14 Types of Numbers and Number Patterns

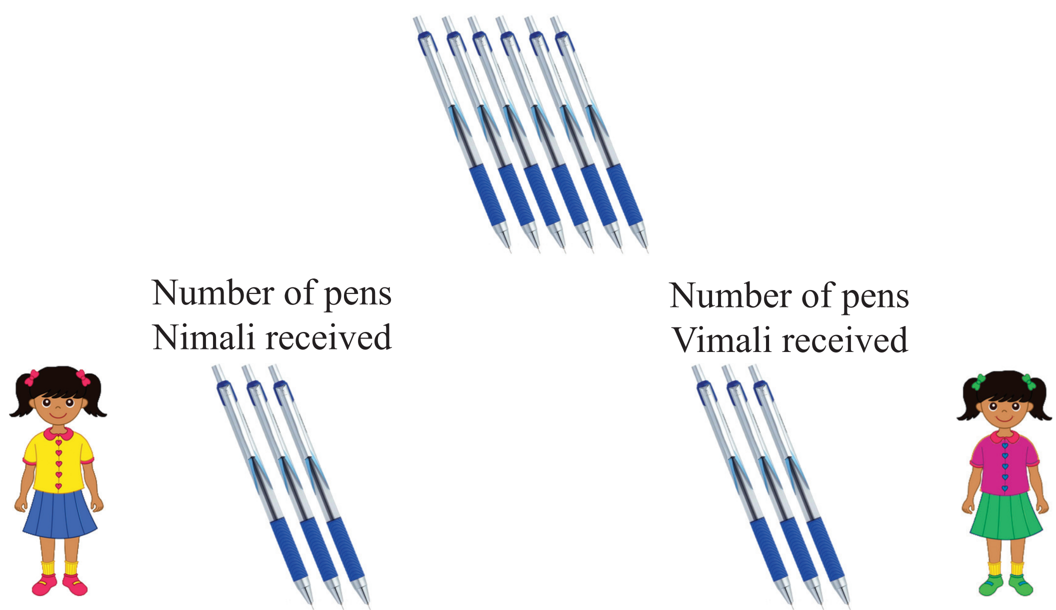
By studying this lesson, you will be able to,

- identify odd numbers, even numbers, prime numbers, composite numbers, square numbers and triangular numbers among the whole numbers and
- identify number patterns formed by the above types of numbers.

14.1 Even numbers and odd numbers

Let us identify even numbers and odd numbers among the whole numbers.

Six pens can be equally divided between Nimali and Vimali in the following manner.



Let us see whether the number of pens in the table below can be divided equally between the two.

Number of pens	Number of pens pictorially	Number of pens Nimali received	Number of pens Vimali received	Remainder
2				No remainder
3				
5				
4				No remainder
8				No remainder

This shows that, if the quantity is 2, 4, 6 or 8, then it can be divided into two equal parts without a remainder. That is, these numbers are divisible by 2. Numbers such as 2, 4, 6 and 8, which when divided by 2 have zero remainder, are called **even numbers**. 0 is also an even number.

When a whole number is divided by 2, if the remainder is zero, then the number is an even number.

Accordingly, even numbers starting from zero can be written as 0, 2, 4, 6, 8, 10, 12, ...

From the above information we see that when the number of pens is 3 or 5, then the pens cannot be divided equally between Nimali and Vimali. When an equal number of pens are given to both, then in the end, there is one remaining.

Numbers such as 1, 3, 5, 7, 9 and 11, which when divided by 2, have a non-zero remainder, are called **odd numbers.**

When a whole number is divided by 2, if the remainder is one, then the number is an odd number.

Accordingly, odd numbers starting from one can be written as 1, 3, 5, 7, 9, 11, 13, ...

Note

- When the two even numbers 2 and 6 are added, the result is 8, which is also an even number. When we add any two even numbers in this manner, the result is also an even number.
- We can verify the following statements by using the examples.
- When two odd numbers are added together, the result is an even number.
- When an even number and an odd number are added together, the result is an odd number.
- When an even number is subtracted from an even number, the result is an even number.
- When an odd number is subtracted from an odd number, the result is an even number.
- When an odd number is subtracted from an even number, the result is an odd number.
- When an even number is subtracted from an odd number, the result is an odd number.
- When two odd numbers are multiplied together, the result is also an odd number.
- When any whole number is multiplied by an even number, the result is an even number.

Example 1

Write whether each number below is an even number or an odd number.

(i) 8 (ii) 13 (iii) 32 (iv) 17 (v) 100 (vi) 351 (vii) 1001

(i) $8 \div 2 = 4$. That is, 8 is an even number.

(ii) $13 \div 2 = 6$ with a remainder of 1. That is, 13 is an odd number.

(iii) $32 \div 2 = 16$. That is, 32 is an even number.

(iv) $17 \div 2 = 8$ with a remainder of 1. That is, 17 is an odd number.

(v) $100 \div 2 = 50$. That is, 100 is an even number.

(vi) $351 \div 2 = 175$ with a remainder of 1. That is, 351 is an odd number.

(vii) $1001 \div 2 = 500$ with a remainder of 1. That is, 1001 is an odd number.

Exercise 14.1

(1) Copy the table below and complete it.

Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Even		✓																		
Odd	✓																			

(2) From the numbers given below, choose the even numbers. Write them in the first row of the table below. Choose the odd numbers, and write them in the second row.

6, 7, 21, 24, 30, 35, 62, 70, 59, 100, 87, 71, 93, 94

Even numbers							
Odd numbers							

(3) State whether the answer of each of following expressions is even or odd without solving.

(i) $31 + 52$ (ii) $103 + 527$ (iii) $32 - 15$ (iv) $88 + 424$
(v) $101 - 27$ (vi) $298 - 114$ (vii) $89 - 22$ (viii) 32×18
(ix) 153×36 (x) 27×39

• Identifying even numbers and odd numbers by considering the digit in the ones place.

Let us consider another method of finding out whether numbers like 2157 and 34 826 are odd numbers or even numbers without dividing the number by 2.

Now let us consider several whole numbers. Let us write each of them using the numbers its digits represent.

$$124 = 100 + 20 + 4$$
$$230 = 200 + 30 + 0$$
$$395 = 300 + 90 + 5$$
$$761 = 700 + 60 + 1$$
$$842 = 800 + 40 + 2$$
$$2157 = 2000 + 100 + 50 + 7$$
$$34\ 826 = 30\ 000 + 4000 + 800 + 20 + 6$$

The numbers represented by the digits in the tens place, hundreds place and the thousands place of any whole number given above, are multiples of 10. Therefore, those numbers are divisible by 2 (without a remainder). So, if the digit in the ones place is divisible by 2, (with zero remainder) then the given number is divisible by 2.

If the ones place of a number has one of the digits 0, 2, 4, 6 or 8 then the number is an even number.

If the ones place of a number has one of the digits 1, 3, 5, 7 or 9 then the number is an odd number.

Example 1

(i) Write the even numbers between 0 and 10 (Here the answer does not include 0 and 10).
2, 4, 6, 8

(ii) Write the even numbers from 0 till 10 (Here the answer includes 0 but not 10).
0, 2, 4, 6, 8

(iii) Write the even numbers from 0 to 10 (Here the answer includes 0 and 10).
0, 2, 4, 6, 8, 10

Exercise 14.2

(1) Write the even numbers between 10 and 25.

(2) Write the odd numbers from 19 till 35.

(3) Write the even numbers from 13 to 24.

(4) From the list of numbers given below, write the even numbers and the odd numbers separately.
456, 395, 714, 1852, 341, 27 850, 148 400, 397 659, 8 000 008

(5) Write your year of birth, month of birth and the date of birth respectively. Write whether each of them is an odd number or an even number.

(6) In a certain street of a city, the following sign is posted, "Vehicles can be parked here only on odd days of the month". What are the days in the month of June that vehicles can be parked along the street?

(7) Write 5 even numbers and 5 odd numbers that can be written using the digits 4, 2, 3, 1 and 0 exactly once.

14.2 Prime numbers and Composite numbers

Recall what you learnt in the Factors lesson. Now let us find the factors of several numbers.

Number	As a product	Factors
2	1×2	1, 2
3	1×3	1, 3
4	$1 \times 4,\ 2 \times 2$	1, 2, 4
5	1×5	1, 5
6	$1 \times 6,\ 2 \times 3$	1, 2, 3, 6
7	1×7	1, 7
8	$1 \times 8,\ 2 \times 4$	1, 2, 4, 8
9	$1 \times 9,\ 3 \times 3$	1, 3, 9

Each of the numbers 2, 3, 5 and 7 has only two different positive factors.

Each of the numbers 4, 6, 8 and 9 has more than two different factors.

Whole numbers greater than one such as 2, 3, 5 and 7 which have exactly two distinct factors are called prime numbers.

Let us now write the prime numbers from 1 to 20.
They are 2, 3, 5, 7, 11, 13, 17 and 19.

Of these prime numbers only 2 is even. All the others are odd.

Apart from 2, all the other even numbers have more than two factors. Therefore, of all prime numbers, 2 is the only even prime number.

Whole numbers such as 4, 6, 8 and 9 which have more than two distinct factors are called **composite numbers**.

Therefore whole numbers greater than one, other than prime numbers are called composite numbers.

1 is neither a prime nor a composite number.

Activity 1

Fill in the blanks in the table below.

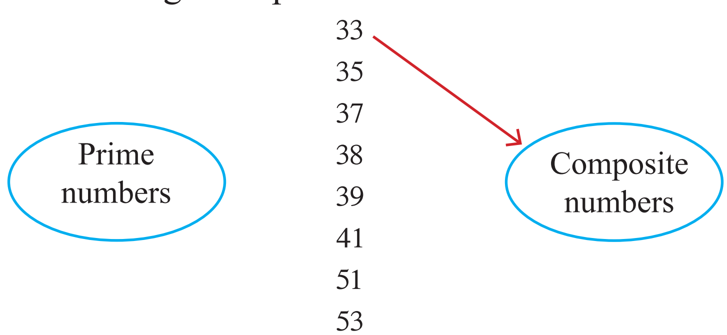
Number	Factors	Number of factors	The number is a prime number (✓) / if not (×)	The number is a composite number (✓) / if not (×)
1				
2	1, 2	2	✓	
3				
4				
5				
6	1, 2, 3, 6	4	×	✓
7				
8				
9				
10				
11				
12				
13				
14				
15				

Exercise 14.3

(1) The month of January of a certain calendar is given below. Draw circles around the prime numbers and draw triangles around the composite numbers.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

(2) Copy the figure given below. From the given numbers, choose the prime numbers. Draw an arrow from each prime number to the circle indicating "prime numbers". From the given numbers, choose the composite numbers. Draw an arrow from each composite number to the circle indicating "composite numbers".



(3) (i) Write two successive prime numbers.
(ii) Write two successive composite numbers.

(4) Find the odd number between 1 and 10 which is not a prime.

(5) (i) Find a pair of prime numbers whose addition is 30.
(ii) Write 14 as a product of two prime numbers.

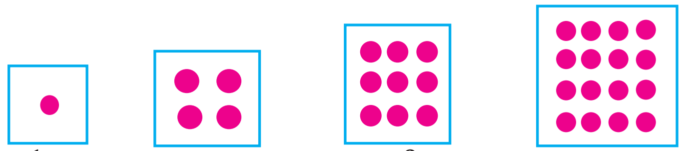
(6) (i) What is the smallest prime number?
(ii) What is the smallest composite number?

(7) Write the composite numbers between 20 and 30.

(8) What is the only even prime number?

14.3 Square numbers

Several numbers that can be represented using dots, arranged in a square formation are given below.



1	4	9	16
1 row	2 rows	3 rows	4 rows
1 column	2 columns	3 columns	4 columns

In each of the given boxes, the number of dots in a row is equal to the number of dots in a column. By multiplying these two numbers the number represented by the dots can be obtained.

That is,

$$1 = 1 \times 1$$
$$4 = 2 \times 2$$
$$9 = 3 \times 3$$
$$16 = 4 \times 4$$

We can write 25, 36 and 49 also in the above manner. Such numbers are called **square numbers**.

Exercise 14.4

(1) Look at the dates in the month of January in a calendar. Which dates are square numbers?

(2) Write down the square numbers from 1 to 100.

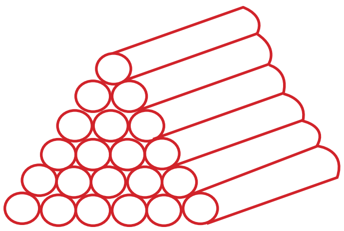
(3) Write down the square numbers between 50 and 150.

(4) Add the odd numbers between 0 and 6. Find whether the sum is a square number.

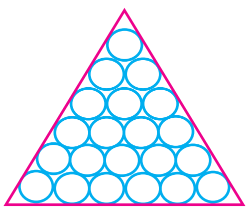
(5) Add the odd numbers between 0 and 10. Find whether the sum is a square number.

14.4 Triangular numbers

In a certain shop, pipes are arranged as in the figure below.



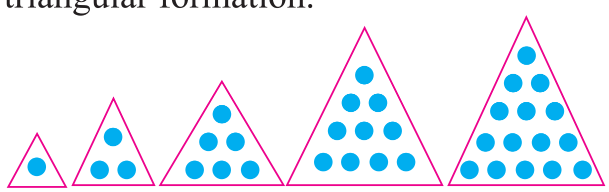
The manner in which the pipes are seen from the front is,



It takes the shape of a triangle. Let us find out the number of pipes here.

The number of pipes in each row from the top is 1, 2, 3, 4, 5 and 6. By adding these numbers, the total number of pipes is obtained as 21. Therefore 21 can be represented using dots, where the dots are placed in a triangular formation.

Let us find out about such numbers that can be represented using dots, arranged in a triangular formation.



Numbers that can be represented as above are called **triangular numbers.**

Considering the number of dots in each row,

Triangular number represented in the first figure $= 1 = 1$

Triangular number represented in the second figure $= 1 + 2 = 3$

Triangular number represented in the third figure $= 1 + 2 + 3 = 6$

Triangular number represented in the fourth figure $= 1 + 2 + 3 + 4 = 10$

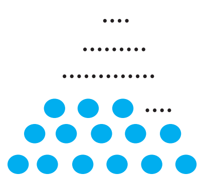
Triangular number represented in the fifth figure $= 1 + 2 + 3 + 4 + 5 = 15$

As explained above, by starting at 1 and adding successive numbers to it, we can obtain triangular numbers. Therefore, if we want to obtain the tenth triangular number, we need to start at 1 and add to it the successive numbers, up to 10.

$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55$

That is, the 10^{th} triangular number is 55.

Exercise 14.5

(1) 

Complete the figure given here in triangular formation and find the triangular number represented by it.

(2) Complete the rows below as given in the first four rows.

1	=	1
1 + 2	=	3
1 + 2 + 3	=	6
1 + 2 + 3 + 4	=	10
....	=	....
....	=	....
....	=	....
....	=	....
....	=	....

(3) What is the smallest triangular number?

(4) A number is represented using dots that are arranged in a triangular formation. Then the bottom row consists of 11 dots. What is the triangular number?

(5) Add two consecutive numbers given in exercise (2). Is the sum a square number?

14.5 Number patterns

Let us write the even numbers in ascending order, starting at 2.

2, 4, 6, 8, 10, ...

This is the even number pattern starting at 2 written in ascending order.

Let us write the odd numbers in ascending order, starting at 1.

1, 3, 5, 7, 9, ...

This is the odd number pattern starting at 1 written in ascending order.

Let us write the square numbers in ascending order, starting at 9.

9, 16, 25, 36, ...

This is the square number pattern starting at 9 written in ascending order.

1, 3, 6, 10, 15 is the triangular number pattern written in ascending order.

Each number in a number pattern which has been arranged as above according to a mathematical rule is called a **term in that pattern.**

Exercise 14.6

(1) 1, 3, 6, 10, ... is the triangular number pattern starting at 1, written in ascending order. Find the 8^{th} term.

(2) (i) 1, 4, 9, 16, … is the square number pattern starting at 1, written in ascending order. Find the 12^{th} term.

(ii) In the above square number pattern, which term is 49?

(iii) Is 65 a term in this number pattern?

(iv) What are the terms that are between 50 and 100 in this pattern?

(3) Write the first 5 numbers in the number patterns given below.

(i) The pattern consisting of all even numbers greater than 5, written in ascending order.

(ii) The pattern consisting of all multiples of 3 greater than 10, written in ascending order.

(iii) The pattern starting at 1 and consisting of triangular numbers that are not primes, written in ascending order.

Miscellaneous Exercises

(1)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32								

(i) As given in the above order, write the numbers from 1 to 50 in your exercise book.

(ii) Now draw a line across 1.

(iii) Draw a circle around 2.

(iv) Now draw lines across all multiples of 2 other than 2.

(v) Draw a circle around 3.

(vi) Now draw lines across all multiples of 3 other than 3.

(vii) Draw a circle around 5. Now draw lines across all multiples of 5 other than 5.

(viii) Draw a circle around 7. Now draw lines across all multiples of 7 other than 7.

(ix) Now draw circles around the remaining numbers. Confirm whether the numbers inside the circles are prime numbers?

(2) Amali says that, when we consider two consecutive whole numbers, one of them is an even number while the other is an odd number. Is this statement true or false?

(3) Jayamini explains that, by adding two consecutive triangular numbers, we obtain a square number, by using the examples below.

$1 + 3 = 4$ $\qquad\qquad$ $3 + 6 = 9$

Check whether her statement is true. Give 3 more examples to verify this.

(4) Consider the sentences below. Mark the ones that are correct as "✓" and the others as "✗".

(i) 1 is a prime number.

(ii) The smallest prime number is 2.

(iii) All square numbers are composite numbers.

(iv) All triangular numbers are composite numbers.

(v) 36 is a composite number which is a square number as well as a triangular number.

Summary

- If the ones place of a number has one of the digits 0, 2, 4, 6 or 8 then the number is an even number.
- If the ones place of a number has one of the digits 1, 3, 5, 7 or 9 then the number is an odd number.
- Whole numbers greater than one, which have only two different factors are called prime numbers.
- Whole numbers greater than one, having more than two different factors are called composite numbers.