

By studying this lesson, you will be able to,

- find the factors and multiples of a whole number,
- solve problems related to factors and multiples and
- examine whether a whole number is divisible by 2, 5 and 10.

### 11.1 Identifying factors

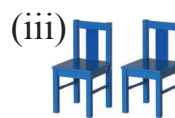
Consider a class consisting of six children. Suppose an equal number of children should sit in each row. The ways in which the 6 chairs can be arranged is shown below.



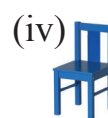
1 row with 6 chairs in the row



2 rows with 3 chairs  
in each row



3 rows with 2  
chairs in each row



6 rows with 1 chair  
in each row

In such arrangements the total number of 6 chairs is obtained by the product of the number of chairs in a row and the number of rows. By that it is clear that, there are many ways of writing 6, as a product of two whole numbers.

$$6 = 1 \times 6$$

$$6 = 2 \times 3$$

$$6 = 3 \times 2$$

$$6 = 6 \times 1$$



Now, let us consider how the chairs in a classroom can be arranged as above if there are twelve chairs. In this arrangement also, the total number of chairs is obtained by the product of the number of chairs in a row and the number of rows.

$$12 = 1 \times 12$$

$$12 = 2 \times 6$$

$$12 = 3 \times 4$$

$$12 = 4 \times 3$$

$$12 = 6 \times 2$$

$$12 = 12 \times 1$$

In this manner, any whole number can be written as a product of two whole numbers in various ways.

When a whole number is written as a product of two whole numbers, those two numbers are known as **factors** of the original number.

Since  $6 = 1 \times 6$ , the numbers 1 and 6 are factors of 6.

Since  $6 = 2 \times 3$ , the numbers 2 and 3 are also factors of 6.

When the products relevant to 6 are considered, we obtain that the factors of 6 are 1, 2, 3 and 6.

Similarly, the factors of 12 are 1, 2, 3, 4, 6 and 12.

Now, let us find the factors of 16.

We can write 16 in the following ways as a product of two whole numbers.

$$16 = 1 \times 16$$

$$16 = 2 \times 8$$

$$16 = 4 \times 4$$

$$16 = 8 \times 2$$

$$16 = 16 \times 1$$

Accordingly, the factors of 16 are 1, 2, 4, 8 and 16.

When we consider the above products relevant to 16, we see that it is sufficient to write only the following products to obtain the factors of 16.

$$16 = 1 \times 16$$

$$16 = 2 \times 8$$

$$16 = 4 \times 4$$

### Example 1

Find the factors of 20.

$$20 = 1 \times 20$$

$$20 = 2 \times 10$$

$$20 = 4 \times 5$$

The factors of 20 are 1, 2, 4, 5, 10 and 20.

### Note

- 0 is not a factor of a whole number.

### Exercise 11.1

(1) Fill in the blanks with the appropriate whole numbers.

(i)  $4 = 1 \times \dots\dots\dots$

$$4 = 2 \times \dots\dots\dots$$

1, 2 and  $\dots\dots\dots$  are the factors of 4.

(ii)  $7 = 1 \times \dots\dots\dots$

1 and  $\dots\dots\dots$  are the factors of 7.

(iii)  $8 = 1 \times \dots\dots\dots$

$$8 = 2 \times \dots\dots\dots$$

1, 2,  $\dots\dots\dots$  and  $\dots\dots\dots$  are the factors of 8.

(iv)  $15 = 1 \times 15$

$$15 = 3 \times \dots\dots\dots$$

1, 3,  $\dots\dots\dots$  and  $\dots\dots\dots$  are the factors of 15.

(v)  $24 = 1 \times \dots\dots\dots$

$$24 = 2 \times \dots\dots\dots$$

$$24 = 3 \times \dots\dots\dots$$

$$24 = 4 \times \dots\dots\dots$$

1, 2,  $\dots\dots\dots$ ,  $\dots\dots\dots$ ,  $\dots\dots\dots$ ,  $\dots\dots\dots$  and  $\dots\dots\dots$  are the factors of 24.

(vi) When the factors of 18 are written, we obtain 1, 2,  $\dots\dots$ , 6, 9 and 18.

(vii) When the factors of 40 are written, we obtain 1, 2,  $\dots\dots$ , 5,  $\dots\dots$ , 10, 20 and  $\dots\dots$ .

(2) Find the factors of each of the following numbers.

- (i) 5      (ii) 27      (iii) 17      (iv) 22      (v) 21  
 (vi) 31      (vii) 32      (viii) 45      (ix) 50      (x) 60

## 11.2 Finding factors using the multiplication table

Now let us consider how factors of a whole number are found by using the  $10 \times 10$  multiplication table.

| $\times$ | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10  |
|----------|----|----|----|----|----|----|----|----|----|-----|
| 1        | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10  |
| 2        | 2  | 4  | 6  | 8  | 10 | 12 | 14 | 16 | 18 | 20  |
| 3        | 3  | 6  | 9  | 12 | 15 | 18 | 21 | 24 | 27 | 30  |
| 4        | 4  | 8  | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40  |
| 5        | 5  | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50  |
| 6        | 6  | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60  |
| 7        | 7  | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70  |
| 8        | 8  | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80  |
| 9        | 9  | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90  |
| 10       | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |

Let us find the factors of 20 using the above multiplication table. For this, let us identify the instances when 20 is obtained as the product.

$$20 = 2 \times 10$$

$$20 = 4 \times 5$$

The numbers 2, 4, 5 and 10 are four factors of 20.

### Example 1

What are the factors of 72 that can be found using the above multiplication table?

$$72 = 8 \times 9$$

8 and 9 are the two factors of 72 that can be found using the above multiplication table.

### Example 2

What are the factors of 18 that can be found using the above multiplication table?

$$18 = 3 \times 6$$

$$18 = 2 \times 9$$

2, 3, 6 and 9 are the four factors of 18 that can be found using the above multiplication table.

## Exercise 11.2

(1) What are the factors of each of the following numbers that can be found using the  $10 \times 10$  multiplication table?

- (i) 48      (ii) 81      (iii) 2      (iv) 28      (v) 40

(2) Obtain different ways in which 36 can be written as a product of two whole numbers using the  $10 \times 10$  multiplication table and fill in the blanks.

- (i)  $9 \times \dots\dots\dots$       (ii)  $4 \times \dots\dots\dots$       (iii)  $6 \times \dots\dots\dots$

Write down in ascending order, the factors of 36 which can be obtained from the above products.

(3) Obtain different ways in which 9 can be written as a product of two numbers using the  $10 \times 10$  multiplication table and fill in the blanks.

- (i)  $\dots\dots\dots \times \dots\dots\dots$       (ii)  $\dots\dots\dots \times \dots\dots\dots$

(4) Write down the different ways in which 30 can be written as a product of two whole numbers using the  $10 \times 10$  multiplication table. Thereby write down factors of 30.

(5) Is 4 a factor of 9? Explain the reason for your answer.

## 11.3 Finding the factors using the method of division

When a number is divided by a factor, there is no remainder. Let us establish this fact through the following examples. The factors of 6 have been obtained before as 1, 2, 3 and 6. When 6 is divided by each of the numbers 1, 2, 3 and 6, there is no remainder.

$$6 \div 1 = 6 \text{ with remainder } 0.$$

$$6 \div 2 = 3 \text{ with remainder } 0.$$

$$6 \div 3 = 2 \text{ with remainder } 0.$$

$$6 \div 6 = 1 \text{ with remainder } 0.$$

Let us divide 6 by 4 and 5 which are not factors of 6.

$$4 \overline{) 6} \begin{array}{r} 1 \\ 4 \\ \hline 2 \end{array}$$

$6 \div 4 = 1$  with a remainder of 2.

$$5 \overline{) 6} \begin{array}{r} 1 \\ 5 \\ \hline 1 \end{array}$$

$6 \div 5 = 1$  with a remainder of 1.

Accordingly, when 6 is divided by any of its factors 1, 2, 3 or 6, there is no remainder. However, when 6 is divided by 4 or 5 which are not factors of 6, there is a remainder of 2 and 1 respectively.

If a certain whole number can be divided by another whole number such that there is no remainder, then we identify the second number as a factor of the first number.

Since any whole number can be divided by 1 and the number itself with no remainder, 1 and the number itself are factors of the given number.

### Example 1

Find three factors of 30 by using the method of division.

$$2 \overline{) 30} \begin{array}{r} 15 \\ 2 \\ \hline 10 \\ 10 \\ \hline 0 \end{array}$$

$$3 \overline{) 30} \begin{array}{r} 10 \\ 3 \\ \hline 30 \\ \hline 0 \end{array}$$

$$5 \overline{) 30} \begin{array}{r} 6 \\ 5 \\ \hline 30 \\ \hline 0 \end{array}$$

The number 30 can be divided by 2, 3 and 5 with no remainder. Therefore, 2, 3 and 5 are three factors of 30.

### Example 2

Is 9 a factor of 12? Explain the reason for your answer.

9 is not a factor of 12.

$12 \div 9 = 1$  with a remainder of 3.

12 cannot be divided by 9 with a zero remainder.

Therefore, 9 is not a factor of 12.

$$9 \overline{) 12} \begin{array}{r} 1 \\ 9 \\ \hline 3 \end{array}$$

### Exercise 11.3

(1) Find three factors of each of the following numbers using the method of division.

(i) 28

(ii) 32

(iii) 54

(iv) 90

(v) 21

(2) Is 6 a factor of 84? Explain the answer by the method of division.

(3) Is 5 a factor of 48? Explain the reason for your answer.

### 11.4 Multiples

The answers that are obtained when 2 is multiplied by the whole numbers 1, 2, 3, 4 and 5 are given below.

$$2 \times 1 = 2$$

$$2 \times 2 = 4$$

$$2 \times 3 = 6$$

$$2 \times 4 = 8$$

$$2 \times 5 = 10$$

A number which is obtained by multiplying 2 by a whole number in this manner is known as a **multiple of 2**.

In the same manner, a number which is obtained by multiplying 3 by a whole number is known as a multiple of 3.

- 3, 6, 9, 12, 15 and 18 are several multiples of three.

Similarly,

- 5, 10, 15 and 20 are several multiples of five.

Also, observe the following results.

- All the multiples of 2 can be divided by 2 with no remainder.
- All the multiples of 3 can be divided by 3 with no remainder.

It is clear that any multiple of a whole number can be divided by that number with no remainder.

Let us discuss more about multiples.

Consider the numbers which can be written as a product of two whole numbers.

For example,  $18 = 3 \times 6$ .

Here 18 is obtained by multiplying 3 by 6. That is 18 is a multiple of 3. We may write  $18 = 6 \times 3$  as well.

There fore 18 is obtained by multiplying 6 by 3 and hence 18 is a multiple of 6.

That is, 18 is a multiple of both 3 and 6.

### Example 1

Check whether 14 is a multiple of 2 by dividing 14 by 2.

$$\begin{array}{r} 7 \\ 2 \overline{)14} \\ \underline{14} \\ 0 \end{array}$$

Since 14 can be divided by 2 with no remainder, 14 is a multiple of 2.

### Example 2

Check whether 42 is a multiple of 3 by dividing 42 by 3.

$$\begin{array}{r} 14 \\ 3 \overline{)42} \\ \underline{3} \\ 12 \\ \underline{12} \\ 0 \end{array}$$

Since 42 can be divided by 3 with no remainder, 42 is a multiple of 3.





## Activity 1

| $\times$ | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10  |
|----------|----|----|----|----|----|----|----|----|----|-----|
| 1        | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10  |
| 2        | 2  | 4  | 6  | 8  | 10 | 12 | 14 | 16 | 18 | 20  |
| 3        | 3  | 6  | 9  | 12 | 15 | 18 | 21 | 24 | 27 | 30  |
| 4        | 4  | 8  | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40  |
| 5        | 5  | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50  |
| 6        | 6  | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60  |
| 7        | 7  | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70  |
| 8        | 8  | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80  |
| 9        | 9  | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90  |
| 10       | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |

- Copy the given multiplication table.
- Draw a circle around each of the multiples of 2 in the table.
- Enclose each multiple of three within a triangle.
- Write down 5 numbers which are enclosed within both a circle and a triangle.
- What is the smallest number which is a multiple of both 2 and 3?
- You will notice that the numbers which are enclosed within both a circle and a triangle are the multiples of 6. Accordingly, which two numbers are the multiples of 6 necessarily a multiple of?
- According to the above conclusions, 15 is a multiple of 15. What other numbers is 15 a multiple of?
- Of which numbers is 45 a multiple?

## Exercise 11.4

- Write down five multiples of 2 which are greater than 10.
- Write down four multiples of 3 between 1 and 20.
- Write down all the multiples of 4 between 1 and 25.

- (4) From the following numbers, select and write down the multiples of three.

26, 60, 115, 48, 26, 14, 27

- (5) (i) How many multiples of 9 are there between 1 and 100?  
(ii) Of these numbers, which is the greatest multiple of 9?

- (6) Write down three multiples of 18.

- (7) What is the largest multiple of 9 which is less than 150?

- (8) Write down five multiples of each of the following numbers.

(i) 4            (ii) 13        (iii) 15        (iv) 18        (v) 20

- (9) Fill in the blanks.

(i) Any multiple of 10 is necessarily a multiple of ..... and .....

(ii)  $11 \times 7 = 77$ . Therefore, 77 is a multiple of .... and of .....

- (10) Write down two numbers which are multiples of both 3 and 4.

- (11) Write down a number which is a multiple of 2, 3 and 4.

### ● Solving problems related to factors and multiples

Now let us solve problems related to factors and multiples.

#### Example 1

There are 30 apples in a bag. They are separated into bags such that each bag has an equal number of apples.

Find out the number of apples in a bag and the number of bags in each instance.

The total number of apples is obtained by the product of the number of apples in a bag and the number of bags.

Hence we can find out the number of bags and the number of apples in each bag by considering the products of two factors of 30.

$$30 = 1 \times 30$$

$$30 = 2 \times 15$$

$$30 = 3 \times 10$$

$$30 = 5 \times 6$$

$$30 = 6 \times 5$$

$$30 = 10 \times 3$$

$$30 = 15 \times 2$$

$$30 = 30 \times 1$$

Accordingly there are eight ways of grouping 30 apples in bags containing the same number of apples.

30 bags with 1 apple in a bag.

15 bags with 2 apples in a bag.

10 bags with 3 apples in a bag.

6 bags with 5 apples in a bag.

5 bags with 6 apples in a bag.

3 bags with 10 apples in a bag.

2 bags with 15 apples in a bag.

1 bag with 30 apples inside it.

### Exercise 11.5

- (1) The price of a ballpoint pen is Rs 12. Find the price of 8 such pens.  
Is it a multiple of 8 and 12?

- (2) A water pump takes a day to fill a household tank of capacity 75 gallons. What is the number of times and the total volume of water it can fill during a week?

- (3) The price of a Rambutan is 6 rupees. Five children bought 2, 3, 4, 5 and 6 Rambutans respectively. Find the amount each student spent.

- (4) A parcel containing the following items needs to be given to each student who participated in a certain event. Find how much it will cost to give 50 students a parcel each.

There are three gingerly rolls (thalaguli), one fish bun, two bananas and a packet of milk in each parcel.

The price of one packet of milk is Rs 30.00

The price of a gingerly roll is Rs 5.00

The price of a fish bun is Rs 30.00

The price of a banana is Rs 10.00

- (5) Separate 50 pupils into groups such that each group has an equal number of pupils. What are the numbers which can be taken in each group?

## 11.5 Divisibility

Through divisibility we can learn about the ability of one whole number to divide another whole number, without a remainder.

Given two numbers, if there is no remainder when one number is divided by the other, then the first number is said to be divisible by the second number.

For example, when 27 is divided by 3, there is no remainder. Therefore, 27 is divisible by 3.

### ● Examining whether a number is divisible by 2



#### Activity 2

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20

- Draw a circle around each of the above numbers which is divisible by 2.
- Write down the digit in the ones place of each number that is divisible by 2.
- Then you will obtain the numbers 2, 4, 6, 8 and 0. Observe that these too are divisible by 2.
- Examine the digits in the ones place of the numbers which are not divisible by 2 (not circled). They are 1, 3, 5, 7 and 9. These are not divisible by 2.

Accordingly, if the value of the digit in the ones place of a number is divisible by 2, then the number is divisible by 2. Also, if the value of the digit in the ones place of a number is not divisible by 2, then the number is not divisible by 2.



- **Examining whether a number is divisible by 5**

We have learnt earlier that multiples of 5 such as 5, 10, 15, 20, 25, 30, 35, 40, 45, ... can be divided by 5 without a remainder.

Examine the digit in the ones place of these numbers.

The digit in the ones place of these numbers is always 0 or 5.

That is, if the digit in the ones place of a number is 0 or 5 then that number is divisible by 5.

- **Examining whether a number is divisible by 10**

We have learnt earlier that multiples of 10 such as 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, ... can be divided by 10 without a remainder.

Examine the digit in the ones place of these numbers.

The digit in the ones place of all these numbers is 0.

That is, if the digit in the ones place of a number is 0, then that number is divisible by 10.

### **Exercise 11.6**

- (1) From the following, select and write down the numbers which are divisible by 2.  
25, 33, 42, 57, 64, 69, 126, 135, 148, 250, 331, 1457, 3263, 4584, 2689, 3150, 2472
- (2) What are the digits that the blank box in  $128\Box$  can be filled with, if this number consisting of four digits is divisible by 2?
- (3) Select and write down the numbers which are suitable for the blank boxes to the right of the box containing numbers. (A number could be included in more than one box)



(a) Numbers which are divisible by 2.

105, 212, 310,  
256, 125, 375,  
420, 860, 1236,  
3245, 5180, 1800

(b) Numbers which are divisible by 5.

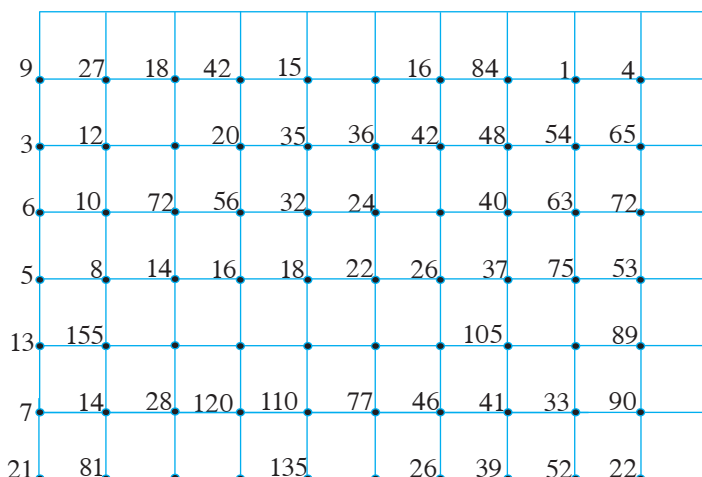
(c) Numbers which are divisible by 10.

- (4) (i) Are there numbers which are common to all three of the boxes on the right in question 3 above? What are these numbers?
- (ii) Are the numbers in box (c) essentially in boxes (a) and (b) ?
- (iii) What are the numbers that are common to both boxes (a) and (b) in question 3 above. Examine whether these numbers are in box (c) as well. Accordingly, write down the conclusion that you can draw.
- (5) Join the points relevant to the numbers which are divisible by 2 in ascending order. Then, join the points relevant to the smallest number and the largest number you joined.

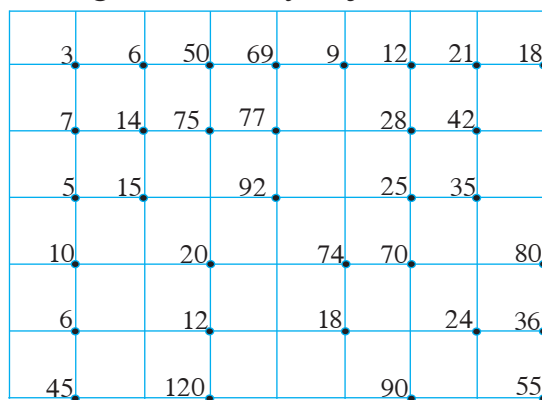
|    |    |    |    |    |    |    |  |
|----|----|----|----|----|----|----|--|
| 3  | 4  | 9  | 23 | 25 | 6  | 37 |  |
| 2  | 16 | 21 | 51 | 1  | 10 | 8  |  |
| 5  | 43 | 19 | 27 | 31 | 19 | 41 |  |
| 31 | 53 | 37 | 29 | 25 | 41 | 23 |  |
| 11 | 14 | 17 | 39 | 33 | 12 | 43 |  |
|    |    |    |    |    |    |    |  |



- (6) Join the points relevant to the numbers which are divisible by 5 in ascending order. Then join the points relevant to the smallest number and the largest number you joined.



- (7) Join the points relevant to the numbers which are divisible by 10 in ascending order. Then join the points relevant to the smallest number and the largest number you joined.



### Miscellaneous Exercise

- (1) Give reasons why 7 is not a factor of 45.
- (2) All the factors of a certain number, apart from the number itself are 1, 2, 3 and 6. What is this number ?

- 
- (3) The number of marbles in a certain box is a multiple of 6. When this number is rounded off to the nearest multiple of ten, the value obtained is 40. Write down the two values that the number of marbles in the box can take.
- (4) The number of biscuits there are in a certain packet of biscuits is a multiple of four which is less than 20. When this number is rounded off to the nearest multiple of 10, the value obtained is 20. How many biscuits are there in the packet?

### Summary

- A whole number is divisible by each of its factors.
- When a whole number is multiplied by another whole number, a multiple of the initial number is obtained.
- If the value of the digit in the ones place of a number is divisible by 2, then the number is divisible by 2.
- The digit in the ones place of a number which is divisible by 5 is either 0 or 5.
- The digit in the ones place of a number which is divisible by 10 is 0.