Grade 10

MATHEMATICS

Competency – 2

Makes decisions for future requirements by investigating the various relationships between numbers.

Competency Level 2.1

Identifies progressions and solves related problems

Competency Level 2.2

Investigates the various behavioral patterns of arithmetic progressions.

Content by : Mathematics Unit of Southern Province.

Prepared by : K.A.Kaushalya Munasinghe, WP/KL/Sri Sumangala Balika Maha Vidyalaya, Panadura.

Arithmetics progressions

❖ Introduction

An arithmetic progression is a sequence of numbers such that a constant value is obtained when any term is subtracted from the term right after that term.

➢ Example :- 1

5, 8, 11, 14, 17, ...

In here 5 and 8,8and 11,11 and 14, 14 and 17 are consecutive terms.

❖ Lets consider the difference between two consecutive terms.

- $8 - 5 = 3$
- $11 - 8 = 3$
- $14 - 11 = 3$
- $17 - 14 = 3$

It is clear, the difference between any two consecutive terms is a constant.
Therefor it is an Arithmetic Progression

➢ Example:- 2

3 , 8 , 13 , 18 , . . . Lets see whether this a arithmetic progression or not.

- $8 - 3 = 5$
- $13 - 8 = 5$
- $18 - 13 = 5$

The difference between any two consecutive terms is 5.

Therefore it is a arithmetic progression.

➢ Excersise-1

Determine whether each of the following sequences is an arithmetic progression.

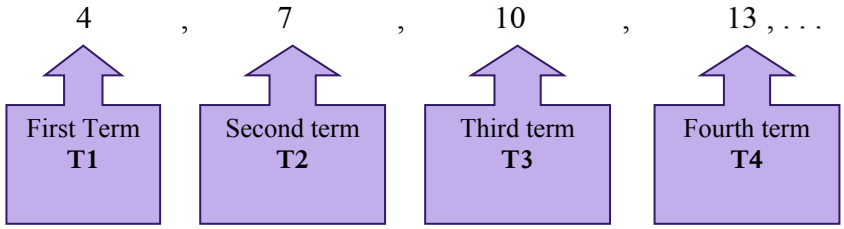
I. 5 , 10 , 15 , 20 , . . .
II. 2 , 6 , 10 , 14 , . . .
III. 4 , 8 , 16 , 32 , . . .
IV. 30 , 24 , 18 , 12 , . . .
V. 50 , 45 , 35 , 15 , . . .
VI. 1 , -8 , 15 , -22 , . . .

Content by : Mathematics Unit of Southern Province.

Prepared by : K.A.Kaushalya Munasinghe, WP/KL/Sri Sumangala Balika Maha Vidyalaya, Panadura.

The following notations used to denote the terms of an arithmetic progression.

This is an arithmetic progression
4 , 7 , 10 , 13 , . . .



Let's identify the terms related to arithmetic progressions

First term	*a*
Common difference	*d*
Last term	*l*
Number of terms	*n*

❖ Lets find the Common difference of progression

As it is mentioned above, difference between two consecutive terms is called common differnce.

➢ Excessive :-1

Let's find the common difference of this progression 3 , 6 , 9 , 12 , . . .

- d = 6 – 3 = 3
- d = 9 – 6 = 3
- d = 12 – 9 = 3

Common differnce of this progression is 3,
d=3

➢ Excersice- 2

Find the common difference of the arithmetic progressions given below.

I. 7 , 13 , 19 , 25 , . . .
II. 3 , 3.5 , 4 , 4.5 , . . .
III. 20 , 18 , 16 , 14 , . . .
IV. 2 , 3 – , 5 , 6 – , . . .

Refer the exercise 24.1 of your textbook

Content by : Mathematics Unit of Southern Province.

Prepared by : K.A.Kaushalya Munasinghe, WP/KL/Sri Sumangala Balika Maha Vidyalaya, Panadura.

❖ Let's find the pattern of which the terms of an arithmetic progression is built up

T_1	T_2	T_3	T_4	T_5
5 ,	7 ,	9 ,	11 ,	13
5 a a	5+2 a+d a+d	5+(2x2) a+dx2 a+2d	5+(2x3) a+dx3 a+3d	5+(2x4) a+dx4 a+4d

According to the above pattern

T_6 = a+5d
T_7 = a+6d
T_{10} = a+9d
If T_{25} = a+24d,
and T_n = **a+ (n-1)d**

If the term of an arithmetic progression is 'a', common difference is 'd',then 'n' th term would be given by Tn=a+(n-1)d

➢ Example :-1

The first term of an arithmetic progression is 6 & common difference is 5
let's find the 12th term of this progression

T_n = a+ (n-1)d
T_{12} = 6+ (12-1)5
T_{12} = 6+ 11x5
T_{12} = 6+55
T_{12} = 61

Here a=6 ,d=5 , n=12
Substitute the above values in
T_n=a+(n-1)d to obtain the answer

➢ Example :-2

Let's find the 20th term of the arithmetic progression 35 , 32 , 29 , 26

T_n = a+ (n-1)d
T_{20} = 35+ (20-1)(-3)
T_{20} = 35+19x(-3)
T_{20} = 35-57
T_{20} = -22

Here a=35
d=-3
n=20

Content by : Mathematics Unit of Southern Province.
Prepared by : K.A.Kaushalya Munasinghe, WP/KL/Sri Sumangala Balika Maha Vidyalaya, Panadura.

➢ Exercise - 3

➢

Find the relevant term according to the data given below.

1. When a=12 , d= 4 , n=16, T_{16}
2. When a=50 , d= -5 , n=11, T_{11}
3. 13th term of the arithmetic progression 10 , 16 , 22 , 28 , . . .
4. 13 T_{15} of the arithmetic progression 84 , 76 , 68 , 60 , . . .
5. 8th term of the arithmetic progression 10 ,12.5 , 15 , 17.5 , . . .

See the first question of exercise 24.2 of your text book

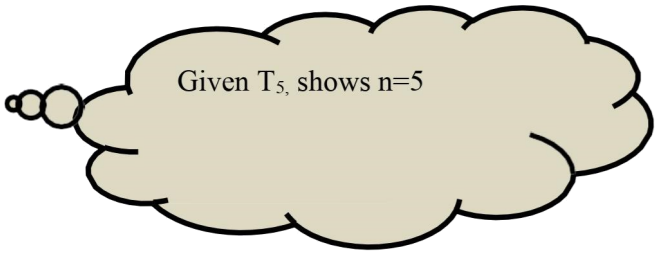
Finding the first term (a = T)and the common difference (d) when the nth term is known

Example :-1

Let's find the first term of the arithmetic progression with d=7 and T_5=29

$T_n = a + (n-1)d$
$29 = a + (5-1)(7)$
$29 = a + 4x(7)$
$29 = a + 28$
$29-28 = a$
$1 = a$

Value of the first term is 1



Find the first term of the following arithmetic progressions,

I. When d = 3 and $T_8 = 26$
II. When d = 10 and $T_{11} = 115$
III. When d = -4 and $T_6 = 12$
IV. When d = -12 and $T_4 = 64$
V. When $T_4 = 15$ and $T_5 = 21$ (As two consecutive terms are given here we can find the common difference)
VI. When $T_9 = 43$ and $T_{10} = 48$

Content by : Mathematics Unit of Southern Province.

Prepared by : K.A.Kaushalya Munasinghe, WP/KL/Sri Sumangala Balika Maha Vidyalaya, Panadura.

➢ Example :-2

Let's find the common difference of the arithmetic progression with a=5 and T_{10} =41

T_n = a + (n-1)d
41 = 5+ (10-1)d
41 = 5 + 9d
41-5 = 9d
36 = 9d
4 = d

Here a=5,T_{10}=41, n=10,Substitute these values to the formulae

When we have to write down the first 5 terms of this arithmetic progression,we can start by adding the common difference to the first term and proceed on.

$T_1 = 5$
$T_2 = 5+4 = 9$
$T_3 = 9+4 = 13$
$T_4 = 13+4 = 17$
$T_5 = 17+4 = 21$

➢ Example 3 :-

Find the 15th term of the arithmetic progression with a=75 and T_7 = 45.

First find the common difference.

T_n = a + (n-1)d
45 = 75+ (7-1)d
45 = 75 + 6d
45-75 = 6d
-30 = 6d
-5 = d

Let's find the 15th term

T_n = a + (n-1)d
T_{15} = 75 + (15-1)ෘ-5ඍ
T_{15} = 75 – 70
T_{15} = 5

➢ Exercise - 5

Find the common difference, first three terms and the given term of the following arithmetic progressions

I. When a = 7 and T_5 = 19 , find T_7
II. When a = 8 and T_5 = 24 , find T_{12}
III. When first term is -4 and the eighth term is 36, Find T_{12}
IV. When a = 65 and T_6 = 15,find T_{10}
V. When a = 44 and T_8 = 23 find T_6
VI. When the first term is 12 and seventh term is 24, find T_{12}

Content by : Mathematics Unit of Southern Province.
Prepared by : K.A.Kaushalya Munasinghe, WP/KL/Sri Sumangala Balika Maha Vidyalaya, Panadura.

Complete the Excersices 24.2 of your textbook

❖ Arithmetic mean

When considering the three successive terms of an arithmetic progression, the second term is an arithmetic mean of the first term and the third term.

If a,b and c are the successive terms of an arithmetic progression, b is an arithmetic mean of a and c. Then common difference can be written as follows.

In an arithmetic progression a, b, c

common difference = b – a or
common difference = c – b

Therefore $b - a = c - b$

$b + b = a + c$

$2b = a + c$

$$b = \frac{a+c}{2}$$

The Arithmetic mean of two terms is obtained by dividing the total(sum) of the two terms by 2

➢ Example :-1

Find the arithmetic mean between 5 and 15.

Let the arithmetic mean be p. Then the progression can be written as 5, p, 15

$$P = \frac{5+15}{2}$$

$$p = \frac{20}{2} = 10$$

Arithmetic mean = 10

If necessary, arithmetic progression can be written as 5, 10, 15

➢ Exercise- 6

Find an arithmetic mean of the following.

I. Between 4 and 12
II. Between 7 and 19
III. Between 60 and 40
IV. Between 75 and 35
V. Between 15 and -5
VI. Between 6 and -12

There may be more than one arithmetic means between two numbers. Let's consider such problems.

➢ Example :-2

Let's find 3 arithmetic means between 12 and _ _.

Content by : Mathematics Unit of Southern Province.

Prepared by : K.A.Kaushalya Munasinghe, WP/KL/Sri Sumangala Balika Maha Vidyalaya, Panadura.

Then the arithmetic progression can be written as 12, p, q, r, 36.

Here, as the first term is 12 and T_5 is 36, we can find the value of 'd'.

$T_n = a + (n-1)d$
$T_5 = 12 + (5-1)d$
$36 = 12 + 4d$
$36-12 = 4d$
$24 = 4d$
$6 = d$

Therefore arithmetic mean

$p = 12 + 6 = 18$

$q = 18 + 6 = 24$

$r = 24 + 6 = 30$.

Then the progression is 12 , 18 , 24 , 30 , 36

➢ Exercise -7

I. Find three arithmetic means between 3 and 31

II. Find four arithmetic means between 15 and 45

III Find five arithmetic means between 55 and -11

IV. The mean difference between two numbers is 14. First number is 7. Find the second number

Complete the exercise 24.3 of your textbook.

❖ Let's find the sum of the first n terms of an arithmetic progression

We use s_n denote to the sum of the first n terms and l for the last term of an arithmetic progression.

When the terms of an arithmetic progression are a , a+d , a+2d , a+3d , a+4d ,....., a+(n-1)d , the last term can be written as a+(n-1)d

Considering the last term l , the reversing terms can be written as (l - d), (l - 2d),...

Now arithmetic progression can be written as follows.
a , a+d , a+2d , ... , l- 2d , l- d , l
Since Sn means sum of all the terms,

$$Sn = a + (a+d) + (a+2d) + \ldots\ldots + (l- 2d) + (l- d) + l . \quad \text{——— (1)}$$

and then reversing the terms,

$$Sn = l + (l- d) + (l- 2d) + \ldots\ldots + (a+2d) + (a+d) + a \quad \text{——— (2)}$$

Let's add above ① &②, ① + ②

$S_n = a + (a+d) + (a+2d) + \ldots\ldots\ldots\ldots + (l-2d) + (l-d) + l$

$+ \quad S_n = l + (l-d) + (l-2d) + \ldots\ldots\ldots\ldots + (a+2d) + (a+d) + a$

$2S_n = (a+l)+(a+l)+(a+l)+\ldots\ldots\ldots\ldots+(a+l)+(a+l)+(a+l)$

$2S_n = n(a+l)$

$S_n = \frac{n}{2}(a+l)$ Build the relation as this

This formula can be used to find the sum of first n terms of arithmetics progression when first term, last term and the number of terms are known.

➢ Example :1

In the arithmetic progression 4 , 7 , 10 , 13 , . . , $T_8 = 25$. Find the sum of the first 8 terms.

$S_n = \frac{n}{2}(a+l)$

$S_8 = \frac{8}{2}(4+25)$

$S_8 = x\ 29$

$S_8 = 116$

Substitute a = 4 , $l = 25$ and number of terms as n =8

➢ Example:-2

Find the sum of the first 6 terms, when a=12 and T_6=32

$S_n = \frac{n}{2}(a+l)$

$S_6 = \frac{6}{2}(12 + 32)$

$S_6 = x\ 44$

$S_6 = 132$

➢ Exercise - 8

Find the sum of the first 10 terms when a = 4 and $T_{10} = 22$

Find the sum of the first 6 terms when a = 80 and $T_6 = -20$

Find the value of S_6 when a = 10 and $T_5 = 70$

Find the value of S_7 when first term is 15 and 7th term is 51

Content by : Mathematics Unit of Southern Province.

Prepared by : K.A.Kaushalya Munasinghe, WP/KL/Sri Sumangala Balika Maha Vidyalaya, Panadura.

- Let's find the sum of the first n terms of an arithmetic progression when last term is unknown

According to the above relationship

$$S_n = \frac{n}{2}(a+l)$$

$$S_n = \frac{n}{2}\{a+ a+(n-1)d\}$$

$$S_n = \frac{n}{2}\{2a+(n-1)d\}$$

If last term is unknown let's use a+(n-1)d instead of l

Example :-1

Find the sum of the first 11 terms of the arithmetic progression 6 , 9 , 12 , . . .

$$S_n = \frac{n}{2}\{2a+(n-1)d\}$$

$$S_{11} = \frac{11}{2}\{ 2 \times 6 + (11-1)3 \}$$

$$S_{11} = \frac{11}{2}\{ 12 + 30 \}$$

$$S_{11} = \frac{11}{2} \times 42$$

$$S_{11} = 231$$

Substitute a = 6 , d = 9 - 6 = 3 and n = 11

Example :-2

Find the sum of the arithmetic progression with first term 60 and common difference -4.

$$S_n = \frac{n}{2}\{2a+(n-1)d\}$$

$$S_8 = \frac{8}{2}\{ 2 \times 60 + (8-1)\text{ඟ}-4\text{ඊ} \}$$

$$S_8 = 4\{ 120 - 28 \}$$

$$S_8 = 4 \times 92$$

$$S_8 = 368$$

Exercise - 9

Find the sum of the number of terms stated in each of the following arithmetic progressions.

I. 3 , 6 , 9 , 12 , . . . first 9 terms
II. 5 , 7 , 9 , 11 , . . . first 12 terms
III. 25 , 20 , 15 , . . .first 15 terms
IV. -20 , -10 , 0 , . . first 14 terms

Content by : Mathematics Unit of Southern Province.

Prepared by : K.A.Kaushalya Munasinghe, WP/KL/Sri Sumangala Balika Maha Vidyalaya, Panadura.

❖ Find the number of terms of an arithmetic progression when sum of the given terms are known

How many terms (starting from first term) are there in the arithmetic progression 20 , 25 , 30 , . . … add to 195 ?

$S_n = \frac{n}{2}\{2a+(n-1)d\}$

$195 = \frac{n}{2}\{ 2 \times 20 + (n-1)5 \}$

$195 \times 2 = n\{ 40 + 5n - 5 \}$

$390 = n(5n + 35)$

$390 = 5n^2 + 35n$

$5n^2 + 35n - 390 = 0$

$n^2 + 7n - 78 = 0$

Obtain a quadratic equation of n

By solving the equation

$n^2 + 7n - 78 = 0$

$(n + 13)(n - 6) = 0$

$n = -13 \quad n = 6$

N cannot be -13 sinke n denotes the number of terms

$n = 6$ Sum of the first 6 terms of the arithmetic progression is 195.

➢ **Exercise - 10**

I. Find the number of terms of which the sum of first n terms of the arithmetic progression 2 , 4 , 6 , 8 , . . . is 110

II. Find the number of terms of which the sum of first n terms of the arithmetic progression 9 , 6 , 3 , . . . equals 9.

III. Find the number of terms of which the sum equals 36 of the arithmetic progression with the first term -12 and the common difference 4 .

IV. Find the value of n when a = 25 , d = -4 , Sn = 76

*Finding the necessary information of an arithmetic progression when the General term has been given.

➢ Example :-1

n th term of an arithmetic progression is given by 2n + 1. Find the,

I. First term

II. First three terms

III. Common difference

IV. Eight term

V. Sum of the first eight terms

The n th term of the above sequence is 2n + 1.
When n= 1,
first term = 2 x 1 + 1
= 2 + 1
first term = 3
second term = 2 x 2 + 1
= 4 + 1
second term = 5

By assuming the number of terms of progression as 1 substitute 1 for n to find the first term of the progression

third term = 2 x 3 + 1
= 6 + 1
third term = 7

Above arithmetic progression can be written as 3, 5, 7 ,…

common difference
d = 5-3
common difference d = 2

Any two consecutive terms can be used to find common differnce

8 th term $T_n = a + (n-1)d$
$T_8 = 3 + (8-1)2$
$T_8 = 3 + 14$
$T_8 = 17$

8 th term = 17

Sum of the first eight terms $Sn = \frac{n}{2}(a + l)$

$S_8 = 4(3+17)$

$S_8 = 4 \times 20$

$S_8 = 80$

Sum of the first eight terms = 80

➢ Exersies- 11

1. The nth term of an arithmetic progression is 3n+5.Find
   - I. First term
   - II. First four terms
   - III. Common difference
   - IV. Tenth term
   - V. Some of first 10 terms
2. The n th term of an arithmetic progression is 5n – 2.
   - I. Find a
   - II. Find T_2 , T_3 , T_4
   - III. Find the value of d
   - IV. Find T_{11} and T_{20}
   - V. Some of S_{11} and S_{19}

complete the exercise 24.4

Content by : Mathematics Unit of Southern Province.
Prepared by : K.A.Kaushalya Munasinghe, WP/KL/Sri Sumangala Balika Maha Vidyalaya, Panadura.