# Grade 10

# MATHEMATICS

# Inequalities

**Grade** : 10

**Subject** : Mathematics

**Competency** : 18 - Analyzes the relationships between various quantities related to real-life problems.

**Competency Level** : 18.1- Solves problems in daily life related to the inequality of two quantities.

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# Lesson – Grade 10 Mathematics

Unit – 25 – Algebraic Inequalities

Number of Periods - 06

By studying this unit, you will be able to

solve inequalities and represent the set of solutions on a number line.

represent inequalities on a Cartesian plane.

Figure 1

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# Grade 10 – Mathematics - Inequalities

## Activity 01

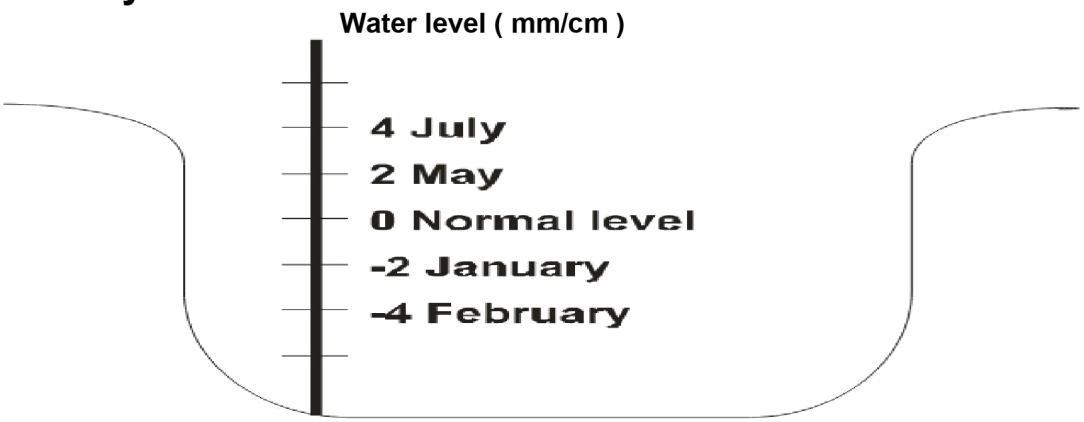


Figure 2

The water level of a certain reservoir had been marked as the above diagram in different months of a year.

- What are the months which shows a higher water level than the normal level?
- In which months the water level has been reduced from the normal level?
- How many months are there with a lower water level than the water level in May?

## Activity 02

Inequality signs ⟶ < or >

| |
|---|
| Smaller number < Larger number |
| Larger number > Smaller number |

Let us put the correct inequality sign in followings.

| | | | | | |
|---|---|---|---|---|---|
| 2 | ..... | 4 | 4 | ..... | -3 |
| 4 | ..... | 0 | -3 | ..... | 0 |
| -2 | ..... | 2 | -2 | ..... | -5 |

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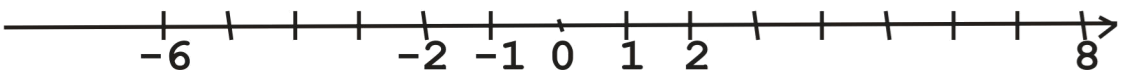
## Review Exercise

Let us revise what we learnt in Grade 09 under the unit Inequalities.

1. Mark the remaining numbers in the number line given.




2. Match the correct answers which are related to the above number line.

| | |
|---|---|
| Smallest positive integer | -1 |
| Largest positive integer | -6 |
| Largest negative integer | 8 |
| Smallest negative integer | 1 |

3. Select the suitable value which satisfy the following inequalities from the numbers given within brackets.

(**Important** – When we consider two numbers on the number line, the number which is in the right side is greater than the other)

i) 5 >..... (7, 3)

ii) (-4) <..... ((-2),(-7))

iii)..... >0 ((-4), 4)

iv) (-5) <.....(0, (-7))

v)..... > ( -3) (2, (-4))

vi) 2> .....(5, (-5))

4. Write the number suitable for the □

I □ + 2 < 6 (3, 4, 5)

II 3 + □ <2 (4,(-4),(-1))

III (-4 )+□ >2, ((-1), 4, 8)

IV -3 > (-2) + □ (3,(-2), 5)

V (-6) >(-4)+ □ (3,(-3), (-1))

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5 Select the suitable value for the variables given. Find one is done for you.

| | | |
|---|---|---|
| I | $x > 5$ | (4, 5, 6) |
| II | $y < (-3)$ | (0, (-2), (-4)) |
| III | $p + 2 > 5$ | (2, 3, 4) |
| IV | $m - 3 < 2$ | (4, 5, 6 ) |
| V | $7 - a > 2$ | (4, 5, 6 ) |
| VI | $x + 8 < 5$ | ((-4),(-3), (-2)) |
| VII | $2x > 7$ | (3, 4, 5) |
| VIII | $3x - 2 < 4$ | (1, 2, 3) |

**I If we add / subtract the same value to / from the both sides of the inequality, the inequality remain same.**

6. Put any positive value for the following inequalities and verify the above statement.

I 5 < 7

5+ ☐ < 7 +☐

…… < ……

II (-3) < 2

(-3) + ☐ < 2 + ☐

…… < ……

III 8 > 6

8 - ☐ > 6 - ☐

……> ……

IV (-3) < 2

(-3) - ☐ < 2 - ☐

……< ……

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**The inequality remains the same if we multiply or divide the both sides of the inequality from the same positive value.**

7. By taking any positive value that you like, investigate on the above statement.

I $3 < 5$

$3 \times \square < 5 \times \square$

..... < .....

II $12 < 24$

$\frac{12}{\square} < \frac{24}{\square}$

..... < .....

III $(-2) < (-1)$

$(-2) \times \square < (-1) \times \square$

..... < .....

IV $(-3) < 2$

$(-3) \times \square < 2 \times \square$

..... < .....

V $(-6) > (-12)$

$\frac{-6}{\square} > \frac{(-12}{\square}$

.....> .....

VI $(-8) < 4$

$\frac{(-8)}{\square} < \frac{4}{\square}$

..... < .....

**When we multiply or divide the both sides of the inequality by a negative number, the *sign* of the inequality will be *interchanged*.**

8. Engage in the following activity by using negative integers.

I. Ex:- $3 < 5$

$3 \times \square > 5 \times \square$

$-6 > -10$

ii $2 > (-3)$

$2 \times \square < (-3) \times \square$

... < ...

iii $(-4) < (-1)$

$(-4) \times \square > (-1) \times \square$

.....> .....

iv $10 < 20$

$\frac{10}{\square} > \frac{20}{\square}$

..... > .....

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v $(-12) > (-20)$

$\frac{(-12)}{\square} < \frac{(-20)}{\square}$

……. < ……

vi $(-12) < 6$

$\frac{(-12)}{\square} > \frac{6}{\square}$

……. > ……

9 Let us solve the following inequalities.

i $x + 3 > 5$

$x + 3 - 3 > 5 - 3$

……. > ……

ii $a + 5 < 7$

……<……

……<……

iii $p - 3 < 1$

……<……

……<……

iv $m - 4 > (-3)$

…… >……

…… >……

v $3y < 12$

……<……

……<……

vi $8x > (-16)$

……>……

……>……

vii $\frac{x}{3} < 2$

…… < ··· ... ....

……<……

viii $\frac{y}{2} > (-3)$

…… >……

……>……

ix $2p + 3 < 7$

…… <……

……<……

…… <……

x $3x - 4 > 5$

…… >……

…… >……

…… >……

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xi $5x - 4 > (-14)$

……>……

……>……

……>……

xii $-2x < 6$

……>……

……>……

xiii $7 - 3x > 1$

……>……

……>……

……<……

……<……

xiv $2 - 5x < (-8)$

……<……

……<……

……>……

……>……

10. Solve the following inequalities and represent the integral solutions and all the solutions on the provided number lines.

I $x > 2$

- Integral solutions
- all the solutions

ii $2x > (-6)$

- Integral solutions
- all the solutions

iii $x - 4 < 2$

- Integral solutions
- all the solutions

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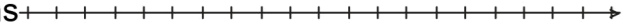
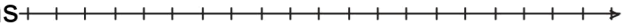
iv $-x+3<(-1)$

- Integral solutions 
- all the solutions

v $-x+5\geq 3$

- Integral solutions 
- all the solutions

## Inequaqlities of the form of $ax+b \begin{matrix} > \\ < \end{matrix} c$

Example 01:- Let us take the inequality of $2x+3>9$

In the above inequality, the value obtained by adding three to the two times of x is greater than nine.

We have to do the following steps to solve that inequality.

Step 1 – Remove 3 from the both sides

Step 2 – Divide the both sides by 2 and obtain the value of $x$.

Step 1 $2+3>9$

- $2x+3-3>9-3$

Step 2 $2x>6$

- $\frac{2x}{2}>\frac{6}{2}$
- $x>3$

(When both sides of the inequality are divided by a positive value, sign of the inequality does not change )

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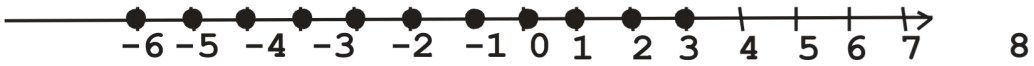
Example 02: - Let us consider the inequality of $3x - 4 \leq 8$

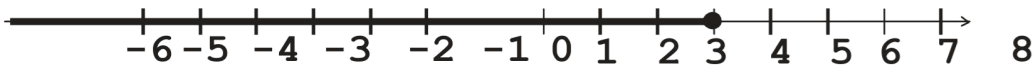
As it is in the above inequality when four is subtracted from the value obtained by the three times of $x$ is less than or equals to eight. The special thing that we should consider in this is that the value can be also equals to eight.

$$3x - 4 \leq 8$$
$$3x - 4 + 4 \leq 8 + 4 \ (adding\ 4\ to\ the\ both\ sides)$$
$$3x \leq 12$$
$$\frac{3x}{3} \leq \frac{12}{3} (dividing\ both\ sides\ by\ 3)$$
$$x \leq 4$$

The integral solutions of the above inequality is as follows




All the solutions of $x$ is



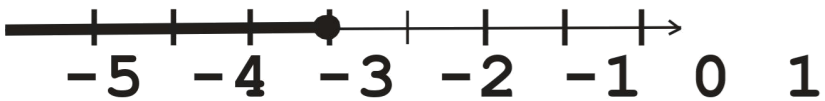

Example :- 03 Let us solve the inequality of $3 - 2x \geq 7$

When two times of $x$ is subtracted from three, that value is greater than equals to seven.

$$3 - 2x \geq 7$$
$$3 - 2x - 3 \geq 7 - 3 \ (Subtracting\ 3\ from\ the\ both\ sides)$$
$$-2x \geq 4$$
$$\frac{-2x}{(-2)} \leq \frac{4}{(-2)} \ (Dividing\ both\ sides\ by\ (-2)$$
$$x \leq -2$$

Integral solutions


All the solutions

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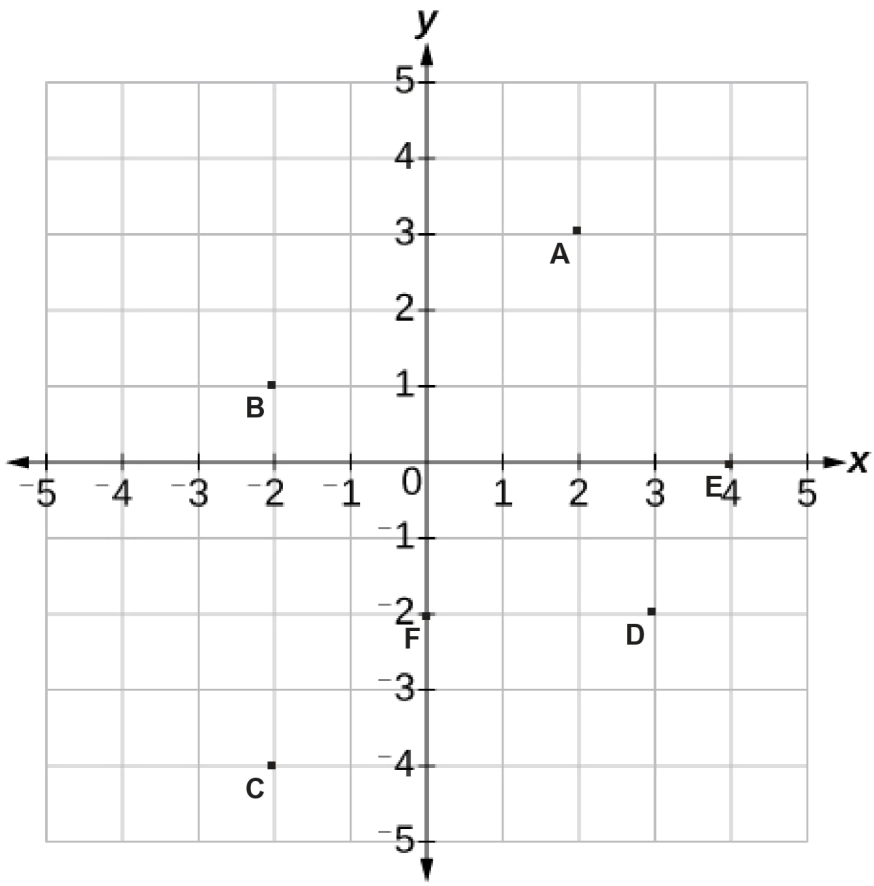
Solve the following inequalities

1. $3x + 2 > 5$
2. $2x + 1 \geq -5$
3. $4x - 3 \leq 5$
4. $6x + 5 \leq -7$
5. $5 - 3x \geq 2$

Do the exercise 25.1 in your text book

## Inequalities related to the lines parallel to the axes

01.

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Join the relevant coordinates of the points marked in the above Cartesian plane.

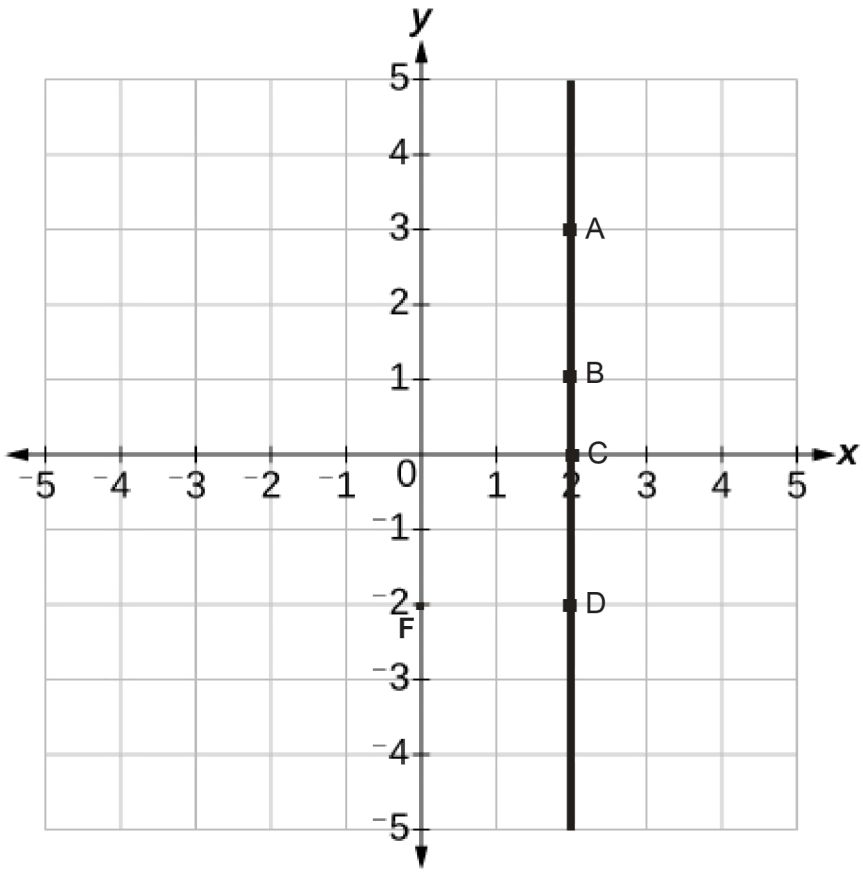
Let us find the coordinates of the following points marked in the above Cartesian plane.

A D

B E

C F

02.




Let us find the coordinates of the following points marked in the above Cartesian plane.

A C

B D

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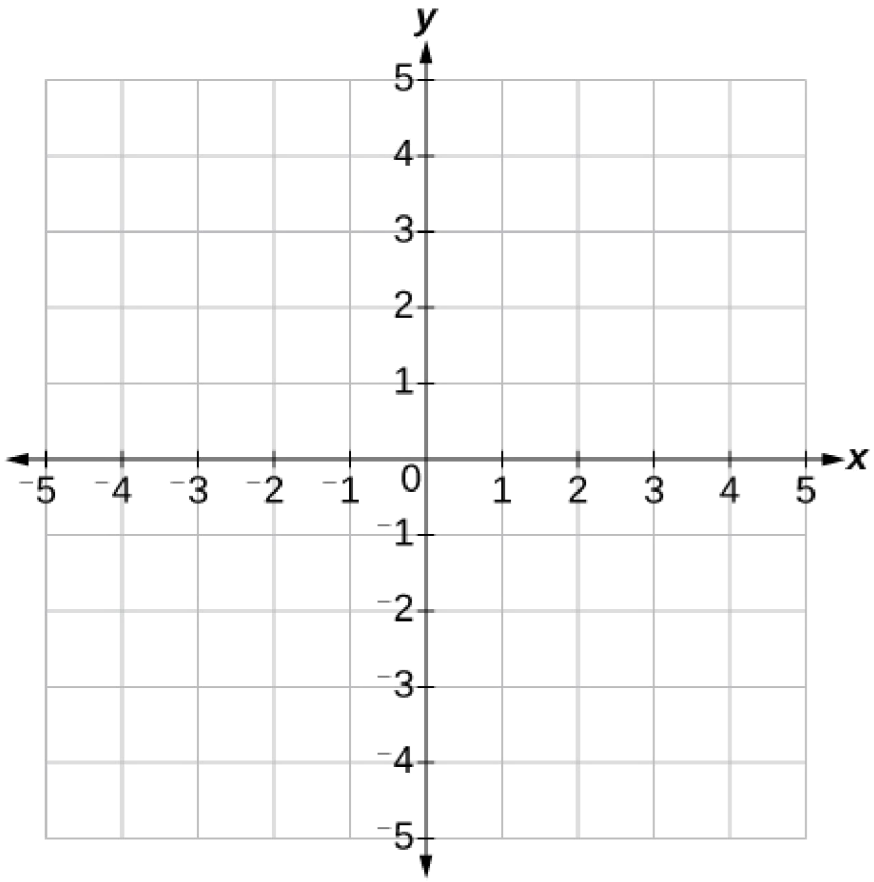
The common property of these points is that the $x$ coordinates of the above points is 2.

At the same time that line pass through the $x$ axis at the point 2

The equation of the line is $x = 2$

03.




Draw the following equations on the Cartesian plane given above. Write the equation on the relevant lines.

*i.* $x = -3$

*ii.* $x = 1$

*iii.* $= 0$

*iv.* $= 2$

*v.* $y = -3$

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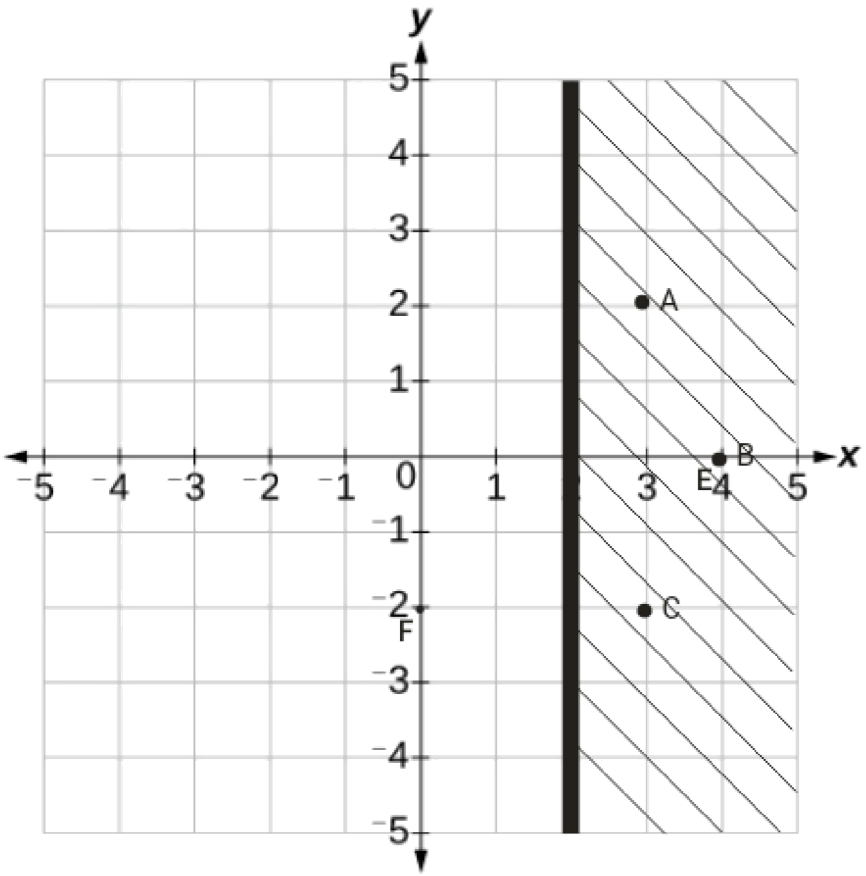
04.




What is the equation of the line?

The $x$ coordinates of the points A, B and C are greater than the $x$ coordinates of the marked line. The shaded region is $x > 2$

At the same time as the line is not a dotted line, the inequality which satisfy the above shaded region is $x \geq 2$

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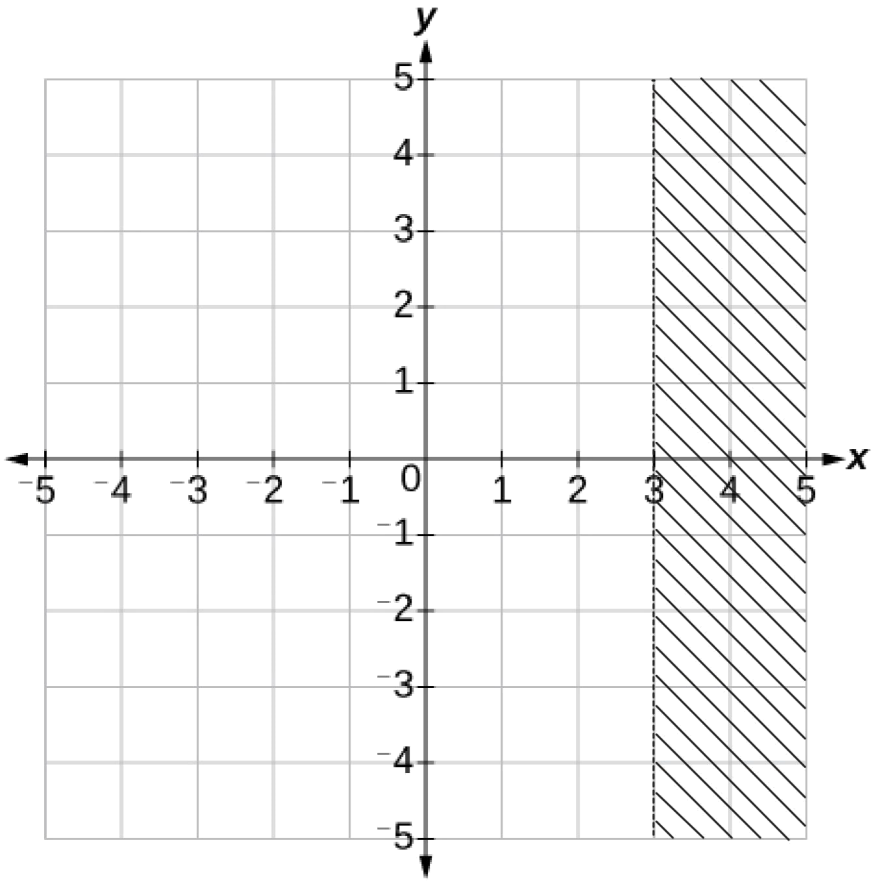
05.




In the above Cartesian plane as the $x = 3$ is marked as a dotted line. The inequality marked in the above Cartesian plane is $x > 3$

Shade the region which satisfy the inequality of $x > (-2)$ in the above Cartesian plane.

6. Shade the region of $y \leq 2$ in a Cartesian plane.

7. Mark the both regions of $y > 3$ $and$ $y \leq 3$ in a Cartesian plane.

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## Graphs of the inequalities of the form of $y \gtrless x$

We have to follow the steps given below, when solving an inequality of the form mentioned above.

- By taking the relevant coordinates of the given graph and mark them on a Cartesian plane.
- If the = sign is existing, draw the straight line or else draw the dotted line.
- Take any point which is clearly outside that line and substitute the coordinates of that point for the given inequality.
- If that point satisfy the inequality than shade that region of else shade the other side of the line.

Example: - Drawing the inequality of $y < x$

The values which satisfy the equation $y = x$

| $x$ | -1 | 0 | 1 |
|---|---|---|---|
| $y = x$ | -1 | 0 | 1 |

But the inequality is $y < x$, so we have to draw a dotted line.

Now let us substitute the point (2,1) to the above inequality

$y < x$

$1 < 2$, it satisfy the given inequality.

We should color the region which includes the point (2,1)

If we select the point as (-1,2)

$y < x$

$2 < (-1)$, this is an incorrect statement.

We should color the other side of the line which means the side which include (2,1)

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Exercise

Shade the following inequalities in different Cartesian planes.

(You should take at least three points to plot the graph and substitute one point for the inequality to check the relevant side)

1. $y \geq x$
2. $< 2x + 1$
3. $y \geq 2x - 1$
4. $y < -x + 1$
5. $y \leq -2x + 3$

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