24 Indices

By studying this lesson, you will be able to,

- recognize the index notation
- write a number as a power of another number and
- expand a power and write its value.

24.1 Index notation

In mathematics there are instances when a number has to be multiplied repeatedly and written.

In the lesson on factors you wrote $16 = 4 \times 4$
In the same manner we can write $16 = 2 \times 2 \times 2 \times 2$

There is a method of writing a number which is multiplied repeatedly as above in a concise way.

Activity 1

In the following table the first few examples show how a number which is multiplied repeatedly is written in a concise way. After understanding the method, fill in the blanks in the other examples.

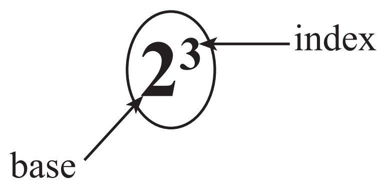
Product	Written concisely
$2 \times 2 \times 2$	2^3
3×3	3^2
2×2	2^2
4×4	....
$5 \times 5 \times 5$	....
....	6^2
....	7^3
$3 \times 3 \times 3 \times 3 \times 3$	....

When a number which is multiplied repeatedly is written in the above manner in a concise way, we say that it is written using indices or that it is written in index form.

$2 \times 2 \times 2$ is written using indices as 2^3.

The fact that 2 is multiplied three times over in this product is indicated by the small digit 3 written at the top, on the right hand side of 2.

In 2^3, 2 is defined as the base and 3 is defined as the index. This is read as two to the power three (or two to the power of three).



The value of 2^3 is equal to the value of $2 \times 2 \times 2$. That is, its value is 8.

A number to the power two is also called the square of the number.
Example: Five to the power two, or 5^2, is also five squared.

A number to the power three is also called the cube of the number.
Example: Eight to the power three, or 8^3, is also eight cubed.

Activity 2

For each of the numbers given below in index form, write down the base, the index, and how it is read.

Number	Written using index notation	Base	Index	How it is read
25	5^2	5	2	Five to the power two
81	3^4	....	....	....
64	2^6	....	....	....
1000	10^3	....	....	....
243	....	3	5	....
625	....	....	....	Five to the power four

Example 1

Write $3 \times 3 \times 3 \times 3$ using index notation.

$3 \times 3 \times 3 \times 3 = \underline{\underline{3^4}}$

Example 2

Find the value of 2^6.

$2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = \underline{\underline{64}}$

Example 3

Write $2 \times 2 \times 2 \times 5 \times 5$ using indices.

$2 \times 2 \times 2 \times 5 \times 5 = \underline{\underline{2^3 \times 5^2}}$

Example 4

Find the value of $5^2 \times 7^3$.

$5^2 \times 7^3 = 5 \times 5 \times 7 \times 7 \times 7 = \underline{\underline{8575}}$

Example 5

Find the value of $2^4 \times 3^2$.

$2^4 \times 3^2 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = \underline{\underline{144}}$

Exercise 24.1

(1) Fill in the blanks in the following table.

Product	Index Form	Base	Index	Value of the Product
7×7	7^2	7	2	49
$4 \times 4 \times 4$	....	4	....	64
$5 \times 5 \times 5 \times 5$	....	....	....	625
....	2^3	....	....	....
....	....	2	2	....
....	5^3	....	....	....

(2) Write down using index notation, each of the following numbers which has been expressed as a product. Find also the value of each product.

(i) 5×5 (ii) $10 \times 10 \times 10$
(iii) $3 \times 3 \times 3$ (iv) $1 \times 1 \times 1$
(v) $1 \times 1 \times 1 \times 1 \times 1$ (vi) $7 \times 7 \times 7 \times 7$
(vii) $5 \times 5 \times 7 \times 3 \times 3$ (viii) $6 \times 3 \times 3 \times 3 \times 4 \times 4$

(3) Write each of the following numbers which has been expressed in words, in index form and as a product, and then write down its value.

(i) Two to the power two (ii) Four cubed
(iii) Six squared (iv) Three to the power four
(v) Two to the power six (vi) Three cubed

(4) Evaluate:

(i) $2^2 \times 3$ (ii) $2^3 \times 3$ (iii) $2^2 \times 3^2$
(iv) 3^4 (v) $2^2 \times 3^2 \times 4^2$

24.2 Representing a number as a power of a given number

To express the number 16 as a power of 2, it is necessary to know how many times 2 should be multiplied by itself to obtain the value 16.

$$16 = 2 \times 2 \times 2 \times 2$$

That is, $16 = 2 \times 2 \times 2 \times 2 = 2^4$

It is easy to use division to find the number of times a particular number should be multiplied to obtain a given number. In the above example, when the base is identified as 2, to find the index, the number 16 should be divided repeatedly by 2.

$$\begin{array}{r|l} 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array} \qquad 16 = 2 \times 2 \times 2 \times 2 = \underline{\underline{2^4}}$$

Example 1

Express 81 as a power of 3.

$$\begin{array}{r|l} 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$81 = 3 \times 3 \times 3 \times 3 = 3^4$

Example 2

Express 125 as a power of 5.

$$\begin{array}{r|r} 5 & 125 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$125 = 5 \times 5 \times 5 = 5^3$

Exercise 24.2

(1) (i) What is two times 5 equal to?
(ii) What is 5 to the power two equal to?

(2) (i) What is three times 4 equal to?
(ii) What is 4 to the power three equal to?

(3) Write 32 as a power of 2.

(4) Write 144 as a power of 12.

(5) Write down 64

(i) as a power of 2. (ii) as a power of 4.
(iii) as a power of 8.

(6) Write down 81

(i) as a power of 3. (ii) as a power of 9.

(7) Indicate the truth/falsehood of the following statements.

(i) $2^3 = 8$ (ii) $3^2 = 6$ (iii) $3^2 = 8$ (iv) $5^2 = 10$
(v) $2^5 = 32$ (vi) $3^2 = 9$ (vii) $2^4 = 4^2$ (viii) $2^4 = 8$
(ix) $7^3 = 21$ (x) $5^3 = 15$ (xi) $3^5 = 243$

Miscellaneous Exercises

(1) What are the base and the index of 7^2?

(2) Write down the following products using index notation.

(i) $5 \times 5 \times 5 \times 5$
(ii) $4 \times 4 \times 7 \times 7 \times 7$
(iii) $3 \times 3 \times 3 \times 3 \times 3 \times 8$
(iv) $2 \times 2 \times 2 \times 3 \times 3 \times 5$
(v) $4 \times 4 \times 4 \times 5 \times 7 \times 7 \times 7 \times 7$
(vi) $2 \times 3 \times 3 \times 2 \times 5 \times 2 \times 3 \times 5$

(3) Find the value of each of the following expressions.

(i) $2^4 \times 5^2$
(ii) $3^2 \times 7^2$
(iii) $11^2 \times 10^2$
(iv) $2^3 \times 5^2 \times 7$
(v) $2^2 \times 3^3 \times 5^2$

(4) Fill in the blanks.

(i) $36 = 6^{\square}$
(ii) $8 = 2^{\square}$
(iii) $125 = 5^{\square}$
(iv) $\square = 10^2$
(v) $\square = 3^4$

(5) Write down 256

(i) as a power of 2.
(ii) as a power of 4.
(iii) as a power of 16.

(6) Write down 729

(i) as a power of 7.
(ii) as a power of 9.
(iii) as a power of 27.

(7) Fill in the blanks appropriately with either the symbol “<” or the symbol “>”.

(i) 2^3 ...... 3^2
(ii) 3^4 ...... 4^3
(iii) 2^4 ...... 4^2
(iv) 8^1 ...... 1^8
(v) 2^4 ...... 4^2
(vi) 3^2 6

Summary

- To represent a number using index notation is to write a repeated product of a number in a concise way.

$$2 \times 2 \times 2 \times 2 \times 2 = 2^5$$

- In the expression 2^5, the base is 2 and the index is 5.
- A number can be written as a power of a given number.
- Division is used to find the index when expressing a number as a power of another number.

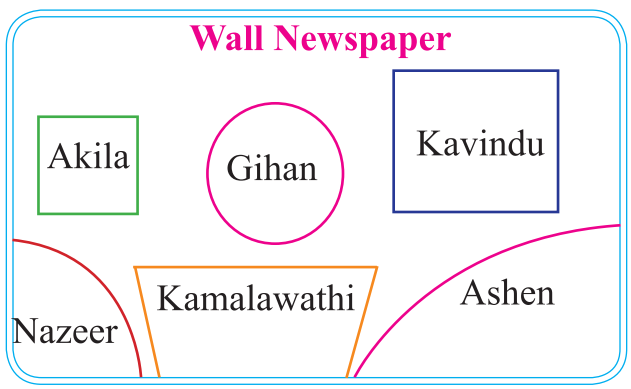
25 Area

By studying this lesson, you will be able to,

- identify the amount of space occupied by a surface as its area,
- measure the area using arbitrary units,
- recognize cm^2 as a unit of measurement of area,
- measure the areas of squares and rectangles using a $1\ cm \times 1\ cm$ square grid and
- create figures of given area using $1\ cm^2$ square laminas.

25.1 Identifying what area is

The space allocated for six students to display their creations on a wall newspaper is shown in the figure.



The space allocated to each student can be identified as a surface which is bounded by line segments. The space occupied by a surface is known as its **area**.

Observe the amount of space each student has been allocated.

We can easily say that the space allocated to Kavindu on the wall newspaper is more than the space allocated to Akila because both have been allocated similar shaped spaces.

That is, Kavindu has been allocated a space with a larger area than the area of the space Akila has been allocated.

25.2 Measuring the area using arbitrary units

Activity 1

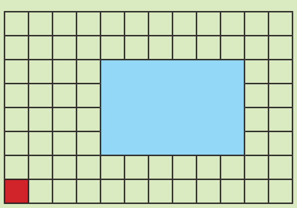
Step 1 - Cut out a square shaped lamina of side length 6 cm from a piece of cardboard.

Step 2 - By taking the area of the lamina to be 1 unit, determine how many units the area of the following surfaces are by placing the lamina on each of them.

1. The front page of your mathematics textbook.
2. The front page of your mathematics exercise book.
3. The top surface of your desk.

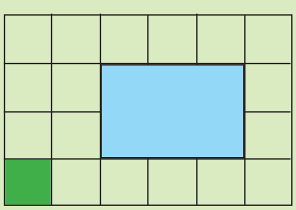
Step 3 - Cut out a rectangular shaped lamina of length 8 cm and breadth 3 cm from a piece of cardboard.

Step 4 - As before, find the area of the above surfaces using this lamina.



The figure illustrates how a student has placed a rectangular shaped lamina on a page of his mathematics exercise book to determine the area of the lamina.

Let us take the area of the square which has been shaded in red as one unit. Then the area of the rectangular lamina is 24 of these units.

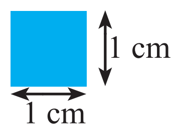


This figure illustrates how another student has placed the same rectangular lamina on a new square grid of a different size to find its area.

Let us take the area of the square which is shaded green as 1 unit. Then the area of the rectangular lamina is 6 of these units.

Two values which are numerically different to each other and which depend on the unit that has been used, are obtained for the area of one and the same rectangular lamina, namely 24 units of the red square and 6 units of the green square. Therefore, when the area is given, the unit used to measure the area too has to be mentioned.

$1\text{cm} \times 1\text{cm}$ square lamina

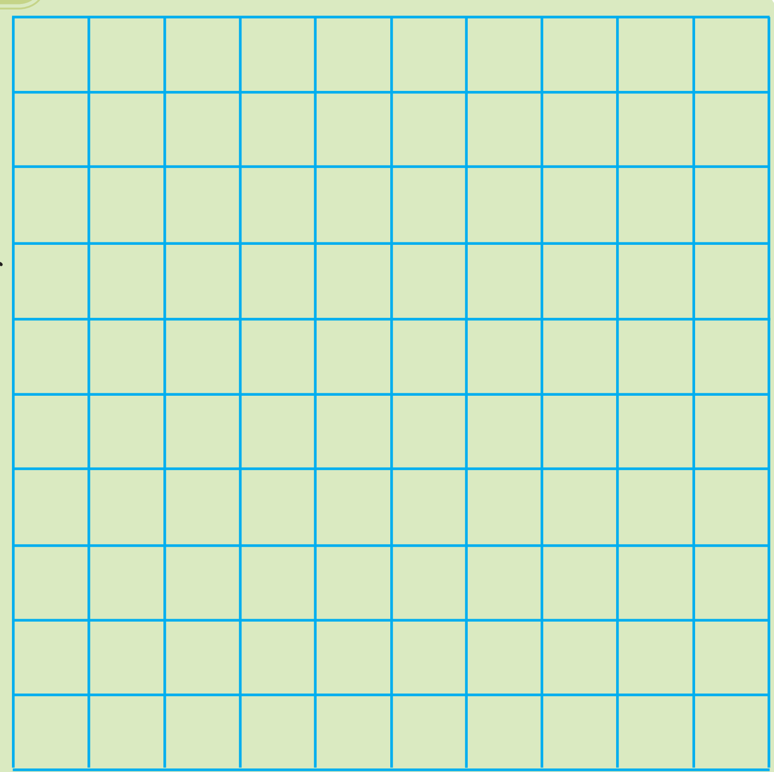


Area of lamina $= 1\ \text{cm}^2$

The area of a square lamina of side length 1 cm is used as a standard unit to measure the area of surfaces. It is defined as one square centimetre and is denoted by $1\ \text{cm}^2$.

Activity 2

Step 1 -
On a tissue paper, draw a $1\ \text{cm} \times 1\ \text{cm}$ square grid as shown in the figure. (Supply yourself with a transparent sheet of paper on which a $1\ \text{cm} \times 1\ \text{cm}$ square grid has been printed)



Step 2 - Draw figures of squares and rectangles of the following dimensions on a suitable piece of paper.

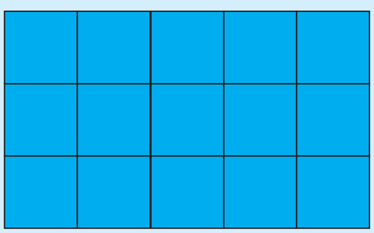
Squares of side length 3 cm/ 5 cm/ 10 cm and rectangles of length 3cm and breadth 2 cm/ length 6 cm and breadth 4 cm/ length 10 cm and breadth 6 cm

Step 3 - Place the square grid that you prepared on each of the above drawn plane figures and find the area of each figure by counting the squares.

Step 4 - Write down the area you found out beside each figure.

Example 1

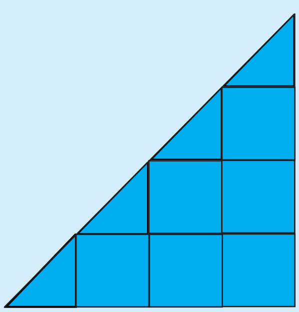
Find the area of the following figure by counting the squares. Take the area of each small square as 1 cm^2.



Number of squares in the figure = 15
Area of a square is 1cm^2.
Therefore, Area of the figure = 15 cm^2

Example 2

Find the area of the following figure by counting the squares. Take the area of each small square as 1 cm^2.

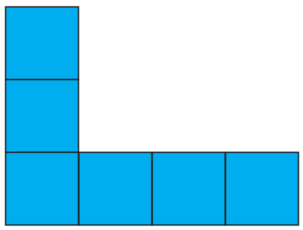


In this figure there are 6 small squares and four parts which are half a square each. Since these four parts together make up two squares, the total area is 8 cm^2.

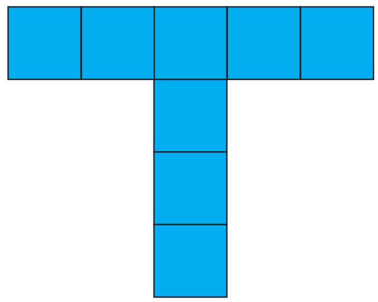
Exercise 25.1

(1) By taking the area of each small square as 1 cm^2, find the area of each of the following figures by counting the squares.

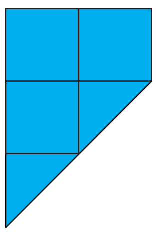
(i)



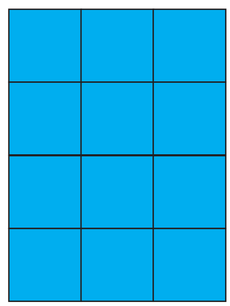
(ii)



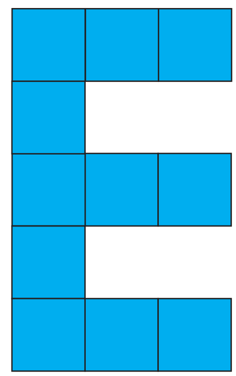
(iii)



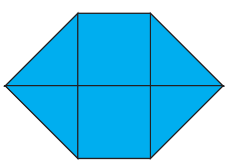
(iv)



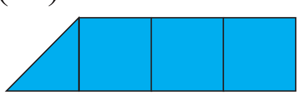
(v)



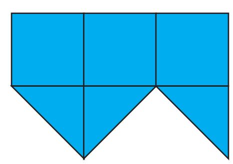
(vi)



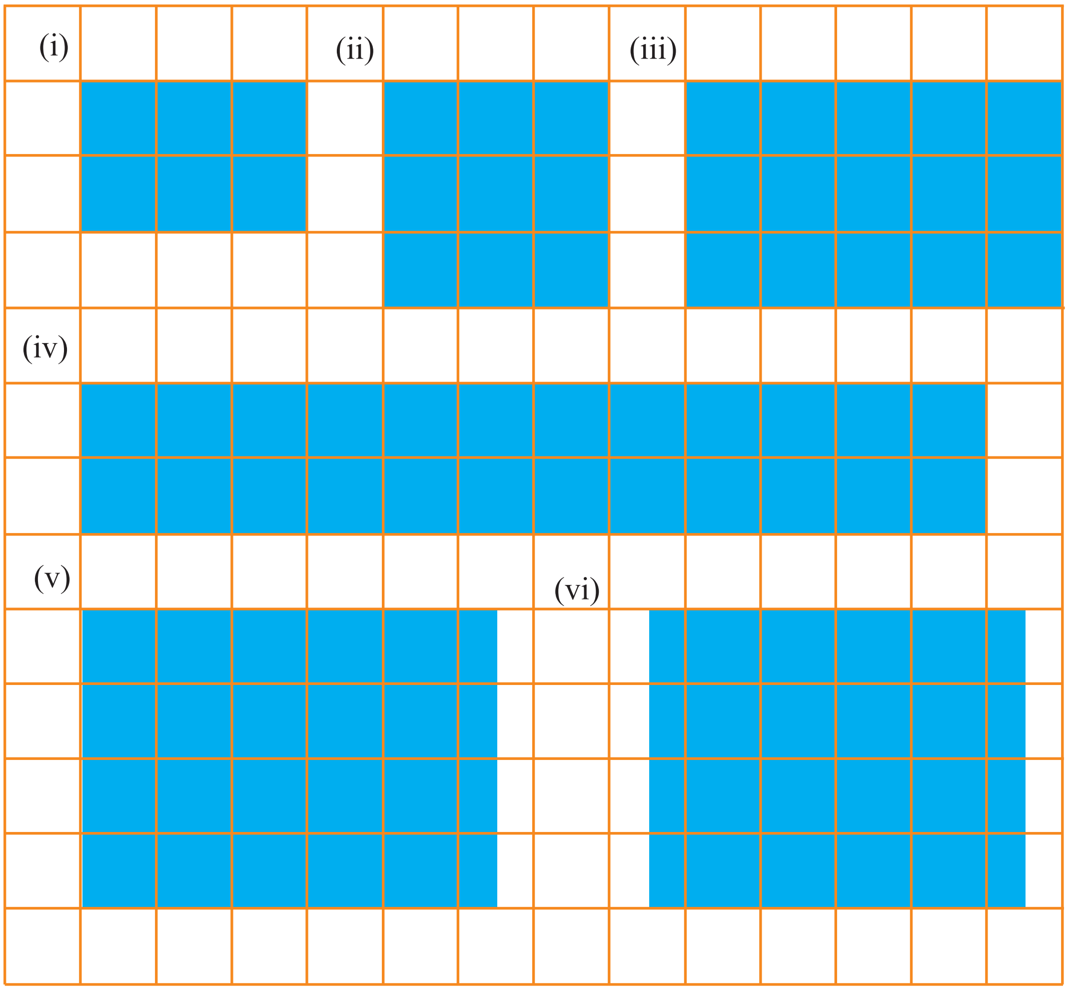
(vii)



(viii)

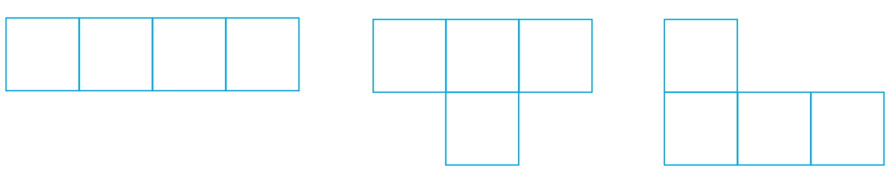


(2) Find the area of each of the following figures which have been drawn on a 1 cm × 1 cm square grid.



25.3 Constructing plane figures using 1 cm² laminas

Cut out 4 square laminas of area 1 cm^2 each. Illustrated below are various composite figures that have been created by joining together such laminas. What can you say about the area of each figure?

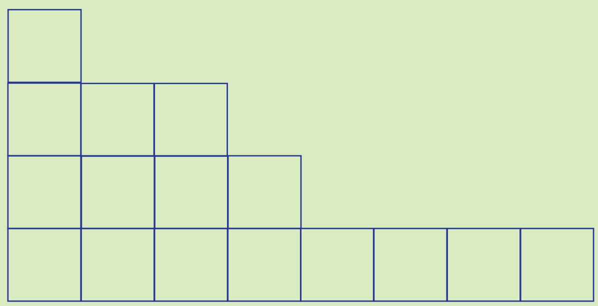


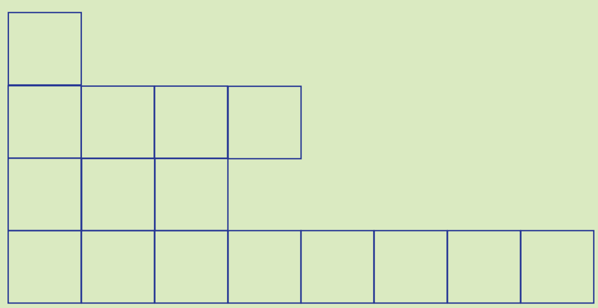
You may have noticed that although the shapes of the above laminas are different to each other, the area of each lamina is 4 cm^2.

Activity 3

Step 1 - Cut out 16 square laminas of area 1 cm^2 each.

Step 2 - Create various plane figures by joining all of these laminas together. Some plane figures which have been created in this manner are shown below.





Find the area of these two figures by counting the squares.

What can you say about the area of each of the figures?

Step 3 - Create squares of side length 2 cm, 3 cm and 4 cm respectively, and find the area of each of these squares by counting the number of 1 cm^2 squares that are in them.

Activity 4

Step 1 - Cut out about 100 square laminas of area 1 cm^2 each from various coloured paper.

Step 2 - Using the laminas that have been cut out, create the following figures and paste them in your exercise book.

(i) A square figure of area equal to 25 cm^2.
(ii) A rectangular figure of area equal to 24 cm^2.
(iii) A rectangular figure of length 5 cm and breadth 4 cm.

Step 3 - Create meaningful figures that you desire and paste them in your book. Next to each figure, write down the area and the name of the figure.

Summary

- The space occupied by a surface is known as its area.
- An arbitrary unit can be used to measure area.
- When the area is given, the unit used to measure the area too has to be mentioned.
- cm^2 is a unit that is used to measure area.
- Measuring the area of a given figure and creating figures of a given area can be carried out using square laminas of area 1 cm^2 each.

Revision Exercise - 3

(1) What is the number represented by  ?

(2) Find the value of the algebraic expression $x - 2$ when $x = 14$.

(3) Write 2085 g in grammes and kilogrammes.

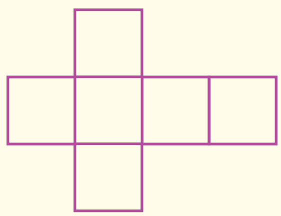
(4) Write two ratios that are equivalent to 2 : 7.

(5) If the price of 6 mangoes is Rs. 72, what is the price of 3 mangoes?

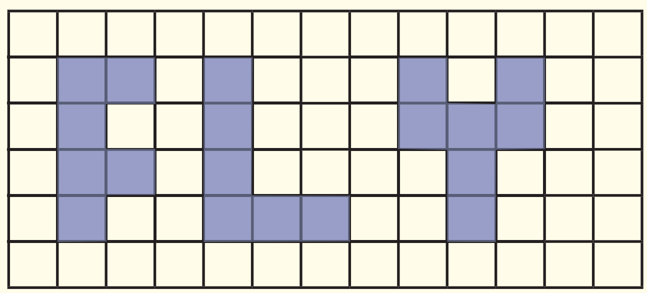
(6) Find the value of $2^3 \times 3^2$.

(7) Write 81 as a power of 3.

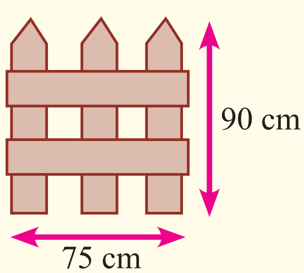
(8) (i) The following is a net of 1 cm × 1 cm squares that can be used to make a cube. What is the area of all the faces of the cube made with this net?



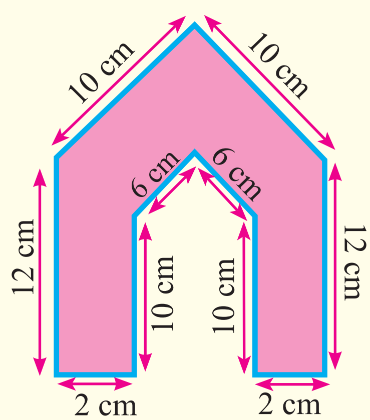
(ii) A board with the word "Fly" is shown here. If each small square is 1cm × 1 cm, what is the total area of the letters in cm^2?



(9) (i) A panel of a small gate is made of wooden planks. Indicate the total length of the wooden planks used to make the gate in metres and centimetres.



(ii) Find the perimeter of the following figure.



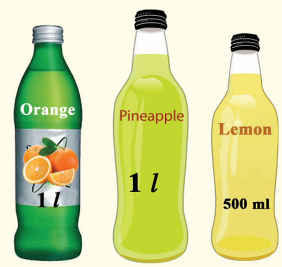
(10) (i)

$$\begin{array}{rrr} & \text{kg} & \text{g} \\ & 2 & 750 \\ + & 3 & 475 \\ \hline\hline \end{array}$$

(ii)

$$\begin{array}{rrr} & \text{kg} & \text{g} \\ & 6 & 600 \\ - & 3 & 799 \\ \hline\hline \end{array}$$

(11) It is required to prepare a mixed fruit drink for a party. 3 bottles of orange juice having 1 litre each, 2 bottles of pineapple juice having 1 litre each and 500 ml of lemon juice are mixed for this purpose.



(i) What is the amount of orange juice mixed in millilitres?

(ii) What is the amount of pineapple juice mixed in millilitres?

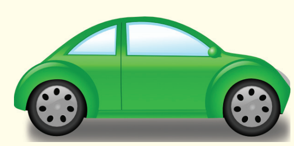
(iii) If 4 litres of water is added to the mixture, express the total amount of mixed drink in litres and millilitres.

(iv) Find the ratio of the amount of orange juice to the amount of pineapple juice in the mixture.

(v) Find the ratio of the amount of orange juice to the amount of lemon juice in the mixture. Express this ratio in the simplest form.

(vi) If 38 persons participate in the party, find out whether this amount of drink is adequate to serve 250 millilitres each.

(12) The price of a petrol litre is x rupees.



(i) If the price of a petrol litre is increased by 12 rupees, write down an expression in terms of x for the price of a petrol litre after the increase in price.

(ii) Write down an expression in terms of x for the balance that a person receives after purchasing a petrol litre before the increase in price, by giving 200 rupees.

(iii) A vehicle can travel y kilometres with a petrol litre. If the vehicle travels 10 kilometres after pumping one petrol litre, write down the number of kilometres in terms of y, that the vehicle can travel further by using the remaining petrol.

(iv) If $y = 14$, find the value of the expression obtained in (iii).

(v) If $x = 160$, find the values of the expressions obtained in (i) and (ii) separately.

(13)

Sugar 750 g

Margarine 750 g

Flour 1 kg

A certain type of sweet is prepared by mixing the items given above and then adding water.

(i) Express the total mass of the items mixed other than water in kilogrammes and grammes.

(ii) The mass of the mixture after adding water is 3 kg 75 g. What is the mass of the added water?

(iii) Sudeepa says that the number of sweets made will exceed 200, if a mass of 15 g is used from the mixture (with water) to prepare one sweet. Show that her statement is true.

(14) The marks received by 30 students in an evaluation which carried 10 marks are given below.

3	6	5	7	1	8	6	7	8	4
7	8	3	0	9	7	2	3	5	6
5	9	7	8	10	4	1	6	7	6

(i) Represent this data in the following table by using tally marks.

Marks	The number of students who obtained the marks in the first column denoted by tally marks	The number of students
0, 1, 2, 3, 4		
5, 6, 7		
8, 9, 10		

(ii) Represent this data by a picture graph.

(iii) How many students obtained 5 marks or higher?

(iv) If the students are categorized as,

8 - 10 marks : Reached the competency level

5 - 7 marks : Approaching the competency level

0 - 4 marks : Direct to remedial learning

Write the number of students that belong to each category separately.

(15) (i) If $16 = 2^{\square} = 4^{\square}$, write the suitable numbers in the boxes.

(ii) Expand $3^2 \times 2^2$ and find its value.

(iii) Write suitable numbers in the boxes.

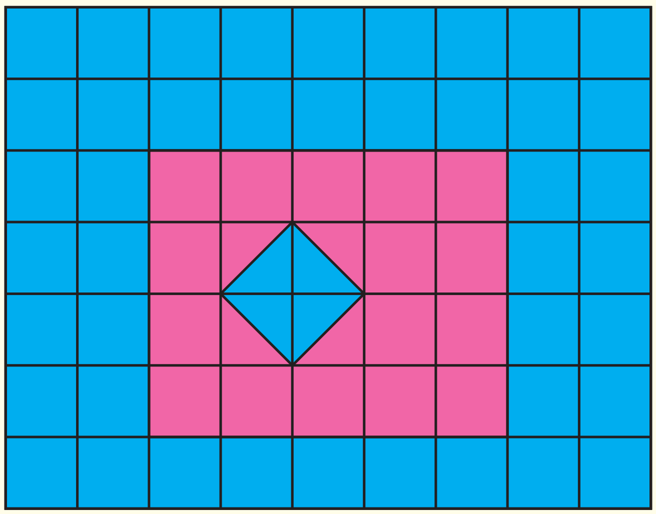
(a) $64 = 2^{\square}$ (b) $64 = 4^{\square}$ (c) $64 = 8^{\square}$

(iv) Write 1024 as,

(a) a power of 2. (b) a power of 4.

(v) Find the larger number from 2^6 and 6^2.

(16) A cushion cover is made of 1 cm × 1 cm squares in the two colours red and blue as follows.

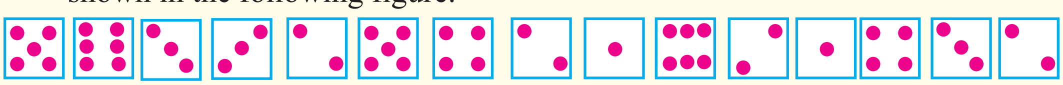


(i) What is the total area of the parts coloured in blue in cm^2?

(ii) What is the total area of the parts coloured in red in cm^2?

(iii) Find the ratio of the total red area to the total blue area.

(17) The results that were obtained when a cubic die was thrown 15 times are shown in the following figure.



(i) Express the number of occurrences of the numbers of each of the following types as a fraction of the total number of occurrences.

(a) Even numbers (b) Odd numbers (c) Prime numbers
(d) Composite numbers (e) Triangular numbers (f) Square numbers

(ii) The fractions relevant to which two types of numbers when added results in the sum $\frac{14}{15}$?

(iii) Show that the fraction resulting from subtracting the fraction relevant to composite numbers from the fraction relevant to odd numbers is equivalent to $\frac{14}{15}$.