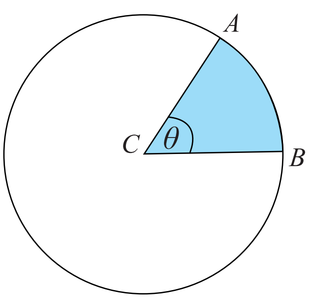


2. Find the perimeter of the following figure.



While doing the above exercise you would have recalled facts on finding the perimeter of some basic plane figures as well as of compound figures.
Now let us consider the perimeter of sectors of circles.

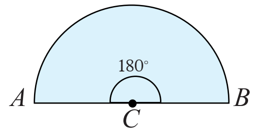
Sector of a circle



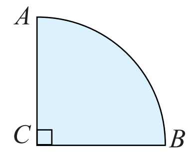
The region which is shaded in this figure is a portion of a circle with centre C which is bounded by two radii and a part of the circumference. Such a portion is called a **sector of a circle**. The angle θ ($A\hat{C}B$) which is the angle between the two radii is called the **angle at the centre**.

The angle at the centre can take any value from 0^{o} to 360^{o}.

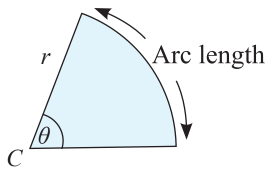
- The sector that is obtained when the angle at the centre is 180º is a semi-circle.



- The sector that is obtained when the angle at the centre is 90º is a quarter of the circle.



1.1 Finding the arc length of a sector of a circle



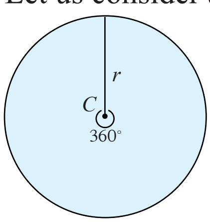
The figure illustrates a sector of a circle of radius r. Such a sector is called a sector of radius r and angle at the centre θ. Let us now consider how the arc length of such a sector of a circle is found.

Let us first find the arc length of a semi-circle of radius r.
We know that the circumference of a circle of radius r is $2\pi r$.

Therefore, due to symmetry, the arc length of a semi-circle of radius r is given by

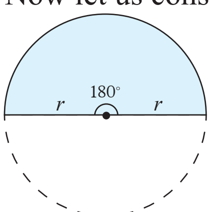
$$\frac{2\pi r}{2} = \pi r$$

Here, the reason for taking the arc length of the semi-circle to be πr, that is, the value of $2\pi r$ divided by 2, is the symmetry of the circle. The expression πr for the arc length of a semi-circle of radius r can be obtained by reasoning out in the following manner too.
Let us consider a circle and a semi-circle, both of radius r.



The angle at the centre of the circle is 360º. The arc length corresponding to this angle is the circumference of the circle which is $2\pi r$.

Now let us consider the semi-circle.



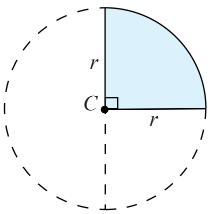
The angle at the centre of the semi-circle is 180º, which is $\frac{1}{2}$ of 360º. Therefore, the arc length of the semi-circle should be $\frac{1}{2}$ of the arc length of the circle.

That is, the arc length of the semi-circle is $\frac{1}{2} \times 2\pi r = \pi r$

Writing this in more detail,

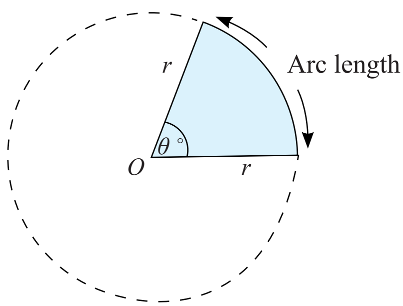
$$\text{the arc length of the semi-circle} = \frac{180}{360} \times 2\pi r = \frac{1}{2} \times 2\pi r$$
$$= \underline{\underline{\pi r}}$$

In the same manner, since the angle at the centre is 90º for a sector which is $\frac{1}{4}$ of the circle,



$$\text{arc length of a sector which is } \frac{1}{4} \text{ of the circle} = \frac{90}{360} \times 2\pi r$$
$$= \frac{1}{4} \times 2\pi r$$
$$= \underline{\underline{\frac{\pi r}{2}}}$$

Reasoning out in this manner, an expression can easily be obtained for the arc length of a sector of a circle of radius r with angle at the centre θ°.

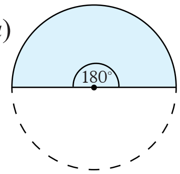
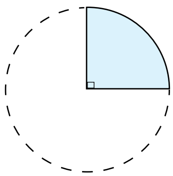
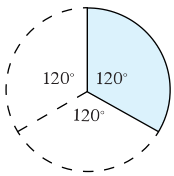
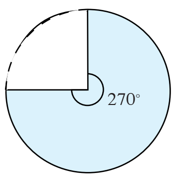
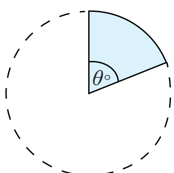


The circumference of the circle $= 2\pi r$

Arc length $= \frac{\theta}{360}$ of the circumference.

$$\boxed{\therefore \text{ Arc length} = \frac{\theta}{360} \times 2\pi r}$$

Study the following table to understand further about finding the arc length of a sector of a circle.

Sector of a circle	Length of the arc as a fraction of the circumference (According to the figure)	Angle at the centre	Angle at the centre as a fraction of the total angle at the centre
(*a*) 	$\frac{1}{2}$	180°	$\frac{180}{360} = \frac{1}{2}$
(*b*) 	$\frac{1}{4}$	90°	$\frac{90}{360} = \frac{1}{4}$
(*c*) 	$\frac{1}{3}$	120°	$\frac{120}{360} = \frac{1}{3}$
(*d*) 	$\frac{3}{4}$	270°	$\frac{270}{360} = \frac{3}{4}$
(*e*) 	$\frac{\theta}{360}$	θ°	$\frac{\theta}{360}$

Observe the 1st and 2nd columns of the table. If the length of the arc of the circle can be identified from the figure as a fraction of the circumference, then the length of the arc can easily be found. When the angle at the centre is in degrees, this fraction is

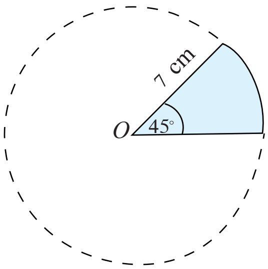
$$\frac{\text{angle at the centre}}{360}\text{, as can be observed from column 4.}$$

Accordingly, it should be clearer to you now that the length of an arc with angle at the centre θ° and radius r is $\frac{\theta}{360} \times 2\pi r$.

Now let us consider several examples.

In the following examples and exercises it is assumed that the value of π is $\frac{22}{7}$.

Example 1



(i) What fraction of the circumference of the circle in the figure is the arc length of the shaded sector?

(ii) Find this arc length.

(i) $\frac{45}{360} = \underline{\underline{\frac{1}{8}}}$

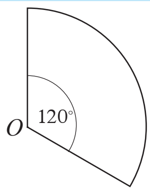
(ii) Arc length $= \frac{1}{8} \times 2\pi r$

$= \frac{1}{\cancel{8}_{\cancel{4}_2}} \times \cancel{2}^{1} \times \frac{\cancel{22}^{11}}{\cancel{7}_1} \times \cancel{7}_1$

$= 5.5$

$\therefore$ Arc length is 5.5 cm.

Example 2



The arc length of the sector in the figure is 44 cm. Find the radius of the corresponding circle. (i.e., the radius of the sector)

Let the radius be r cm.

Arc length $= \frac{120}{360}$ of $2\pi r$

$\therefore \quad 44 = \frac{120}{360} \times 2\pi r$

$44 = \frac{\cancel{120}^{1}}{\cancel{360}_3} \times 2 \times \frac{22}{7} \times r$

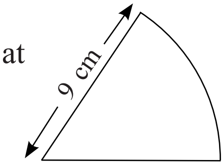
$\therefore \quad r = \frac{\cancel{44}^{\cancel{2}^{1}} \times 3 \times 7}{\cancel{2}_1 \times \cancel{22}_1}$

$r = 21$

$\therefore$ The radius of the circle is 21 cm.

Example 3

The arc length of the sector in the figure is 11 cm. Find the angle at the centre of this sector.



Let θ° be the angle at the centre.

Then,

$$\text{arc length} = \frac{\theta}{360} \times 2\pi r$$

$$\therefore \quad 11 = \frac{\theta}{360} \times 2 \times \frac{22}{7} \times 9$$

$$\theta = \frac{\cancel{11}^{\,1} \times \cancel{360}^{\,10} \times 7}{\cancel{2}_{\,1} \times \cancel{22}_{\,1} \times \cancel{9}_{\,1}}$$

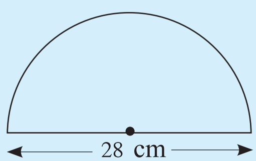
$$\theta = 70$$

Therefore, the angle at the centre is 70°.

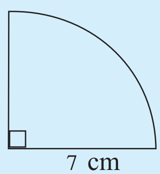
Exercise 1.1

1. Find the arc length of each of the sectors given below.

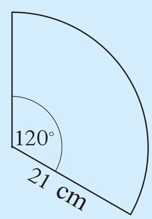
(i)



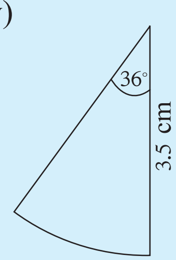
(ii)



(iii)



(iv)



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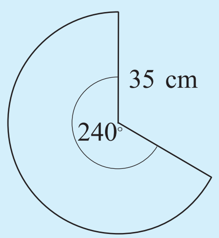
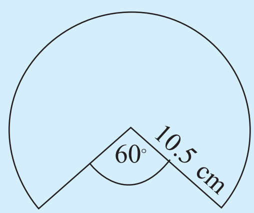


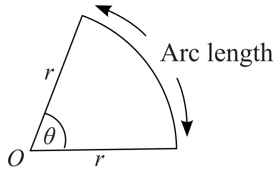
(vi)



1.2 Finding the perimeter of a sector of a circle

To obtain the perimeter of a sector of a circle, the arc length and the lengths of the two radii by which the sector is bounded should be added together.

That is,

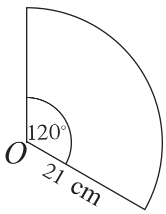


perimeter of the sector = arc length + radius + radius
= arc length + 2 × radius

Therefore,
the perimeter of a sector of a circle of radius r with angle at the centre θ°

$$\text{is } \frac{\boldsymbol{\theta}}{\mathbf{360}} \times \mathbf{2\pi} \boldsymbol{r} + \mathbf{2}\boldsymbol{r}$$

Example 1



The figure denotes a sector of a circle of radius 21 cm with angle at the centre 120°. Find its perimeter.

$$\text{Arc length} = \frac{120}{360} \times 2\pi r$$

$$= \frac{\cancel{120}^{1}}{\cancel{360}_{\cancel{3}_1}} \times 2 \times \frac{22}{\cancel{7}_1} \times \cancel{21}^{\cancel{3}^1}$$

$$= 44$$

i.e., arc length is 44 cm.

$\therefore$ The perimeter of the sector $= 44 \text{ cm} + 2 \times 21 \text{ cm}$
$= \underline{\underline{86 \text{ cm}}}$

Example 2

The perimeter of a sector which is $\frac{2}{3}$ of a circle is 260 cm. Find its radius.

Let us assume that the radius is r cm.

$$\text{Arc length} = 2\pi r \times \frac{2}{3}$$

$$= 2 \times \frac{22}{7} \times r \times \frac{2}{3}$$

$$= \frac{88r}{21}$$

$$\text{The perimeter of the sector } = \frac{88r}{21} + 2r$$

$$\therefore \frac{88r}{21} + 2r = 260$$

$$\therefore 88r + 42r = 260 \times 21$$

$$\therefore \quad 130r = 260 \times 21$$

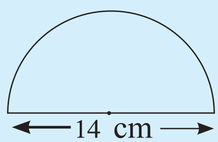
$$r = \frac{\overset{2}{\cancel{260}} \times 21}{\underset{1}{\cancel{130}}}$$

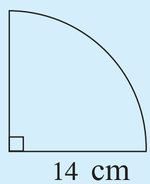
$$= 42$$

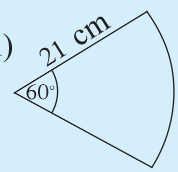
$\therefore$ The radius of the circle $= \underline{\underline{42 \text{ cm}}}$

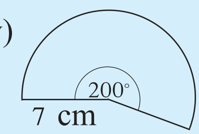
Exercise 1.2

1. Find the arc length of each of the sectors given below.

(i) 

(ii) 

(iii) 

(iv) 

2. Find the radius of the sector

(i) when the angle at the centre is 180° and the perimeter is 180 cm.

(ii) when the angle at the centre is 120° and the perimeter is 43 cm.

3. Find the angle at the centre of the sector,

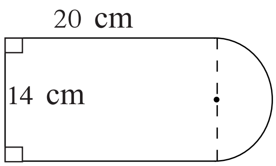
(i) when the perimeter is 64 cm and radius is 21 cm.

(ii) when the perimeter is 53 cm and the radius is 21 cm.

1.3 The perimeter of compound plane figures containing sectors of circles

Let us see how the perimeter of compound plane figures containing sectors of circles is found by considering several examples.

Example 1



The figure illustrates how a semi-circle has been joined to a rectangle of length 20 cm and breadth 14 cm such that the breadth of the rectangle is the diameter of the semi-circle. Find the perimeter of this figure.

Since the arc length of a semi-circle of radius r is $\frac{1}{2}\times 2\pi r$,

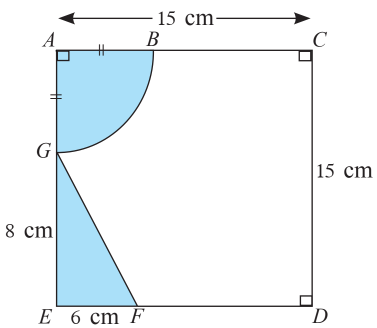
the arc length of the semi-circle of radius 7 cm $= \frac{1}{2}\times 2\times\frac{22}{7}\times 7$ cm

$= 22$ cm

$\therefore$ The perimeter of the figure $= 20 + 20 + 14 + 22$ cm

$= 76$ cm

Example 2



The figure represents a square lamina of side length 15 cm. If the shaded sector (AGB) and the triangular portion (GEF) are cut out, find the perimeter of the portion ($BCDFG$) that is left over.

Perimeter of $BCDFG$ is $BC + CD + DF + FG +$ arc length GB

First let us find the length of FG.

Let us apply Pythogoras' theorem to the right angled triangle GEF.

$$FG^2 = 8^2 + 6^2$$
$$= 64 + 36$$
$$= 100$$
$$\therefore\ FG = \sqrt{100}$$
$$= 10 \text{ cm}$$

Next let us find the arc length GB.
Since the angle at the centre of the sector ABG is 90°,

$$GB = \frac{\overset{1}{\cancel{90}}}{\underset{\cancel{4}\ \cancel{2}\ 1}{\cancel{360}}} \times \overset{1}{\cancel{2}} \times \frac{\overset{11}{\cancel{22}}}{\underset{1}{\cancel{7}}} \times \overset{1}{\cancel{7}}$$

$$GB = 11\text{ cm}$$

Finally let us find the length of BC and DF.

$BC = 15 - 7$ cm
$= 8$ cm

$DF = 15 - 6$ cm
$= 9$ cm

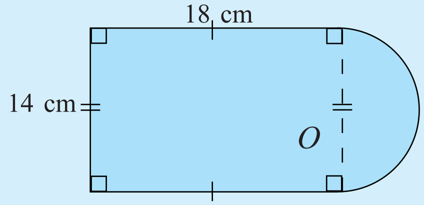
Perimeter of $BCDFG$ $= BC + CD + DF + FG +$ arc length GB
$= 8 + 15 + 9 + 10 + 11$ cm
$= 53$ cm

$\therefore$ The perimeter of the remaining portion of the lamina is 53 cm

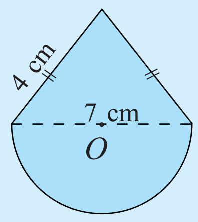
Exercise 1.3

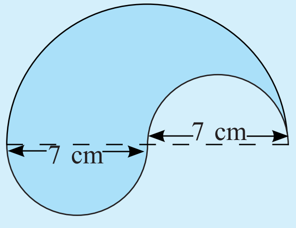
1. Find the perimeter of each of the following plane figures. O denotes the centre of the circle of the sector in the figure.

(i)



(ii)





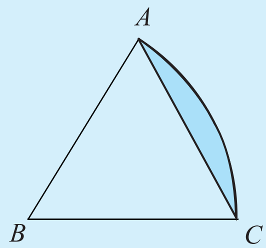
2. A semi-circular portion of diameter 7 cm has been cut out from a semi-circular lamina of radius 7 cm and welded to the lamina again as shown in the figure.

(i) Find the arc length of the sector of the circle of radius 7 cm.

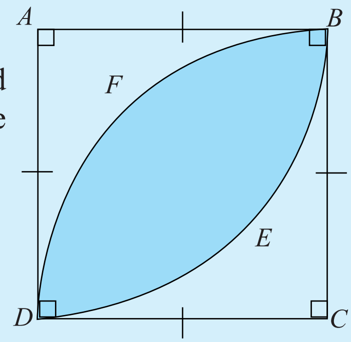
(ii) Find the arc length of the sector of the circle of diameter 7 cm

(iii) Find the perimeter of the shaded region.

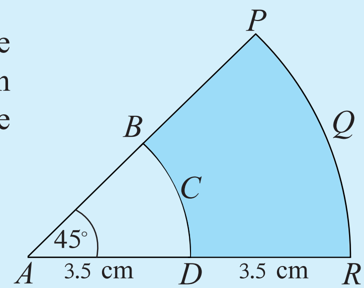
3. The figure illustrates how an equilateral triangle of side length 7 cm is drawn within a sector of a circle of radius the length of a side of the triangle.
(i) Find the arc length of the sector of the circle.
(ii) Find the perimeter of the shaded region.



4. The figure shows two sectors of circles *ABED* and *CDFB*. If *AB* = 10.5 cm, find the perimeter of the shaded region using the given data.

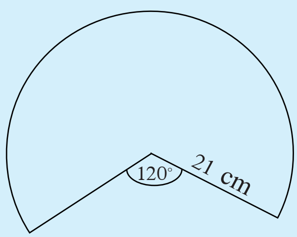


5. The figure shows two sectors of circles of centre *A* and radius *AD* and *AR* respectively. How much greater is the perimeter of the sector *APQR* than the perimeter of the shaded region?



Miscellaneous Exercise

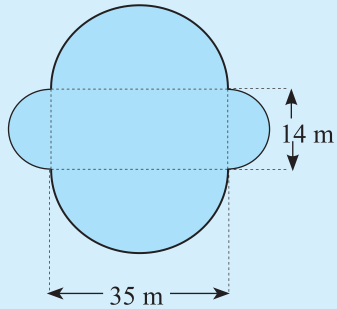
1. A sector of a circle as shown in the figure, with angle at the centre equal to 120° has been cut out from a circular lamina of radius 21 cm. Show that the perimeter of the remaining portion of the lamina is 130 cm.



2. The figure depicts a pond having semi-circular boundaries. A protective fence along the boundary of the pond has been planned.

(i) Find the perimeter of the pond.

(ii) It has been estimated that the cost of constructing 1 m of the fence is Rs. 5000. How much will it cost to complete the whole fence?

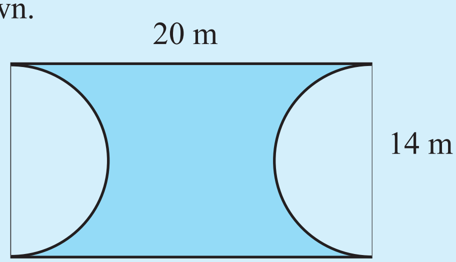


3. The figure depicts a rectangular plot of land with two semi-circular flower beds at the two ends. The shaded region is a lawn.

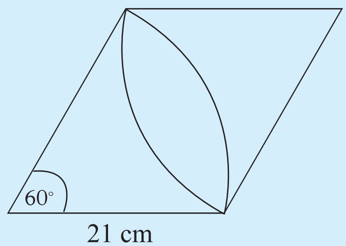
(i) Find the perimeter of the lawn.

It has been decided to lay tiles around the lawn. Each tile is of length 25 cm.

(ii) Find the minimum number of tiles that are required.



4. A portion of a grill to be fixed to a window has been made by combining two equal sectors of circles as shown in the figure. The person making the grill states that based on the given data, a wire of length 128 cm is required for it. Show with reasons that his statement is correct.



Summary

- The arc length of a sector of a circle of radius r with angle at the centre $\theta°$ is given by $\frac{\theta}{360} \times 2\pi r$

- The perimeter of the sector is given by $\frac{\theta}{360} \times 2\pi r + 2r$