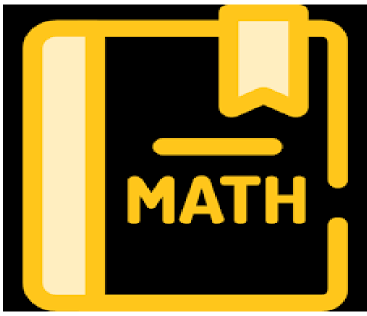
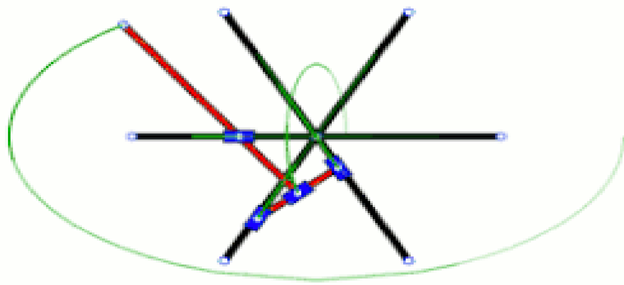
Grade 9 Mathematics

Note





lesson 14

Loci and constructions

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By studying this lesson you will be able to,

- Identify four basic loci,
- Construct a line perpendicular to a given line,
- Costruct the perpendicular bisector of a straight-line segment,
- Construct and copy angles,
- Solve problems related to loci and constructions.

Following materials are needed for the activities,

- Mathematical instrument box.
- Colour pencils.
- A piece of twine rope with 1 m.
- Pieces of ekels.
- A graph paper.

Basic Loci

A set of points satisfying one or more conditions is known as a locus.

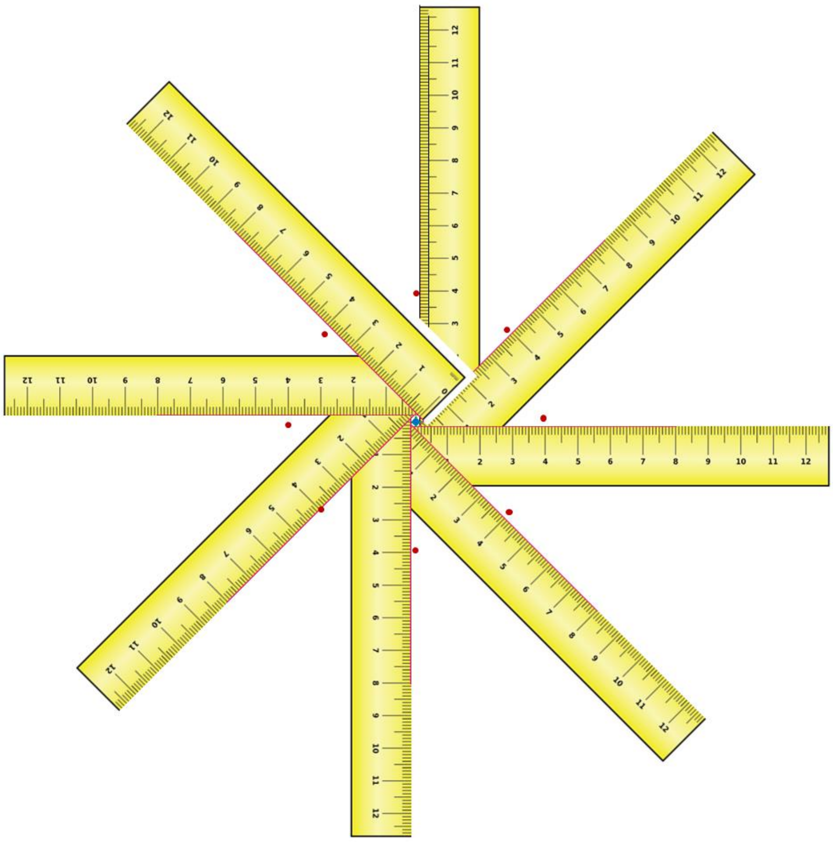
There are four basic loci.

1. The locus of points which are at a constant distance from a fixed point.

Identify the locus of points which are at a distance of 4 cm from the fixed point of O.

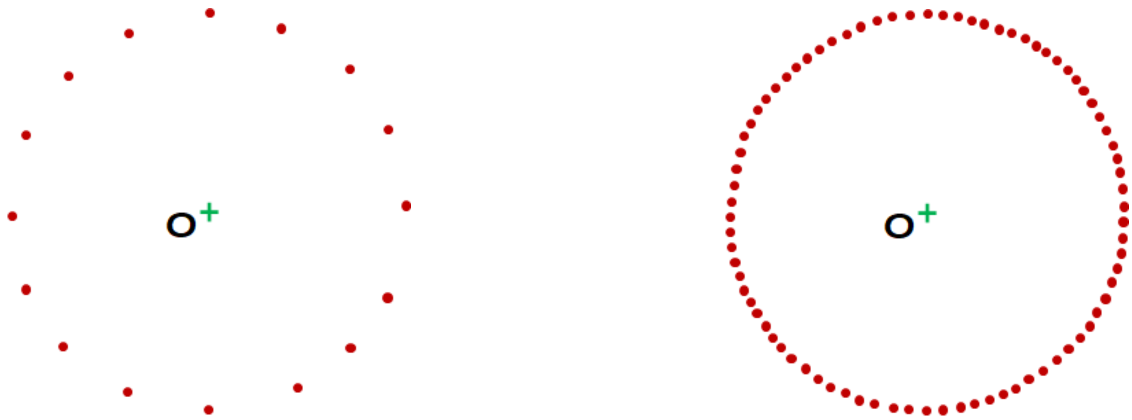
Step **1**

Mark the points which are at a distance of 4 cm from the fixed point.



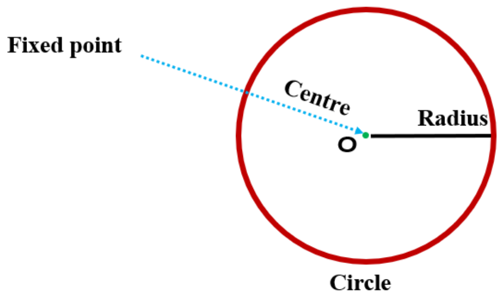
Step **2**

Mark the points as much as possible as follow..



The set of points which are at a distance of 4 cm are lie on a circle.

The locus of a points on a plane which are at a constant distance from a fixed point is a circle.



2 The locus of points which are equidistant from two fixed points

Identify the locus of points which are equidistant from two points A and B.

Step 1

Draw a line segment AB as 10 cm.

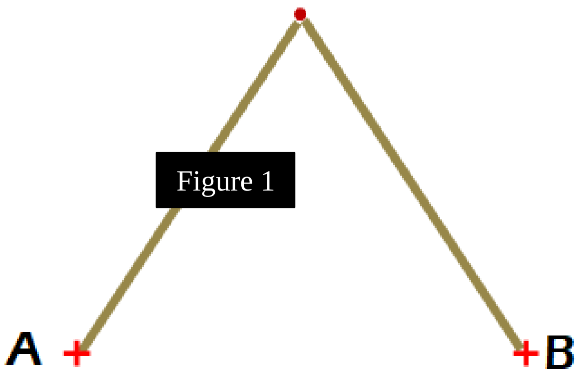


Figure 1

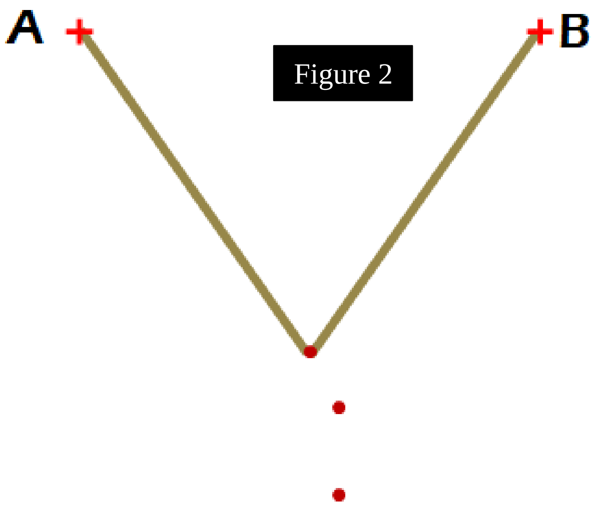
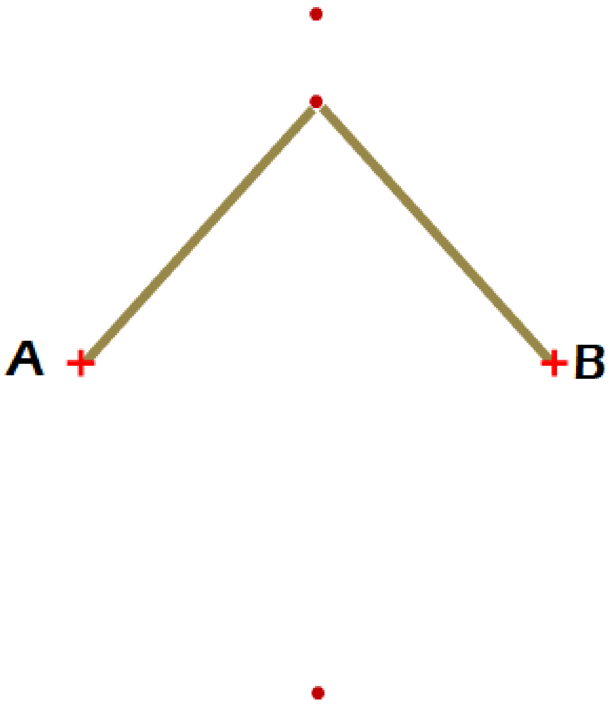
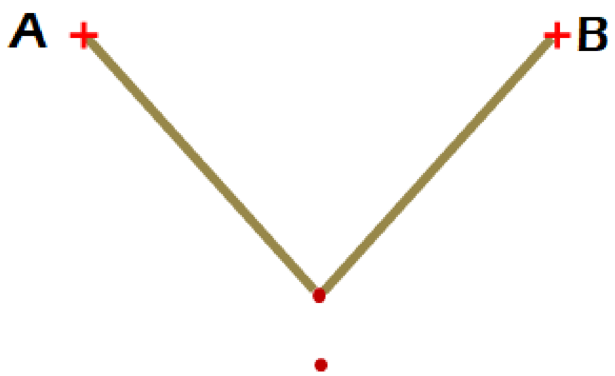
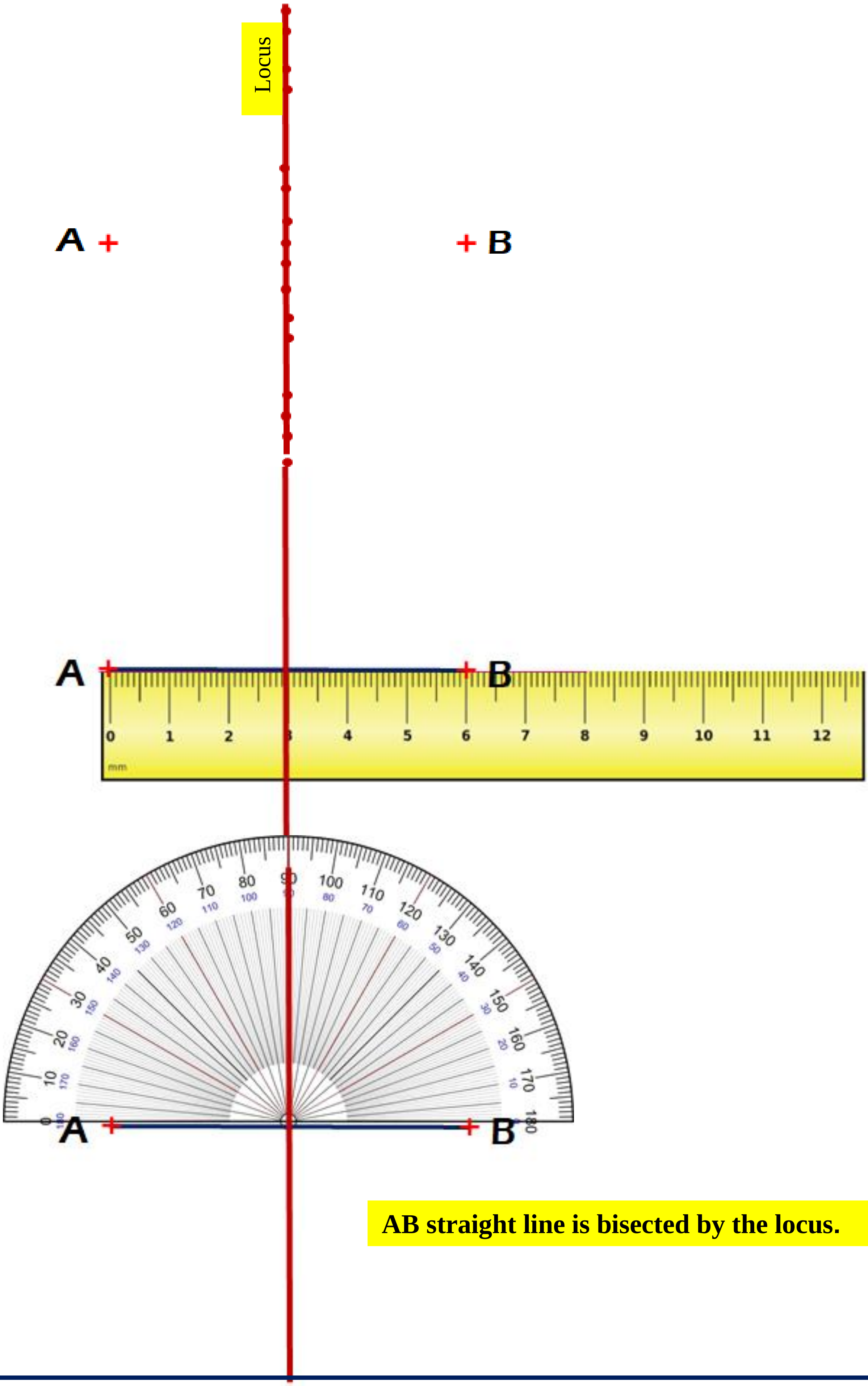


Figure 2





Do the above by using some ekel pieces or any other thing. Mark the points equidistant form A and B.



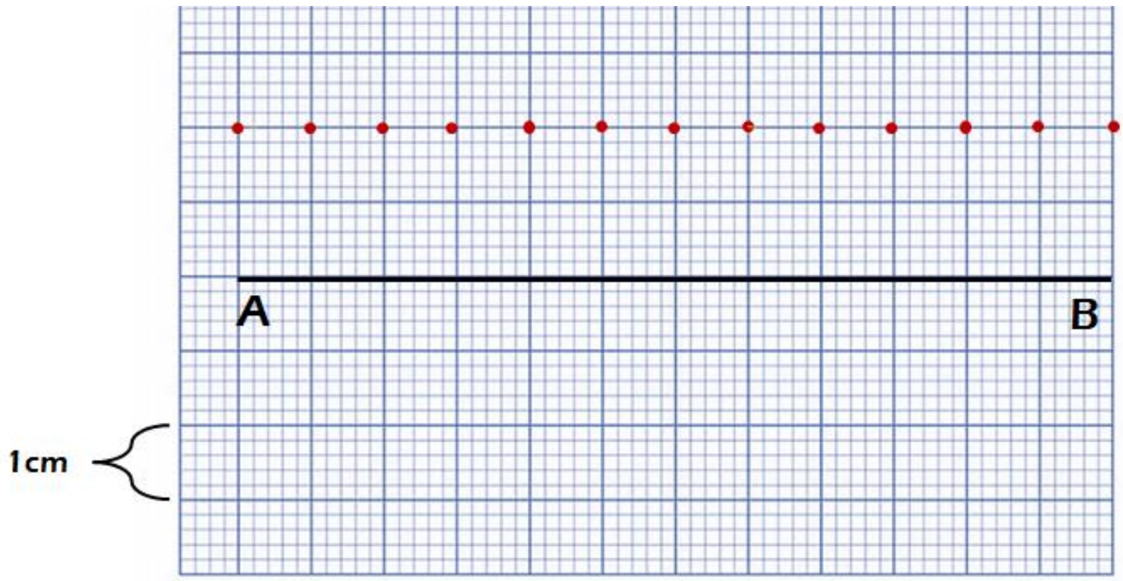
AB straight line is bisected by the locus.

The locus is perpendicular to AB. The locus is the perpendicular bisector of AB.

The locus points which are equidistant from two given points is the perpendicular bisector of the line joining the two points.

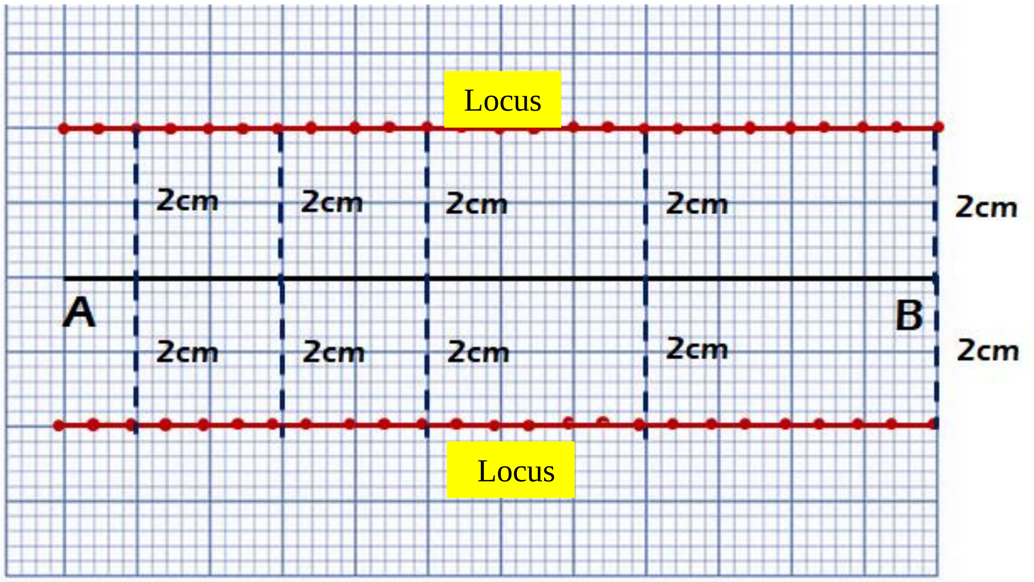
3 .The locus of points which are at a constant distance from a fixed line.

Identify the locus of the points at a distance of 2cm from the straight line of AB.



Step 1

Draw the line segment of Ab as shown below.



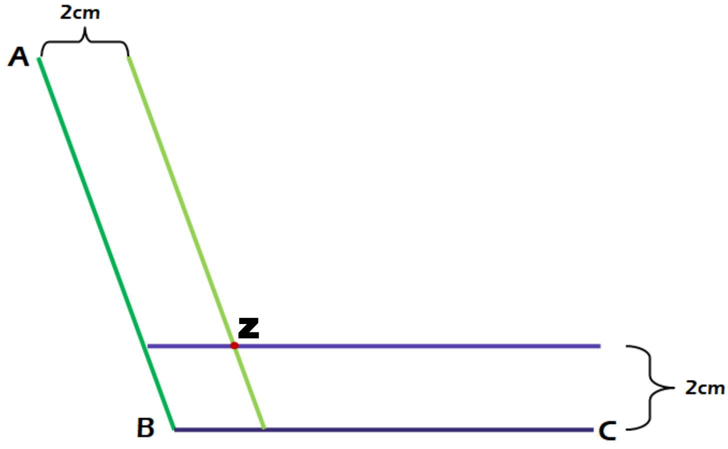
Step 2

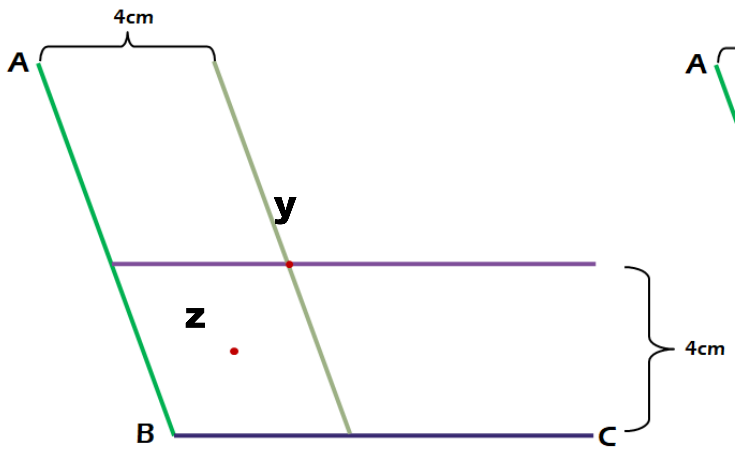
Mark the points as much as possible such that points lie above and below to AB.

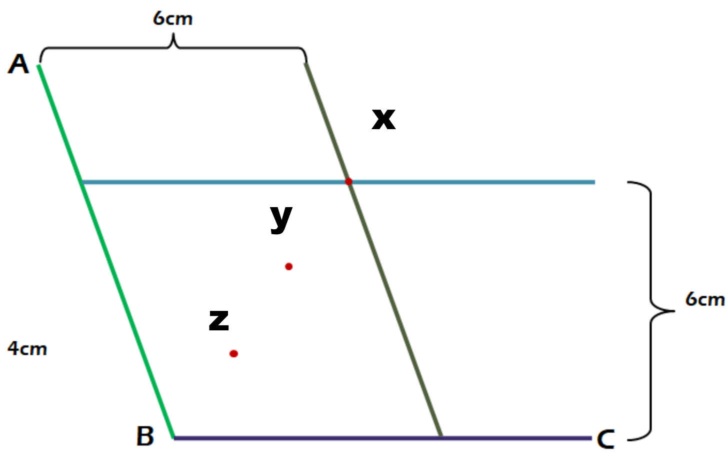
The locus of points which are at a constant distance from a straight Line are the two straight lines parallel to it at the given constant distance from it,on either side of it.

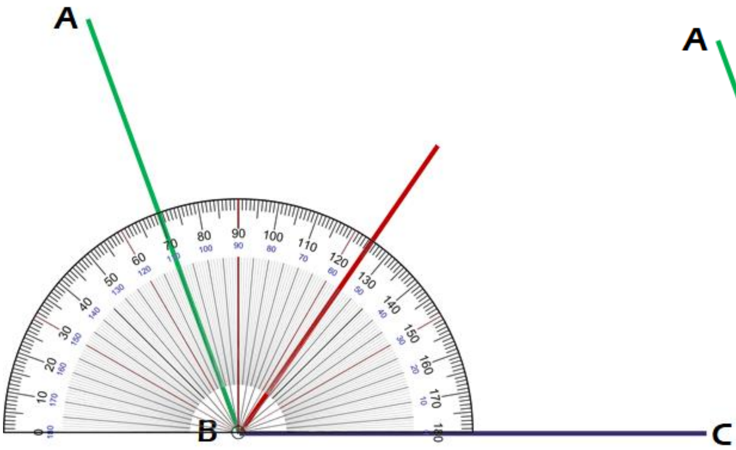
4. The locus of the points equidistant from two interseting straight line

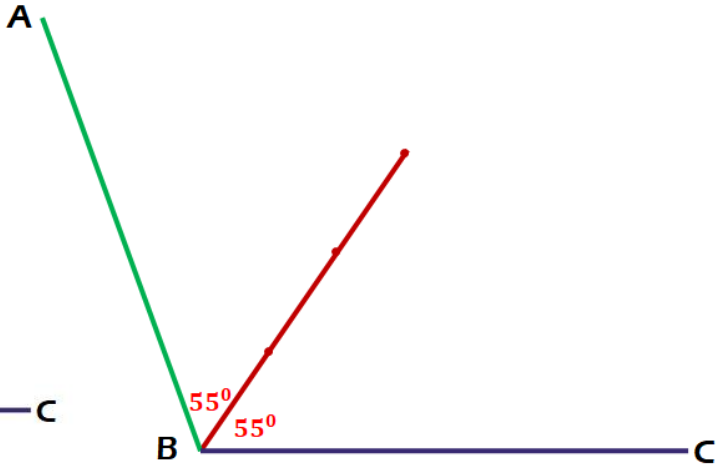
Identify the locus of the points equidistant from two intersecting straight lines of *AB and* BC.











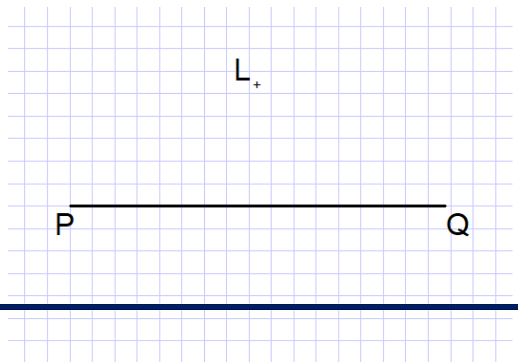
The angle of ABC divides in to two equal angles and that the distance from any two point on the line of bisector to the lines AB and BC are equal.

The locus of points equidistant from two intersecting straight lines is the angle bisector of the angles formed by the intersection of the two lines.

Constructing lines perpendicular to a given straight line.

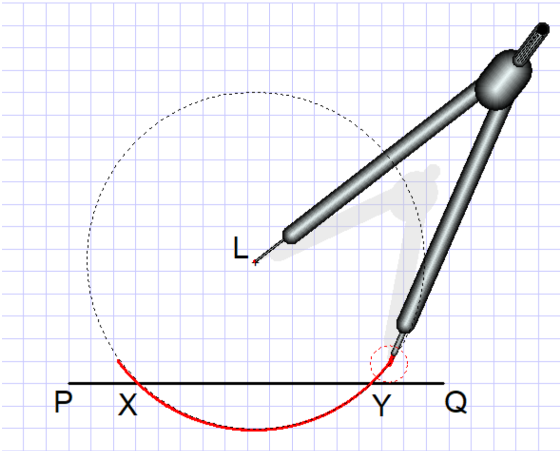
1. Constructing a line perpendicular to a given line from an external point.

Construct a line perpendicular to PQ from the external point P.



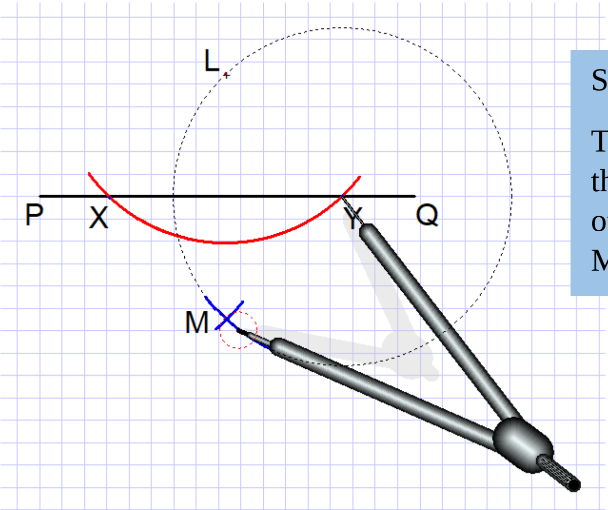
Step 1

Draw a straight line segment in your exersice book and name it PQ.Mark a point external to PQ and name it L.



Step 2

Taking a length which is more than the distance from L to PQ as the radius and L as the centre,drawn arc such that it intersects the line PQ.Name points of intersection X and Y.

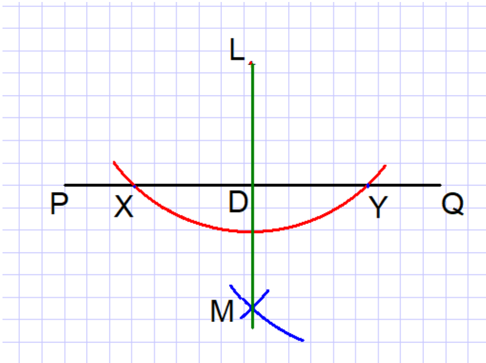


Step 3

Taking each of the points X and Y as the centre and using the same radius ,draw two arcs such that they intersect each other as shown in the figure.name the point of intersection M.

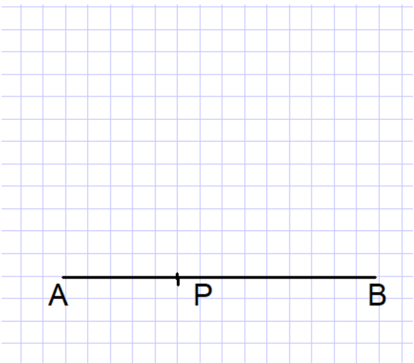
Step 4

Join the points L aand M and name the point at which LM intersects PQ as D.Measure and write the magnitude of $L\widehat{D}P$.



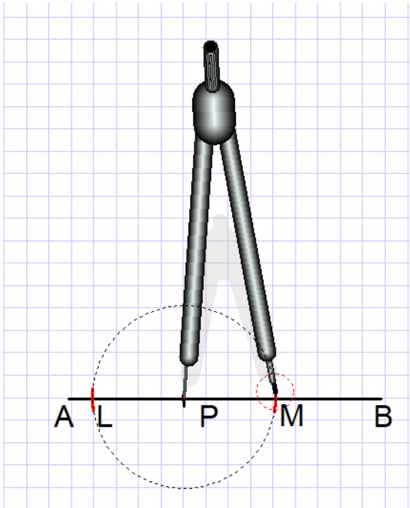
2Constructing a line perpendicular to a given line through a point on the line.

Construct a line perpendicular to AB through the the point P on AB.



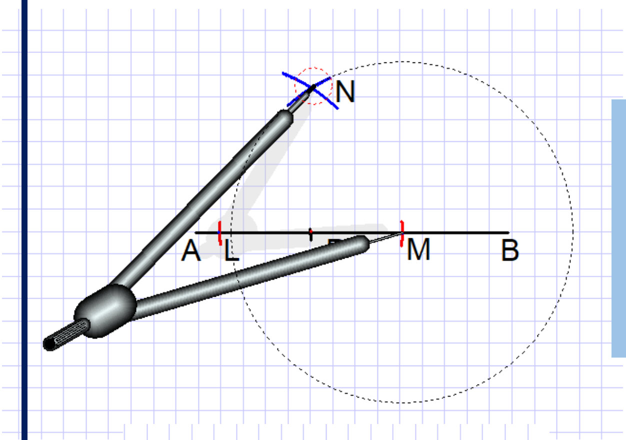
Step 1

Draw a straight line and name it Ab.Mark a point on it and name it P.



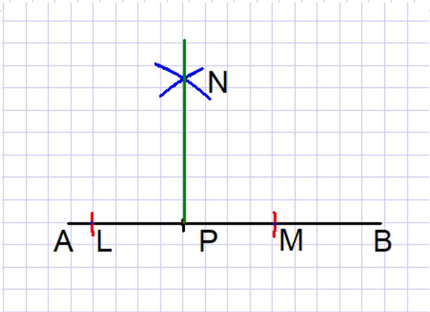
Step 2

Taking a length less than the length of PA as the radius,and taking P as the centre,draw two arcs using the pair of compasses such that they intersect the line segments AB PB.Name the two points of intersection L and M.



Step 3

Taking a length greater than the one taken step 2 as the radius,and taking L and M as the centres,draw two arcs such that they intersect each other as shown in the figure.Name the point of intersection N.



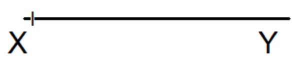
Step 4

Join NP,measure the magnitude of the angle

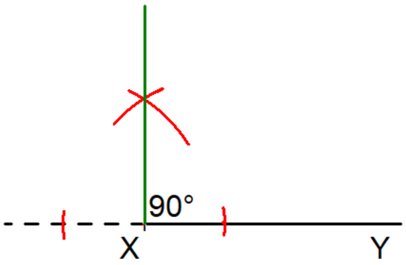
$N\hat{P}A$ and write its value.

3Constructing a line perpendicular to a given straight line segment through an end point.

Costruct a line perpendicular to the line segment XY through the point X.



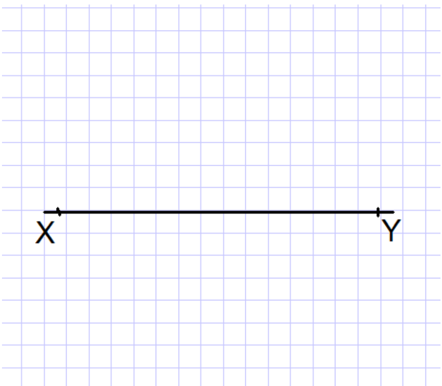
Draw a line perpendicular to the line segment XY through the point X.



Produce the line YX and do this construction using the method identified above.

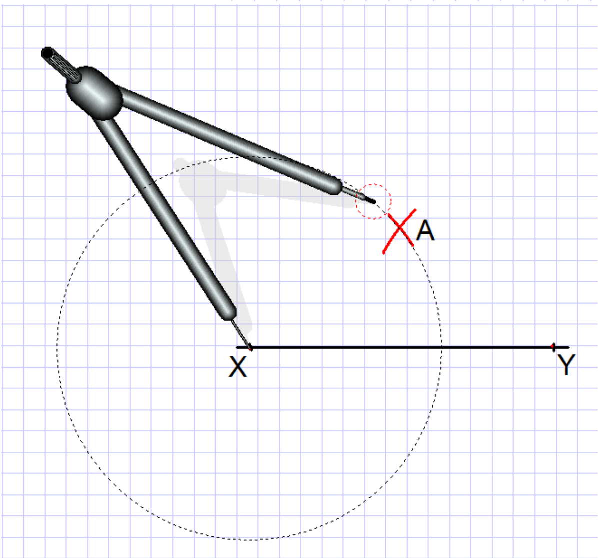
4. Constructing the perpendicular bisector of a straight line segment.

Constuct the perpendicular bisector of AB line segments.



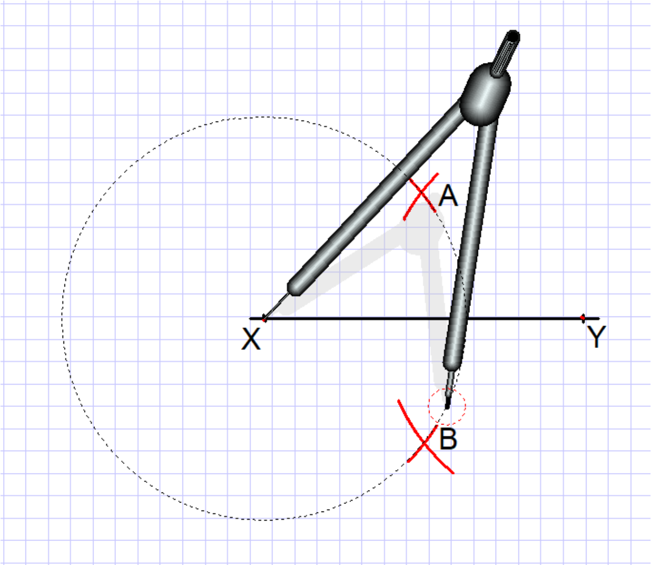
Step 1

Taking a length greater than half of XY as the radius,and without changing it,draw two arcs with X and Y as the centres,such that they intersect each other.Name the point of intersection P.

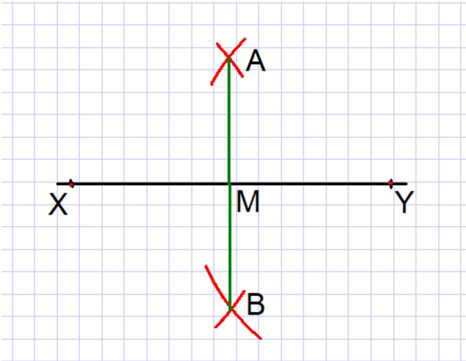


Step 2

As done above,taking X and Y as the centres,draw two other arcs such that they intersect each other on the side of XY opposite to the side on which P is located.Name the point of intersection Q.



It is not necessary to use the same radius in the above two steps.

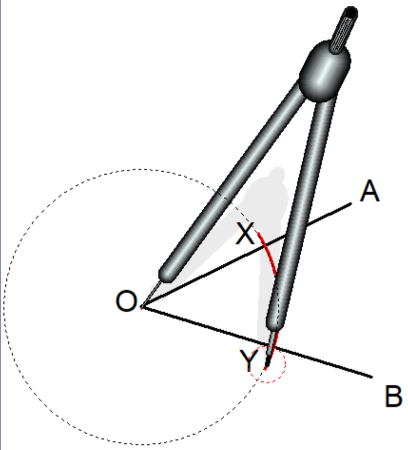


Step 3

Join PQ and name the point at which PQ intersects XY as M.Measure XM and My and magnitude of XMP angle.

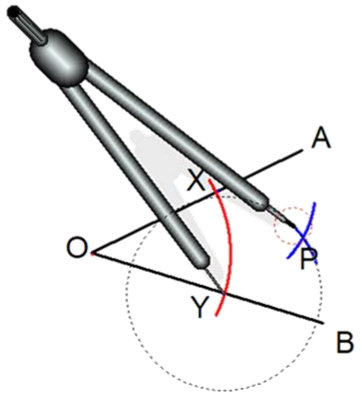
Constructions related to angles.

1. Constructing the angle bisector



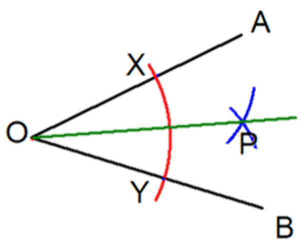
Step 1

Draw an arc with O as the centre such that it intersects the arms OA and OB.Name the points of intersection X and Y.



Step 2

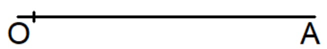
Using a pair of compasses and taking a suitable radius,construct two arcs with X and Y as the centres such that they intersect each other as shown in the figure.Name the point of intersection P.



Step 3

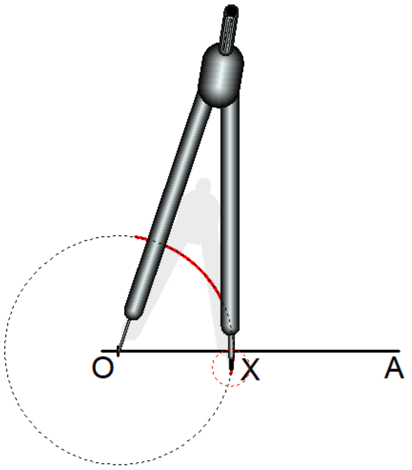
Join OP.Measure $A\hat{O}P$ and $B\hat{O}P$ and check whether they are equal.

2.Constructing angle of 60°



Step 1

Name a straight line segment in your exercise book and name it OA.

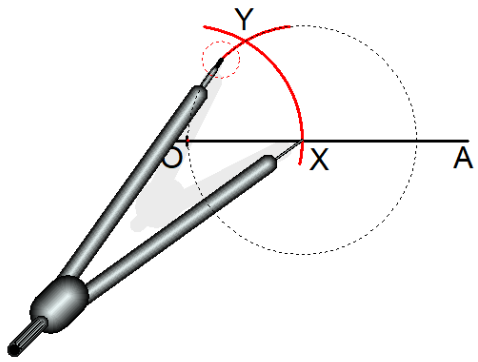


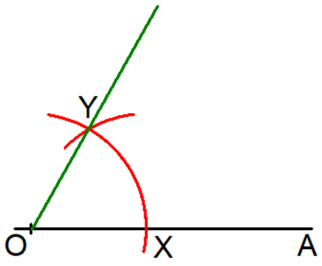
Step 2

Taking O as the centre,construct an arc such that it intersects OA as shown in the figure.Name the point of intersection X.

Step 3

Without changing the length of the radius ,and taking X as the centre,draw another arc using the pair of compasses ,such that it intersects the first arc.Name the point of intersection Y.

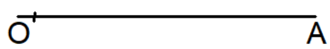




Step 4

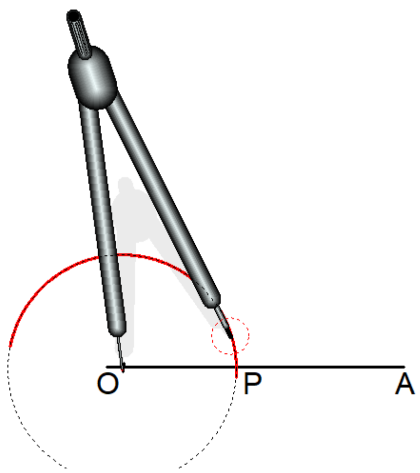
Join the points O and Y and produce it as required.Measure $A\hat{O}Y$ and check whether it is 60°

3.Constructing angle of 120°



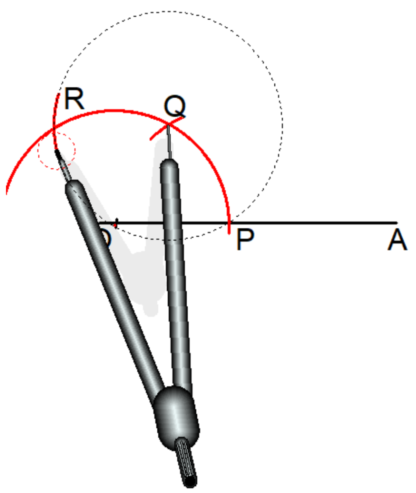
Step 1

Contruct a straightnline segment and name it OA.



Step 2

Taking O as the centre,construct an arc such that it intersects OA as shown in the figure.Name the point of intersection P.

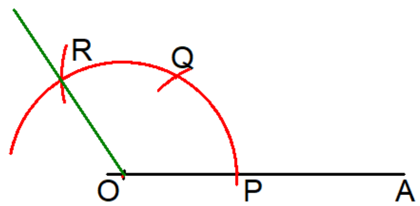


Step 3

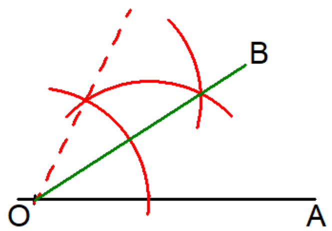
Without changing the length of the radius,and taking P as the centre ,draw a small arc using the pair of compasses,such that it intersects the first arc shown in the figure,and name that point of intersection Q.Now,without changing the radius,take Q as the centre and draw another small arc such that it too intersects the first arc and name that point of intersection R.

Step 4

Join QR and produce it as required.Measure and check the magnitude of $A\hat{O}R$.

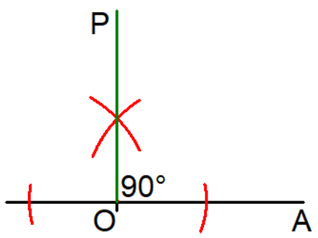


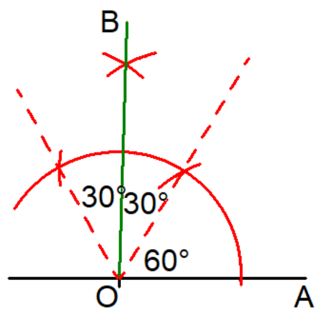
4.Constructing angle of 30°



Construct an angle 60^{0} and construct its bisector.Then $A\hat{O}B = 30^{0}$.

5.Constructing angle of 90°





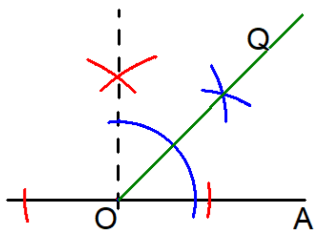
Method 1

At O,construct a line perpendicular to the line segment /ao.Then $A\hat{O}P = 90^{0}$.

Method II

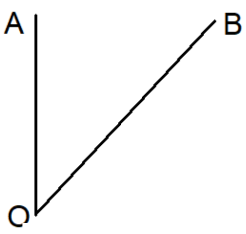
Construct an angle of120^{0} and bisect one 60^{0} angle.Then $A\hat{O}B = 90^{0}$.

6.Constructing angle of 45°.



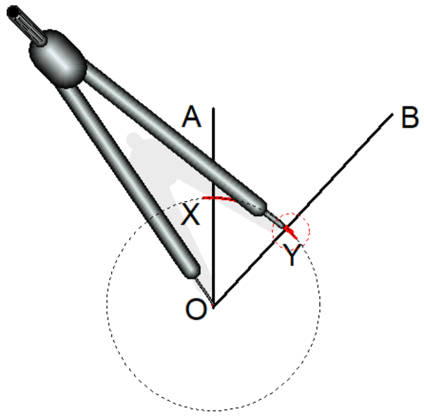
Construct an angle of $90^{0}$and bisect it.Then $A\hat{O}Q = 45^{0}$.

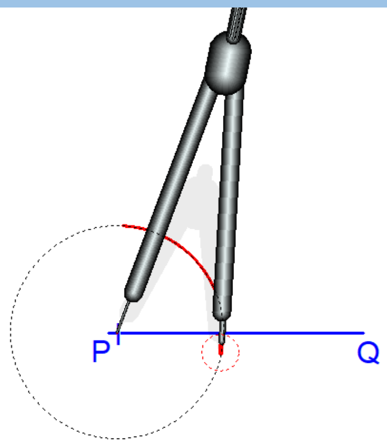
6. Copying a given angle.



Step 1

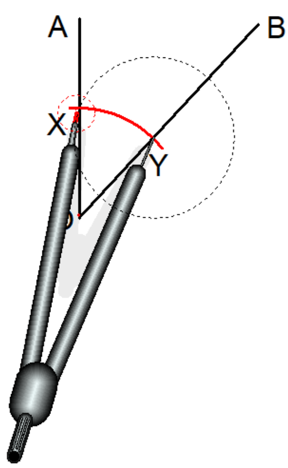
Draw any angle and name it $A\hat{O}B$ .Draw the arm PQ on which $A\hat{O}B$ needs to be copied.

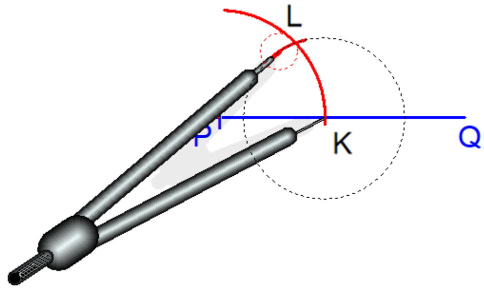




Step 2

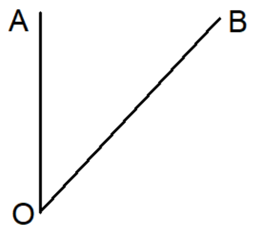
Taking O as the centre ,draw an arc as shown in the figure such that it intersects the arms OA and OB,and name the points of intersection X and Y.Using the same radius and taking P as the centre,draw an arc longer than the previous arc such that it intersects PQ.

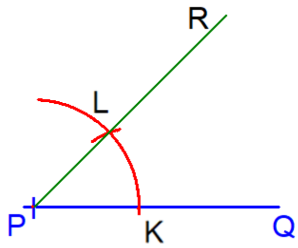




Step 3

Taking XY as the length of the radius and K as the centre,using the pair of compasses,construct a small arc such that intersects the initial arc and name the point of intersection L.

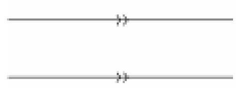
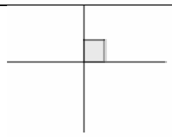
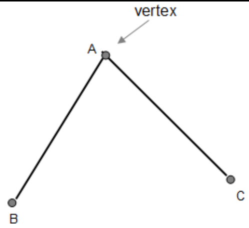
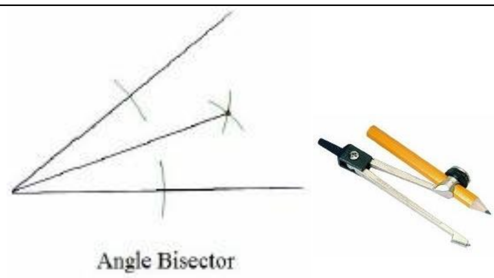


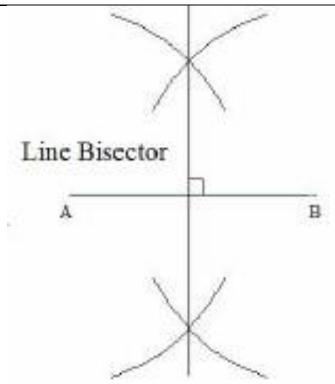
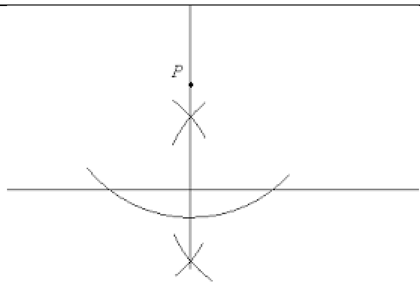


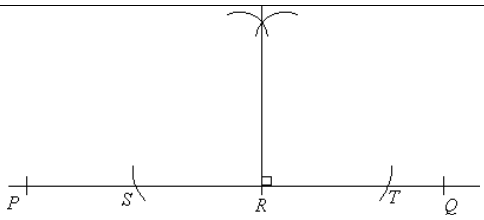
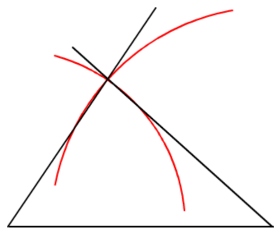
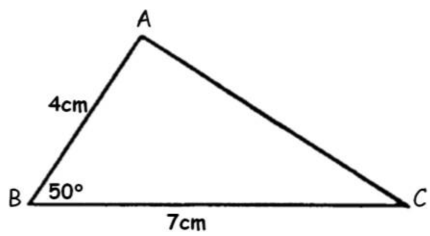
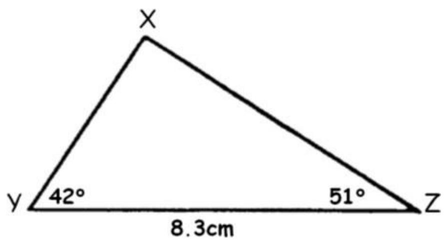
Step 4

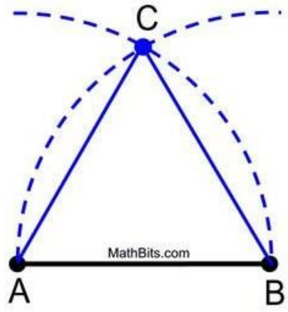
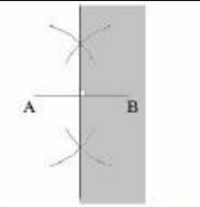
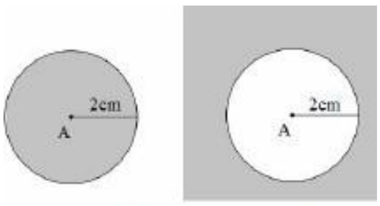
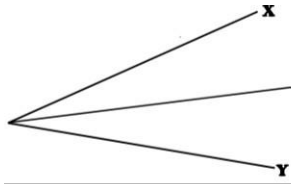
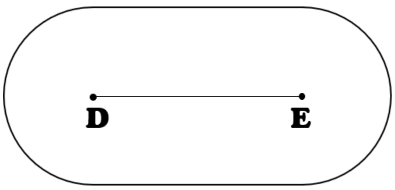
Join PL and produce it as required.Using a protactor(or any other method),check whether $A\hat{O}B$ and $Q\hat{P}L$ are equal.

Topic: Loci and Constructions

Topic/Skill	**Definition/Tips**	**Example**
1. Parallel	Parallel lines never meet.	
2. Perpendicular	Perpendicular lines are at right angles. There is a 90° angle between them.	
3. Vertex	A corner or a point where two lines meet.	
4. Angle Bisector	**Angle Bisector: Cuts the angle in half**. 1. Place the sharp end of a pair of compasses on the vertex. 2. Draw an arc, marking a point on each line. 3. Without changing the compass put the compass on each point and mark a centre point where two arcs cross over. 4. Use a ruler to draw a line through the vertex and centre point.	

5. Perpendicular Bisector	**Perpendicular Bisector: Cuts a line in half and at right angles**. 1. Put the sharp point of a pair of compasses on A. 2. Open the compass over half way on the line. 3. Draw an arc above and below the line. 4. Without changing the compass, repeat from point B. 5. Draw a straight line through the two intersecting arcs.	
6. Perpendicular from an External Point	The **perpendicular distance** from a point to a line is the **shortest distance** to that line. 1. Put the sharp point of a pair of compasses on the point. 2. Draw an arc that crosses the line twice. 3. Place the sharp point of the compass on one of these points, open over half way and draw an arc above and below the line. 4. Repeat from the other point on the line.	

	5. Draw a straight line through the two intersecting arcs.	
7. Perpendicular from a Point on a Line	Given line PQ and point R on the line: 1. Put the sharp point of a pair of compasses on point R. 2. Draw two arcs either side of the point of equal width (giving points S and T) 3. Place the compass on point S, open over halfway and draw an arc above the line. 4. Repeat from the other arc on the line (point T). 5. Draw a straight line from the intersecting arcs to the original point on the line.	
8. Constructing Triangles (Side, Side, Side)	1. Draw the base of the triangle using a ruler. 2. Open a pair of compasses to the width of one side of the triangle. 3. Place the point on one end of the line and draw an arc. 4. Repeat for the other side of the triangle at the other end of the line. 5. Using a ruler, draw lines connecting the ends of the base of the triangle to the point where the arcs intersect.	
9. Constructing Triangles (Side, Angle, Side)	1. Draw the base of the triangle using a ruler. 2. Measure the angle required using a protractor and mark this angle. 3. Remove the protractor and draw a line of the exact length required in line with the angle mark drawn. 4. Connect the end of this line to the other end of the base of the triangle.	
10. Constructing Triangles (Angle, Side, Angle)	1. Draw the base of the triangle using a ruler. 2. Measure one of the angles required using a protractor and mark this angle. 3. Draw a straight line through this point from the same point on the base of the triangle. 4. Repeat this for the other angle on the other end of the base of the triangle.	

11. Constructing an Equilateral Triangle (also makes a 60° angle)	1. Draw the base of the triangle using a ruler. 2. Open the pair of compasses to the exact length of the side of the triangle. 3. Place the sharp point on one end of the line and draw an arc. 4. Repeat this from the other end of the line. 5. Using a ruler, draw lines connecting the ends of the base of the triangle to the point where the arcs intersect.	
12. Loci and Regions	A **locus** is a **path of points that follow a rule**. For the locus of points **closer to B than A**, create a **perpendicular bisector** between A and B and shade the side closer to B. For the locus of points **equidistant from A**, use a compass to draw a **circle**, centre A. For the locus of points **equidistant to line X and line Y**, create an **angle bisector**. For the locus of points a set **distance from a line**, create **two semi-circles** at either end joined by **two parallel lines**.	 Points Closer to B than A  Points less than 2cm from A Points more than 2cm from A  
13. Equidistant	A point is equidistant from a set of objects if the **distances between that point and each of the objects is the same**.