(21) Ratio

By studying this lesson, you will be able to,

- understand the concept of a ratio of two quantities,
- write ratios equivalent to a given ratio,
- write a ratio in its simplest form and
- understand the difference between ratio and rate.

21.1 Introduction to ratio

Fruit juice   Water

Shown above is a label of a fruit juice bottle. It illustrates that a certain amount of fruit juice should be mixed with five equal amounts of water. Accordingly, six bottles of fruit drinks can be prepared by mixing one part of fruit juice with five parts of water.

The number of male workers and female workers who work in an office can be compared numerically.

There are many instances where we have to compare the quantities of certain things or to compare the amounts of several quantities.

The following are some examples.

- A cement mixture is made by mixing certain amounts of cement and sand.
- When making cakes, flour and sugar are mixed together.
- To make a concrete mixture, metal, sand and cement have to be mixed together.
- The number of girls and boys in a school can also be considered.

Let us consider a mixture that we make by mixing several ingredients together.

Irrespective of the amount of the mixture, very often we have to maintain its composition throughout the mixture. In such instances we have to know the relationship between the quantities of ingredients we have to mix. For this reason it is necessary to express all ingredients using the same unit.

The same mixture of the above mentioned fruit juice can also be prepared in the following ways.

1 glass of fruit juice   5 glasses of water

1 *l* fruit juice bottle   5 *l* Water bottle
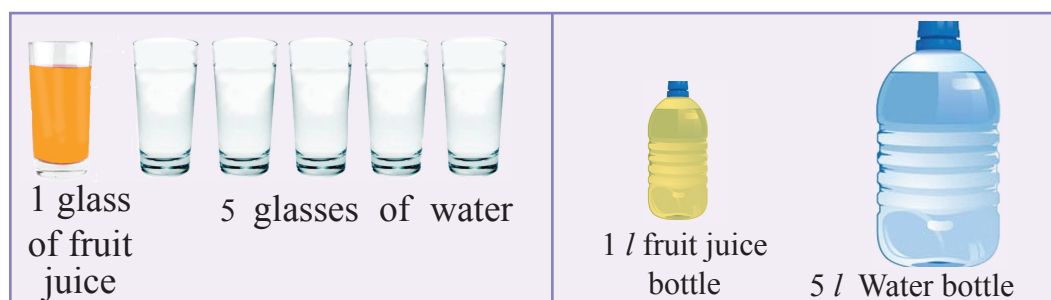
In this instance though the unit used to measure the quantities is bottle or glass or litre, one unit of fruit juice has been mixed with five units of water.

A **ratio** is a numerical relationship between the amounts of two or more quantities that have been represented using the same unit (as in the above case). Further, a ratio is also a numerical relationship between the amounts in two groups which are being compared.

Accordingly, the ratio of the above fruit juice to water is said to be one to five. This can be symbolically expressed as 1 : 5. Here " : " is known as the symbol of ratio. 1 and 5 are the terms. When written as 1 : 5, the first term is 1 and the second is 5. As a ratio does not change with the units used to measure the quantities, it is not necessary to indicate the unit here.

Now let us consider another example.

A person spends 7 rupees and saves 3 rupees from each 10 rupees of his earnings. Let us find the ratio of what he spends to what he saves. Here the quantities are savings and spendings that are expressed in the same unit, rupees. Accordingly, the ratio of spendings to savings is 7 : 1.

Another example is that of making paint of a certain colour by mixing paints of different colours.

Let us consider making light blue paint by mixing one part of dark blue paint with two parts of white paint .
Accordingly, the ratio of the amounts of dark blue to white paint is 1:2.

The following table shows how to make light blue paint by mixing different quantities of white and dark blue paint in the ratio 1 : 2.

Quantity of dark blue paint	Quantity of white paint	Quantity of light blue paint once paints are mixed
1 *l*	2 *l*	3 *l*
2 *l*	4 *l*	6 *l*
3 *l*	6 *l*	9 *l*
5 *l*	10 *l*	15 *l*

To prepare less than 3 *l* of light blue paint, dark blue and white paint can be mixed as follows.

Quantity of dark blue paint	Quantity of white paint	Quantity of light blue paint once paints are mixed
200 ml	400 ml	600 ml
400 ml	800 ml	1200 ml

Example 1

The length of a rectangle is 12 cm and the breadth is 7 cm. Find the ratio of the length to the breadth of the rectangle.

Length of the rectangle = 12 cm
Breadth of the rectangle = 7 cm

The ratio of the length to the breadth of the rectangle is 12 : 7.

Example 2

Three boys and two girls are in a debating team. Find the ratio of the number of boys to the number of girls in the team.

Number of boys = 3
Number of girls = 2

The ratio of the number of boys to the number of girls is 3 : 2.

Example 3

The price of a small envelope is 50 cents and that of a large envelope is Rs. 2. Find the ratio of the price of a small envelope to the price of a large envelope.

Price of a small envelope = 50 cents
Price of a large envelope = Rs. 2

Price of a large envelope should be written in cents, since the units of the given envelopes are not the same.

Price of a large envelope = Rs. 2 = 200 cents

The ratio of the price of a small envelope to the price of a large envelope is 50 : 200.

Example 4

To make an orange drink, two equal quantities of orange juice and three equal quantities of water are mixed together.

(i) Find the ratio of orange juice to water in a glass of orange drink.

(ii) How many litres of water do you need to prepare an orange drink which has four litres of orange juice?

(i) The ratio of orange juice to water is 2 : 3.

(ii) Quantity of water which is mixed with two litres of orange juice $\Big\}$ = 3 litres

Quantity of water which is required for four litres of orange juice $\Big\}$ = 3 × 2 litres

= 6 litres

Exercise 21.1

(1) Select and write down the statements that denote a ratio.

(i) To make milk tea, three tea-spoons of milk powder and two tea - spoons of sugar should be used.

(ii) Sunil is taller than Sarath.

(iii) A rectangular land has a width of 80 m and a length of 117 m.

(iv) To make a cake 250 g of sugar and 500 g of flour are needed.

(v) Sarath has got more marks than Malidu for Mathematics.

(2) Write down how you read the following ratios.

(i) 1 : 2   (ii) 2 : 3   (iii) 10 : 8   (iv) 8 : 7   (v) 9 : 13

(3) Write down the following ratios in symbolic form.

(i) The ratio one to three.   (ii) The ratio two to seven.
(iii) The ratio three to fifteen.   (iv) The ratio eight to one.
(v) The ratio one to one.

(4) A boy has five olives and his sister has seven olives. Find the ratio of the number of olives the boy has to the number the sister has.

(5) The mass of an apple is 200 g and that of an orange is 200 g. Find the ratio of the mass of an apple to the mass of an orange.

(6) A rectangular shaped land has a length of 75 m and a width of 37 m. Find the ratio of the length to the width.

(7) Mother and father bought 500 g and 2 kg of dhal respectively. Find the ratio of the weights of dhal bought by both.

(8) The distance from school to Priyantha's house is 700 m and the distance from school to Lasantha's house is 1 km 300 m. Find the ratio of the distances.

(9) Rajitha has 8 rupees and Vijitha has 5 rupees and 50 cents. Find the ratio of the money that the two have.

(10)

1 *l* 450 ml	2 *l*
A	B
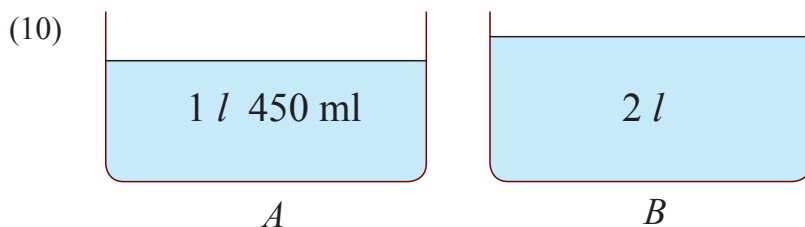
Find the ratio of the amount of water in container *A* to the amount in container *B*.

(11) The time spent by a motor bike to go to Galle from Matara was 1 hour and 10 minutes while a car spent 1 hour. Find the ratio of the time spent by the two vehicles.

(12) The ratio of the number of olives that Nimali has to the number that Amal has is 3:5. If Nimali has only one olive, find the number of olives that Amal has.

(13) To make a cement mixture, two pans of cement and twelve pans of sand are mixed together. Find the ratio of the cement to the sand in this mixture.

 (i) How many pans of cement are needed to be mixed with one pan of sand to make this mixture ?

 (ii) Find the number of cement pans and sand pans that are required to make 28 pans of the mixture.

21.2 Equivalent ratios

A concrete mixture is made with cement and sand according to the ratio 1 : 3. Accordingly, the number of parts of sand that should be used when the number of parts of cement is changed is shown in the table below.

Cement	Sand	Ratios
1	3	1 : 3
2	6	2 : 6 (1 : 3 is multiplied by 2)
3	9	3 : 9 (1 : 3 is multiplied by 3)
4	12	4 : 12 (1 : 3 is multiplied by 4)
5	15	5 : 15 (1 : 3 is multiplied by 5)

The table shows,

$$1 : 3 = 2 : 6 = 3 : 9 = 4 : 12 = 5 : 15$$

In the above example, the ratio of sand to cement can be written in any one of the forms 1 : 3, 2 : 6, 3 : 9. All these are **equivalent ratios.**

Accordingly, a ratio equivalent to a given ratio can be obtained by multiplying each term of the ratio by the same number (greater than zero).

Example 1

Write down two ratios equivalent to 2 : 5.

Multiplying the terms of the ratio by 2,

$2 : 5 = 2 \times 2 : 5 \times 2 = 4 : 10$

Multiplying the terms of the ratio by 3,

$2 : 5 = 2 \times 3 : 5 \times 3 = 6 : 15$

$2 : 5 = 4 : 10 = 6 : 15$

4 : 10 and 6 : 15 are two other ratios equivalent to 2 : 5.

● Another method for finding equivalent ratios

Let us consider a mixture made of 2 l of lime juice mixed with 4 l of water. In this mixture, the ratio of lime juice to water is 2 : 4.
The same composition can be obtained by mixing 1 l of lime juice with 2 l of water.

Hence the ratios 2 : 4 and 1 : 2 are equivalent ratios.

The ratio 1 : 2 can be obtained by dividing the terms of 2 : 4 by 2.
Hence $2:4 = \dfrac{2}{2} : \dfrac{4}{2} = 1:2$

Accordingly, a ratio equivalent to a given ratio can be obtained by dividing each term of the ratio by the same number (except zero).

21.3 Writing a ratio in its simplest form

Let us consider some ratios equivalent to each other.
8 : 12 = 4 : 6 = 2 : 3 = 6 : 9 = 10 : 15

A ratio is in its simplest form when both terms are whole numbers and there is no whole number which both terms can be divided by.

So 2 : 3 is the simplest form of the ratio 8 : 12.
The ratio 2 : 3 is the simplest form of 4 : 6, 6: 9 and 10 : 15 as well.

To write in its simplest form, keep dividing both sides of the ratio by the same number until you can't go any further without going into decimals.

Example 1

Write the ratio 9 : 15 in its simplest form.

9 : 15 = 9 ÷ 3 : 15 ÷ 3

   = 3 : 5

Example 2

Write the ratio 18 : 24 in its simplest form.

18 : 24 = 18 ÷ 2 : 24 ÷ 2

   = 9 : 12

   = 9 ÷ 3 : 12 ÷ 3 = 3 : 4

Example 3

The breadth of a blackboard is 50 cm. The length is 1 m 25 cm. Find the ratio of the breadth to the length of the blackboard.

1 m 25 cm = 125 cm

50 : 125 = 50 ÷ 5 : 125 ÷ 5

   = 10 : 25

   = 10 ÷ 5 : 25 ÷ 5

   = 2 : 5

Exercise 21.2

(1) For each ratio given below, write down an equivalent ratio.

   (i) 2 : 7   (ii) 10 : 30   (iii) 50 : 45

(2) Write each of the ratios given below in its simplest form.

   (i) 40 : 70   (ii) 30 : 35   (iii) 56 : 21   (iv) 63 : 45   (v) 60 : 150

(3) The length of a rectangle is 15 cm and its breadth is 10 cm. Find the ratio of the length to the breadth of the rectangle.

(4) There are 96 boys and 112 girls in a primary school. Write the ratio of the boys to the girls.

(5) There are 12 red flags and 8 blue flags. Write the ratio of the red flags to the blue flags in its simplest form.

(6) Write 6 : 15 and 14 : 35 in their simplest forms. Show that they are equivalent ratios.

21.4 Rate

The picture shows that two table - spoons of milk powder have to be added to make a cup of tea. Here, as the amount of tea and milk powder that should be added cannot be expressed in the same unit, the amounts of tea and milk powder to be mixed cannot be expressed as a ratio. Consider the following.

- 10 eggs are used to make 1 kg of cake.
- A vehicle can travel 12 km on 1 *l* of oil.
- The price of 10 guavas is Rs. 100.

The two quantities in each of the above situations are measured in different units. **Rate** is a relationship between such quantities.

Let us consider some other instances where rate is used.

(1) Price of a pencil is Rs. 10.

(2) Rs. 40 is charged to travel 1 km in a vehicle.

(3) Students present at a function received 3 biscuits each during the interval.

(4) The price of a match box is Rs. 50.

The relationship between two currencies of different countries is a rate. The value of a United States dollar in Sri Lanka was Rs. 130.54 on 11.03.2014. Such a relationship between different currencies is called exchange rate. Since the exchange rate changes daily, we need to write the date when mentioning the exchange rate.

The following table gives the exchange rate of foreign currencies rounded off to the nearest whole number on a particular day.

Name of Currency	Sri Lankan Rupees
1 US Dollar	131
1 Sterling pound	217
1 Euro	181
100 Japanese yen	125
1 Bahrain dinar	346

Example 1

The price of an exercise book is Rs. 20. Find the price of 4 such books.

$$\text{Price of an exercise book} = \text{Rs. } 20$$
$$\text{Price of 4 such books} = \text{Rs. } 20 \times 4$$
$$= \text{Rs. } 80$$

Example 2

If the price of 5 pencils is Rs. 100, find the price of 2 pencils.

Price of 5 pencils $=$ Rs. 100
Price of a pencil $=$ Rs. $100 \div 5 =$ Rs. 20
Price of 2 pencils $=$ Rs. $20 \times 2 =$ Rs. 40

Example 3

The value of a present sent by a friend who is working abroad to his friend who is living in Sri Lanka was 25 US dollars. Express its value in Sri Lankan rupees.

That day, the exchange rate of 1 USD was Sri Lankan Rupees 131.

$$\text{Value of 1 Dollar} = \text{Rs. } 131$$
$$\text{Value of 25 USD} = \text{Rs. } 131 \times 25$$
$$= \text{Rs. } 3275$$

(1) If the price of a pen is Rs. 12, what is the price of 5 such pens?

(2) If a vehicle travels 75 km in 2 hours, what is the distance it travels in 4 hours?

(3) A vehicle travelled 20 km on 1 l of fuel. How many litres of fuel are required to travel 120 km?

(4) 1 kg of sugar is sufficient to make 40 cups of tea. How much sugar is required to make 240 cups of tea?

(5) If the value of a Sterling pound was Rs. 217 on a certain day, what was the value of 8 Sterling Pounds on that day?

(6) If the price of a television set imported from Japan is 10 000 Yen, what is its price in Sri Lankan Rupees? (Use the exchange rate table in page 123.)

(7) The value of 1 US Dollar is 130 Sri Lankan Rupees. What is the value of Sri Lankan Rupees 26 000 in US Dollars?

Summary

- A ratio equivalent to a given ratio can be obtained by dividing each term of the ratio by the same number (greater than zero).

- A ratio equivalent to a given ratio can be obtained by multiplying each term of the ratio by the same number (greater than zero).

- A ratio is in its simplest form when both terms are whole numbers and there is no whole number which both terms can be divided by.