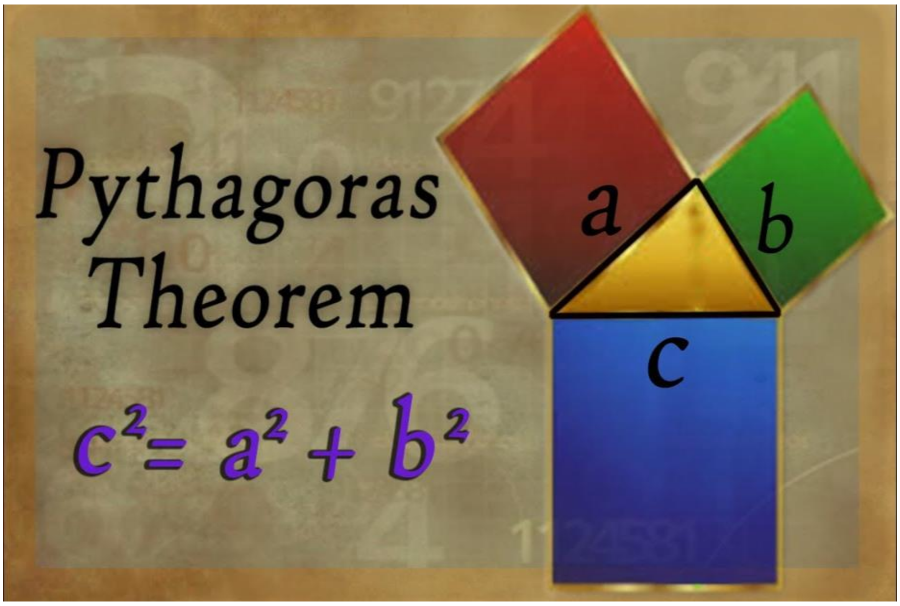
Grade 9

Mathematics

Unit 19

Pythagorean Relation

Reading material



Pythagorean Relation

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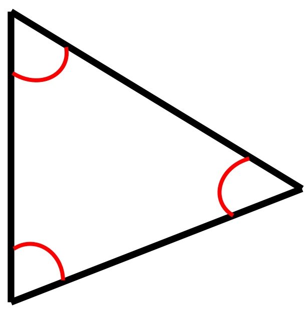
By studying this lesson, you will be able to;

- ❑ Develop the Pythagorean relation by means of a right-angled triangle,
- ❑ Solve problems related to the Pythagorean relation.

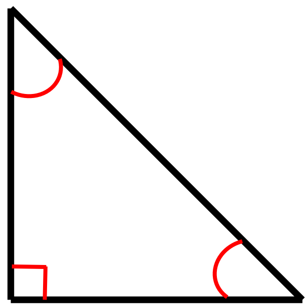
19.1. Classifying triangles according to the magnitude of interior angles.

The triangles can be classified in to three types according to the magnitude of the largest angle in the triangle.



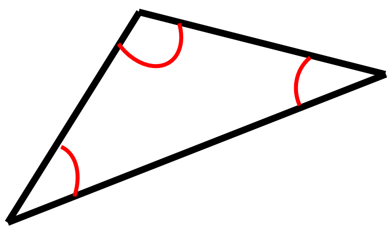
Acute angled triangle

If the largest angle in a triangle is an acute angle, that triangle is known as an acute angled triangle.



Right angled triangle

If the largest angle in a triangle is a right angle, that triangle is known as a right-angled triangle.



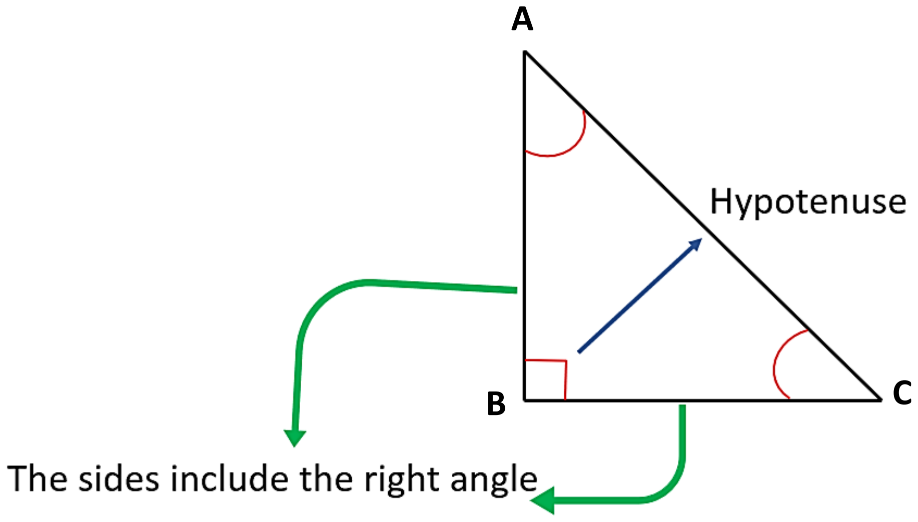
Obtuse angled triangle

If the largest angle in a triangle is an obtuse angle, that triangle is known as an obtuse angled triangle.

19.2. Identifying the characteristics of a right-angled triangle.

The side opposite to the right angle (the longest side) of any right-angled triangle is known as the **hypotenuse.** The other two sides which contain the right angle is known as the **sides include the right angle.**

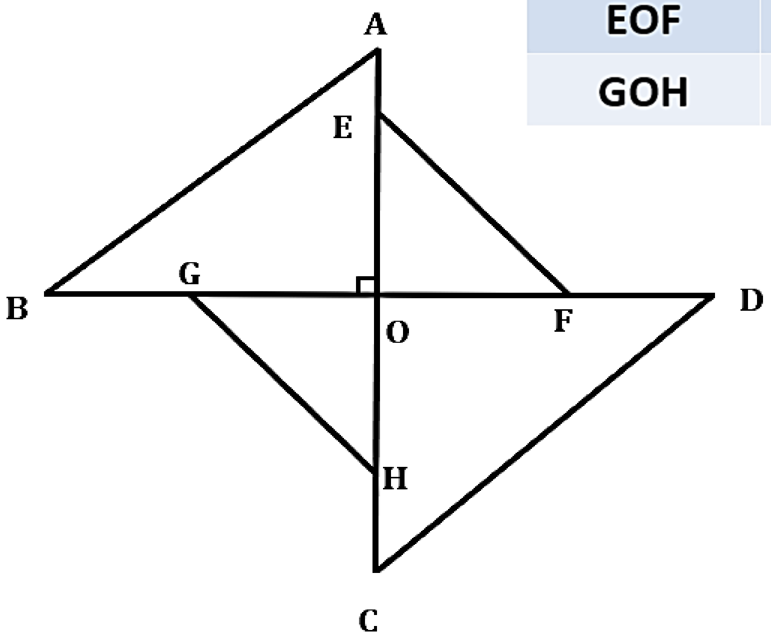
Consider the triangle **ABC** given in the figure.



Activity 1

Complete the table given below. By identifying all of the right-angled triangles given in the figure.

Triangle	Hypotenuse	The sides include the right angle
AOB		
COD		
EOF		
GOH		



19.3. Pythagorean relation.

Once upon a time ....,

There was a Greek mathematician called Pythagoras.

He has identified a special geometrical relationship among the length of sides of a right-angled triangle.

We call that relationship as Pythagorean relation or Pythagoras` theorem.

Pythagorean relation

The area of the square drawn on the hypotenuse of a right-angled triangle is equal to the sum of the areas of the squares drawn on the remaining two sides.

we can write it as follows

The area of square drawn on hypotenuse	**=**	**The area of square drawn on one side which include the right angle**	**+**	**The area of square drawn on the other side which include the right angle**

19.4. Pythagorean relation verification.

Let's learn this relation by doing the activity below.

Activity 2

Step 1

Draw the ABC right angled triangle, as AB=3cm, BC=4cm and AC=5cm, as shown in the figure.

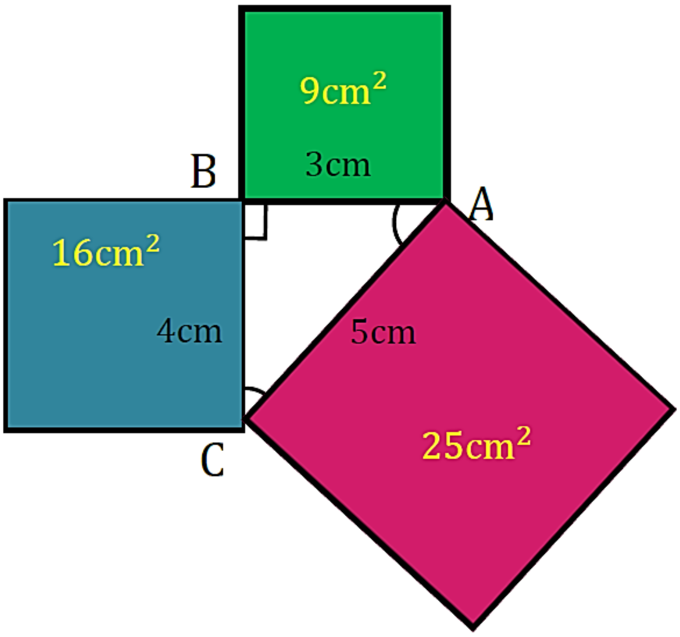
Step 2

Then construct three squares with side lengths 3cm, 4cm and 5cm on a separate paper.

Step 3

Cut out those squares drawn in step 2, and paste them on sides on the right-angled triangle what you have drawn in step 1, as shown in the figure.

Now let`s calculate the area of each square as shown in the figure.



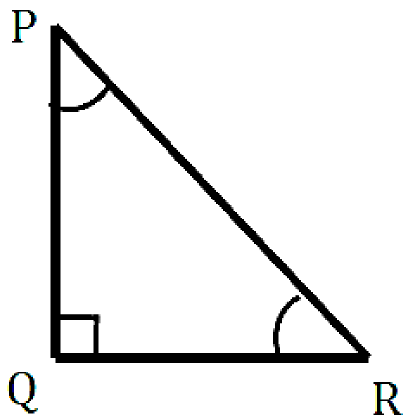
Area of the square drawn on AB	=	3cm x 3cm	=	$9cm^2$
Area of the square drawn on BC	=	4cm x 4cm	=	$16cm^2$
Area of the square drawn on AC	=	5cm x 5cm	=	$25cm^2$

By considering these values we can observe that, there is a relationship as below.

The area of square drawn on side AC	**=**	**The area of square drawn on side AB**	**+**	**The area of square drawn on side BC**

According to the above activity we can verify the relationship presented by Pythagoras.

The way of presenting Pythagorean relation by using the sides of a right-angled triangle.



Area of the square drawn on **PQ**	=	**PQ x PQ**	=	**PQ^2**
Area of the square drawn on **QR**	=	**QR x QR**	=	**QR^2**
Area of the square drawn on **PR**	=	**PR x PR**	=	**PR^2**

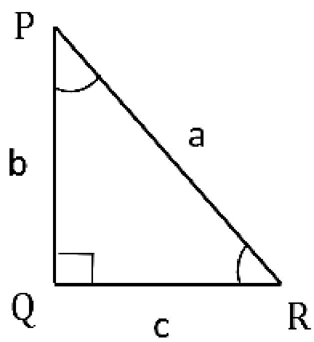
Therefore, we can write the Pythagorean relation as,

$$\boxed{PR^2 = PQ^2 + QR^2}$$

We can write the Pythagorean relation in this way also,

Lets buildup this relationship in another way…

If the side lengths of the right angled triangle PQR are given by a, b and c as below,



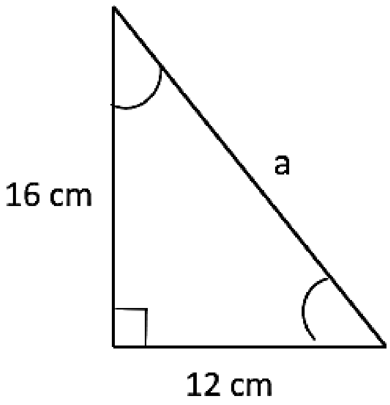
According to the Pythagorean relation

$$\boxed{a^2 = b^2 + c^2}$$

Let`s solve problems related to the Pythagorean relation.

Example 1

Find the lengths of sides denoted by English letters on figures given below.



According to the Pythagorean relation,

$$a^2 = 12^2 + 16^2$$

$$a^2 = 144 + 256$$

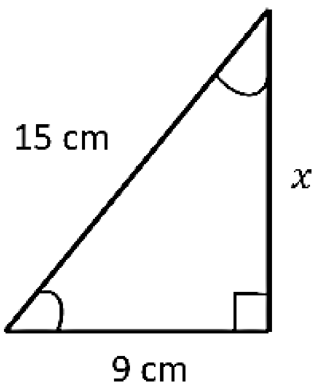
$$a^2 = 400$$

$$a = \sqrt{400} = 20$$

$\therefore$ a = 20 cm

Example 2

According to the information in the figure, find the value of x.



According to the Pythagorean relation,

$$15^2 = 9^2 + x^2$$

$$225 = 81 + x^2$$

$$x^2 = 225 - 81$$

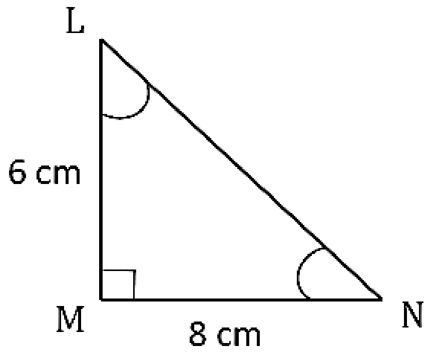
$$x^2 = 144$$

$$x = \sqrt{144} = 12$$

$\therefore x = 12$ cm

Example 3

If the lengths of the sides of LMN right angled triangle are, LM = 6 cm, MN = 8 cm, find the length of LN.



According to the Pythagorean relation,

$$LN^2 = LM^2 + MN^2$$

$$LN^2 = 6^2 + 8^2$$

$$LN^2 = 36 + 64$$

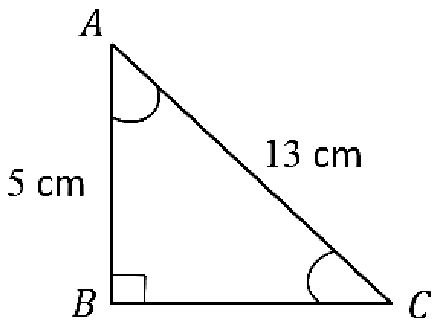
$$LN^2 = 100$$

$$LN = \sqrt{100} \quad = 10$$

$\therefore$ the length of LN is 10 cm

Example 4

In the ABC right angled triangle $AB = 5$ cm , $AC = 13$ cm . Find the length of BC.



According to the Pythagorean relation,

$$AC^2 = AB^2 + BC^2$$

$$13^2 = 5^2 + BC^2$$

$$169 = 25 + BC^2$$

$$BC^2 = 169 - 25$$

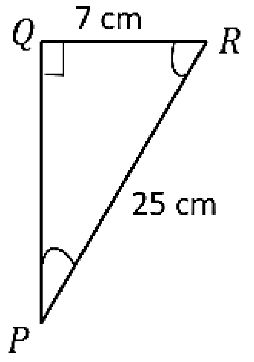
$$BC^2 = 144$$

$$BC = \sqrt{144} \quad = 12$$

$\therefore$ the length of BC is 12 cm

Example 5

If $P\hat{Q}R = 90^0$, QR = 7 cm and PR = 25 cm in PQR tringle. Find the length of PQ.



According to the Pythagorean relation,

$$\mathrm{PR}^2 = \mathrm{QR}^2 + \mathrm{PQ}^2$$
$$25^2 = 7^2 + \mathrm{PQ}^2$$
$$625 = 49 + \mathrm{PQ}^2$$
$$\mathrm{PQ}^2 = 625 - 49$$
$$\mathrm{PQ}^2 = 576$$
$$\mathrm{PQ} = \sqrt{576} \quad = 24$$

$\therefore$ the length of the sidePQ is $24\ \mathrm{cm}$

Example 6

A lamp post is placed vertically on a horizontal ground. One end of a ladder is placed $2\ \mathrm{m}$ below the top of that post, as shown in the figure. And the other end is placed $3\ \mathrm{m}$ away from the bottom of the post. If the ladder is $5\ \mathrm{m}$ long, find the height of the lamp post.

According to the Pythagoras relation,

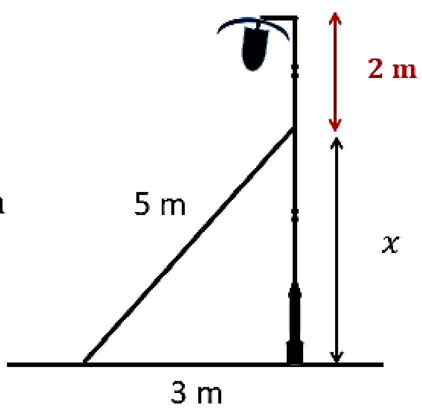
$$5^2 = 3^2 + x^2$$
$$25 = 9 + x^2$$
$$x^2 = 25 - 9$$
$$x^2 = 16$$
$$x = \sqrt{16} \quad = 4$$

Height of the lamp post $= x + 2\ \mathrm{m}$

$= 4\ \mathrm{m} + 2\ \mathrm{m}$

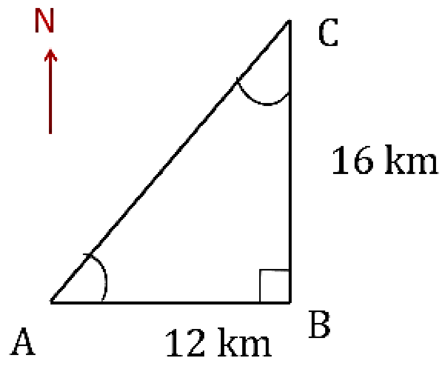
$= 6\ \mathrm{m}$

$\therefore$ the height of the lamp post is $6\ \mathrm{m}$



Example 7

The city B is located 12 km away to the East from A, and the city C is located 16 km away to the North from B. Find the shortest distance between the cities A and C.



According to the Pythagorean relation,

$$\mathrm{AC}^2 = \mathrm{AB}^2 + \mathrm{BC}^2$$

$$\mathrm{AC}^2 = 12^2 + 16^2$$

$$\mathrm{AC}^2 = 144 + 256$$

$$\mathrm{AC}^2 = 400$$

$$\mathrm{AC} = \sqrt{400} = 20$$

$\therefore$ the shortest distance between cities A and C is 20 km