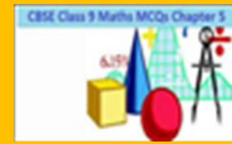
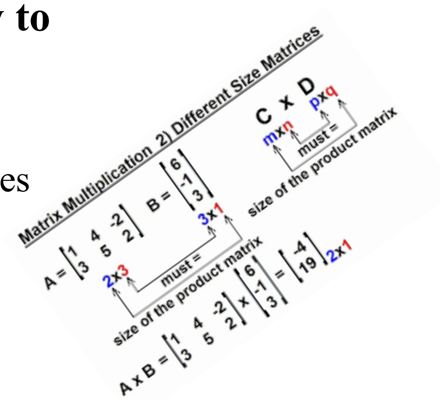
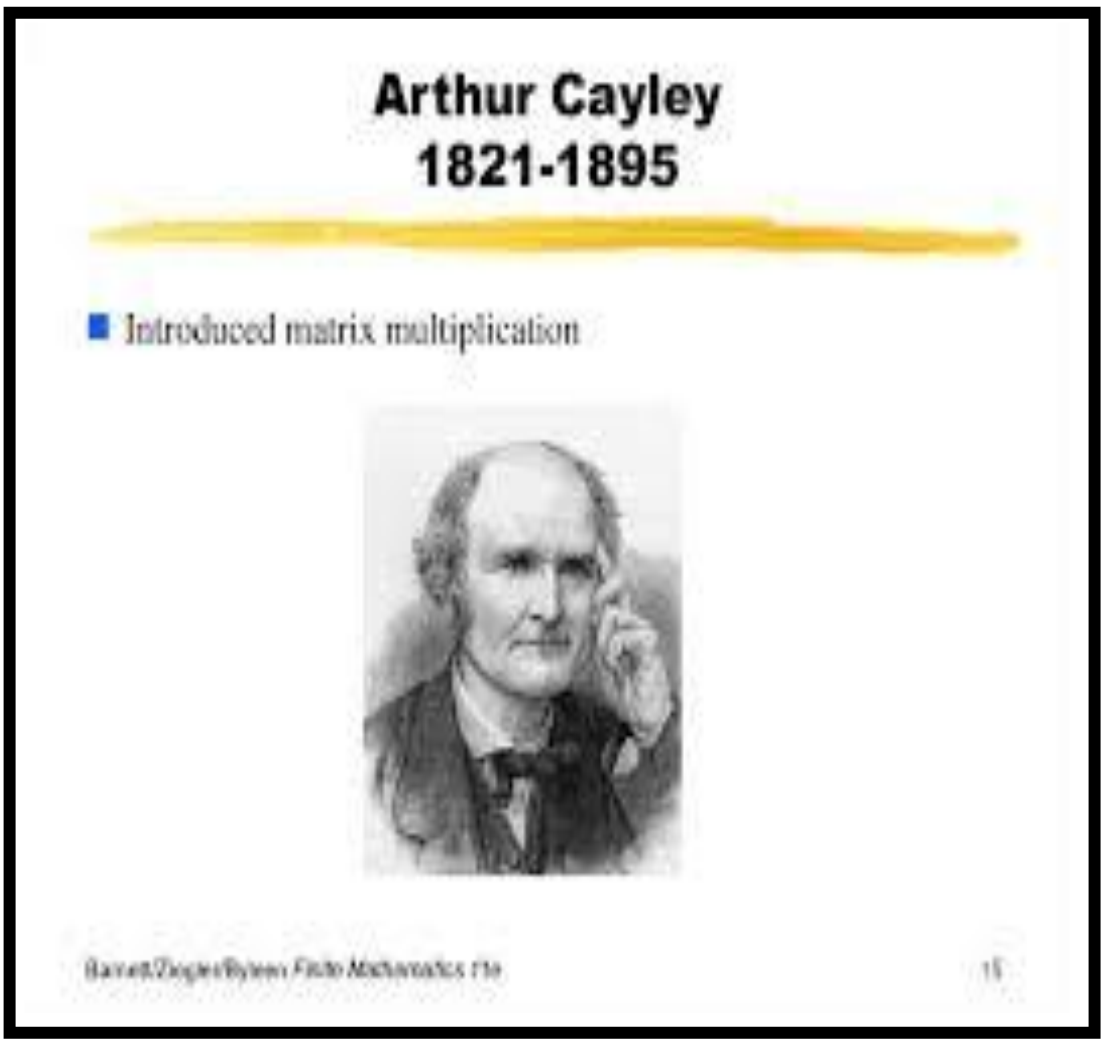
19 Matrices



When this lesson is learnt, you will have ability to

- identify a matrix
- identify elements of a matrix
- identify the addition and the subtraction of matrices
- multiply a matrix by a number
- multiply a matrix by a matrix
- solve problems related to the matrices





19.1 Introduction

- Matrices can be used to represent data shortly and to intreprete data easilly in various ways.
- Let's identify matrices with simple examples.

Example (01).

Information about the rice storages of three shops A , B and C is given below.

	A	B	C
Number of 5 kg rice packets	36	21	43
Number of 10 kg rice packets	27	56	35

Let's represent these data in a matrix as follows.

$$\begin{pmatrix} 36 & 21 & 43 \\ 27 & 56 & 35 \end{pmatrix}$$

* Matrix is represented as a group of numbers included in a pair of brackets with the rows and the columns..

Example 02).

Blue, red and yellow shirts are sold in a shop. Number of shirts sold in a certain day is given in the table.

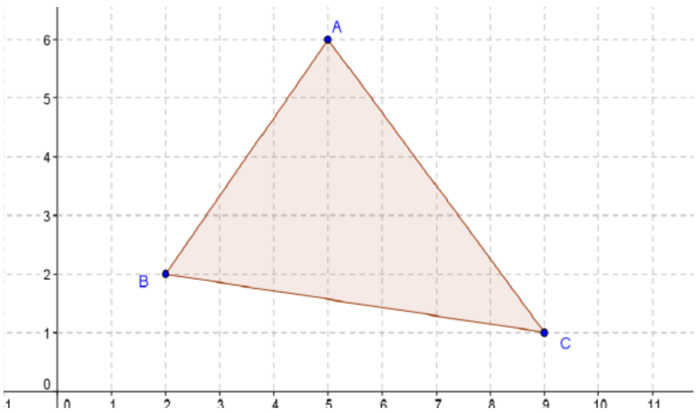
	Large size	Small size
Blue shirts	27	30
Red shirts	12	18
Yellow shirts	35	30

Let's represent this information in a matrix as follows.

$$\begin{pmatrix} 27 & 30 \\ 12 & 18 \\ 35 & 30 \end{pmatrix}$$

Activity (01).

Complete the table given according to the coordinates of the vertices of the triangle drawn on the coordinate plane.

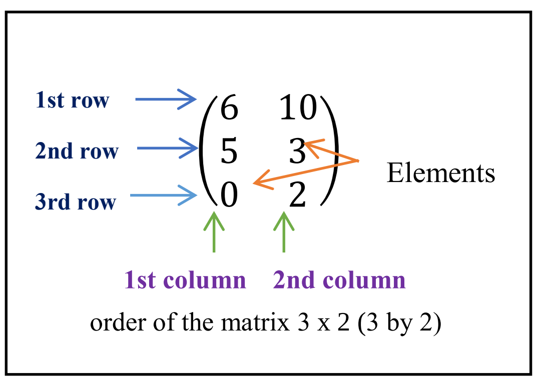


	A	B	C
x coordinate			
y coordinate			

Complete following matrix according to the table.

$$\begin{pmatrix} \dots & \dots & \dots \\ \dots & \dots & \dots \end{pmatrix}$$

19.2 Elements and the order of a matrix



*The order of a matrix is written according to its number of rows and number of columns.

*Elements of a matrix can be numbers, algebraic trems or algebraix expressions.

*English capital letters are used to name the matrices.

Eg: $$A = \begin{pmatrix} 5 & 3 & 1 \\ 4 & 6 & 8 \end{pmatrix}_{2\times 3}$$

*Order of the matrix A can be written as 2×3 . (2 by 3.)

Exercise(01) :-

1. The order of matrix $A = [-9 \quad 6 \quad -3]$ is
   i. 2×1 ii. 3×3 iii. 1×1 iv. 1×3 v. 3×1

2. Find the number of elements of the matrix with the order of 5×3.

3. The sum of the number of rows and the number of columns of a matrix gives the order of the matrix.
   i. this statement is true. ii. this statement is false.

4. If $= \begin{pmatrix} 5 & 3 & 1 \\ 7 & 8 & 0 \\ 9 & 4 & 5 \end{pmatrix}$, The element at the 2nd row and the 3rd column of the matrix B is
   i. 4 ii. 3 iii. 8 iv. 0 v. 1

5. The order of a matrix having equal number of rows and columns with 4 elements is
   i. 1×4 ii. 4×1 iii. 2×2 iv. 2×3 v. 4×4

19.3 Types of the Matrices

Column Matrices

$A = \begin{bmatrix} 7 \\ -1 \end{bmatrix}$

$B = \begin{bmatrix} 9 \\ 0 \\ 7 \end{bmatrix}$

Column matrix** **is a matrix having only one column.

*D is a column matrix with the order of 2×1.

$$D = \begin{pmatrix} -5 \\ 3 \end{pmatrix}$$

* E is a column matrix with the order of 3×1

$$E = \begin{pmatrix} 8 \\ 1 \\ 6 \end{pmatrix}$$

Row Matrices

$A = [5 \quad -1]$

$B = [4 \quad 0 \quad 6]$

***Row matrix** is a matrix having only one row.

*B is a row matrix with the order of 1×2. $B = (1 \quad 2)$

*E is a row matrix with the order of 1×3. $E = (9 \quad -2 \quad 5)$

Square Matrices

$$T = \begin{pmatrix} 6 & 3 \\ 0 & 4 \end{pmatrix} \quad V = \begin{pmatrix} 7 & 1 & 9 \\ 3 & 2 & 5 \\ 2 & 1 & 8 \end{pmatrix}$$

Major Diagonal

The matrices having equal number of rows and columns are square matrices.

Symmetric Matrices

$$R = \begin{pmatrix} 1 & 2 & 6 \\ 2 & 8 & 4 \\ 6 & 4 & 5 \end{pmatrix}$$

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Equally valued elements besides of the major diagonal of the symmetric matrices lie symmetrically.

Identity Matrices

$$I_{2\times2} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad I_{3\times3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

*Elements on the major diagonal of an identity matrix are 1 and all remaining elements are 0.

*Identity matrices are named by I

*They can be catogorized under the square matrices and the symmetric matrices.

*Identity matrix with n number of rows and n number of columns can be given as $I_{n\times n}$

Exercise(02) :-

Answer the exercise 19.1 of the text book.

19.4 Addition and Subtraction of Matrices

$$\begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 5 & -3 \end{bmatrix}$$

$3+4=7$

*The orders must be equal for the addition and subtraction of 2 matrices.

*In these operations, corresponding elements are added and subtracted.

Example(01): - Let's find the total marks obtained by three students for Sinhala, Mtahematics and English question papers by using matrices.

Part i			
	S	M	E
Kmal	40	36	49
Jagath	45	47	38
Amali	48	34	46

Part ii			
	S	M	E
Kmal	42	46	47
Jagath	45	50	46
Amali	40	44	46

$$\begin{pmatrix} 40 & 36 & 49 \\ 45 & 47 & 38 \\ 48 & 34 & 46 \end{pmatrix} + \begin{pmatrix} 42 & 46 & 47 \\ 45 & 50 & 46 \\ 40 & 44 & 46 \end{pmatrix} = \begin{pmatrix} 82 & 82 & 96 \\ 90 & 97 & 84 \\ 88 & 78 & 92 \end{pmatrix}$$

Example(02):- If $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

$$A + B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{1+1} & \mathbf{1+2} \\ \mathbf{0+3} & \mathbf{1+4} \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 3 \\ 3 & 5 \end{pmatrix}$$

Example(03):- If $A = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$ and $B = \begin{pmatrix} 6 & 2 \\ 7 & 1 \end{pmatrix}$

$$A - B = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} - \begin{pmatrix} 6 & 2 \\ 7 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 5-6 & 6-2 \\ 7-7 & 8-1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 4 \\ 0 & 7 \end{pmatrix}$$

<u>Exercise(03)</u>:-

Answer the exercise 19.2 of the text book.

Equal Matrices

If each and every elements of a matrix are equal to the corresponding elements of another matrix, such matrices are equal matrices.

$P = \begin{pmatrix} 2 & 6 & 1 \\ 0 & 5 & 3 \end{pmatrix}$ $Q = \begin{pmatrix} 2 & 6 & 1 \\ 0 & 5 & 3 \end{pmatrix}$

$*$ P and Q are equal matrices.

$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$

$*$ A and B are not equal matrices.

Avctivity(02). Find the values of x , y and z

$$\begin{pmatrix} x+3 & -1 \\ 4 & 5 \end{pmatrix} = \begin{pmatrix} 6 & y \\ z-3 & 5 \end{pmatrix}$$

so that equal matrices (by using obtained relationships)

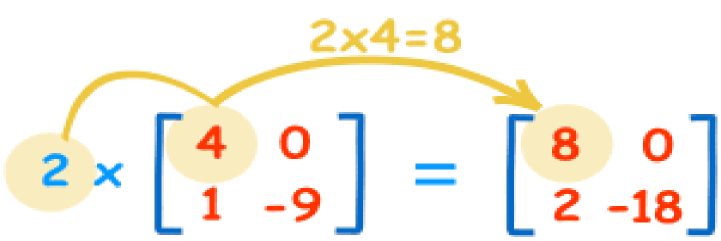
Find the values of x , y and z

$$x + 3 = 6$$
$$x = 6 - \quad \cdots$$
$$x = \cdots$$

$$z - 3 = 4$$
$$z = 4 + \ldots.$$
$$z = \cdots$$

$$y = \cdots$$

19.4 Multiplying a matrix by a number



Exercise **(04)** **:-**

Answer the exercise 19.3 of the text book.

19.5 Multiplication of Matrices

Activity(03).

- Information about the prices of two types of fruits and the number of fruits bought by Samitha and Rawindu is given below.

	one Veralu	one Mango
Price Rs.	6	5

	Samitha	Rawindu
Number of Veralu	6	5
Number of Mango	2	3

- Let's find the amounts paid for bought fruits by two of them seperatly.

The amount paid by Samitha $= (6 \times 6 \quad + \quad 5 \times 2 \quad = \quad ...)$

The amount paid by Rawindu $= (6 \times ... \quad + \quad ... \times 3 \quad = \quad ...)$

- **By writing this as a product of two matrices now**

$$\begin{pmatrix} 6 & 5 \end{pmatrix}_{1x2} \times \begin{pmatrix} 6 & 5 \\ 2 & 3 \end{pmatrix}_{2x2} = \begin{pmatrix} 6 \times 6 + 5 \times 2 & 6 \times ... + ... \times 3 \end{pmatrix}_{1x2}$$

$$= \begin{pmatrix} \cdots & \cdots \end{pmatrix}_{1x2}$$

- Let's find the amounts paid for bought fruits by two of them seperatly by uisng the matrices.

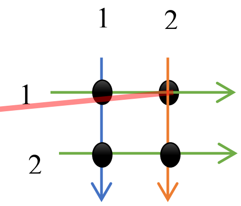
(Fill in the blanks to obtain the answer.)

Example-(01)

$$\begin{pmatrix} 1 & 2 \end{pmatrix}_{1x2} \begin{pmatrix} 3 \\ 4 \end{pmatrix}_{2x1} = (1 \times 3 + 2 \times 4) = (3 + 8) = (11)$$

Example (02)–

$$\begin{pmatrix}1 & 2\\3 & 4\end{pmatrix}\times\begin{pmatrix}5 & 6\\7 & 8\end{pmatrix}=\begin{pmatrix}19 & \dots\\ \dots & \dots\end{pmatrix}$$



$(1\ \ 2)\bullet\begin{pmatrix}5\\7\end{pmatrix}$	$= 1\times5 + 2\times7$	$= 19$	(Row -1, Column-1)
$(1\ \ 2)\bullet\begin{pmatrix}6\\8\end{pmatrix}$	$= 1\times6 + 2\times8$	$= 22$	(Row-1, Column-2)
$(3\ \ 4)\bullet\begin{pmatrix}5\\7\end{pmatrix}$	$= 3\times 5+ 4\times7$	$= 43$	(Row-2, Column-1)
$(3,\ \ 4)\bullet\begin{pmatrix}6\\8\end{pmatrix}$	$= 3\times6 + 4\times8$	$= 50$	(Row-2, Column-2)

The Matrix obtained by multiplying two matrices

$$A=\begin{bmatrix}1 & 4 & -2\\3 & 5 & 2\end{bmatrix} \qquad B=\begin{bmatrix}6\\-1\\3\end{bmatrix}$$

The Order → 2×3 $\qquad$ 3×1

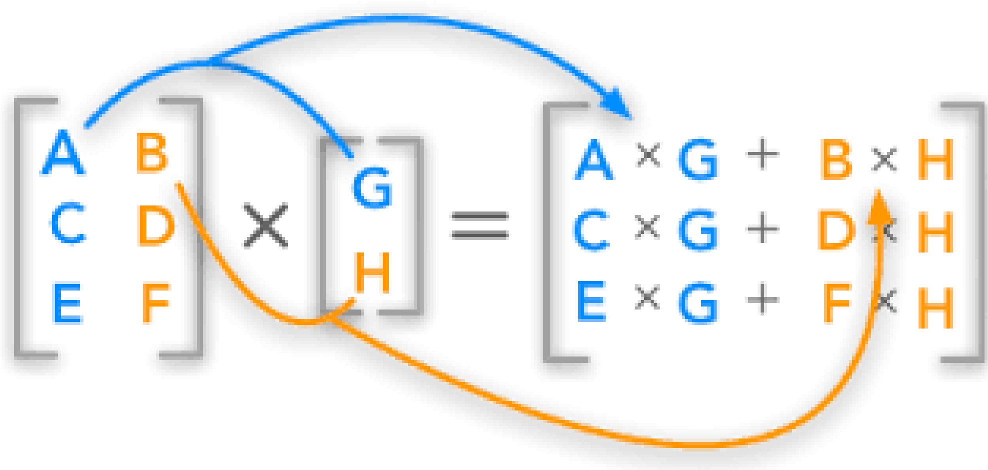
It should be equal to multiply

The order of the matrix obtained for the answer gives. (2×1)

*If the number of columns of the 1st matrix (Multiplicant) and the number rows of the second matrix (Multiplier) are equal, those matrices can be multiplied.

*By using the number rows of the 1st matrix (Multiplicant) and the number of columns of the second matrix (Multiplier), the order of the matrix given for the answer can be obtained.

Follow the following figure to study the multiplication of the matrices further more.



Exercise ***(05)*** **:-**

Answer the exercise 19.4 of the text book.

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