









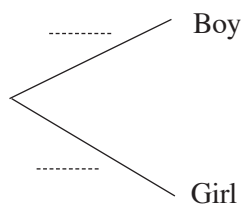




5. Four boys and 2 girls who are able to sing as well as play a musical instrument are to participate in a talent show. The four boys have been given the numbers 1, 2, 3 and 4 while the two girls have been given the numbers 5 and 6, to select the order in which they are to perform. These six numbers are marked on six identical cards such that each card has a different number. A card is picked at random from the box. The child who has the number which is picked has to sing.

(a) (i) Complete the given incomplete tree diagram.

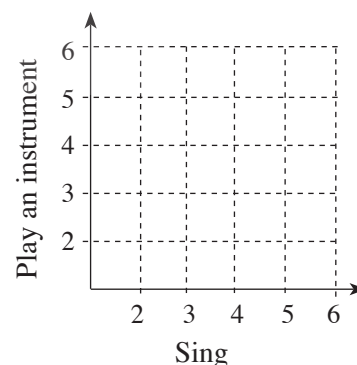
**First picking (Sing)**



(ii) The first card picked is replaced in the box and again a card is picked at random. The child with this number has to play an instrument. Extend the given tree diagram in a suitable way, and find the probability of a boy performing on one occasion and a girl on the other occasion.

(b) Now suppose that the boy who received the number 1 had to withdraw from the show due to another commitment. If the card with the number 1 is removed from the box and children are picked in the same manner as above from the remaining cards, to sing and to play an instrument,

(i) represent the sample space relevant to picking a child to sing and a child to play an instrument in the given grid.



(ii) Mark the event of the same child not being picked to sing and play an instrument on the grid and find its probability.

# G.C.E.(O.L.) Support Seminar - 2016

## Mathematics II

Three hours

- \* Answer **ten** questions, selecting **five** questions from **Part A** and five questions from **Part B**.
- \* Each question carries 10 marks.
- \* The volume of a right circular cone with base radius  $r$  and height  $h$  is  $\frac{1}{3}\pi r^2 h$ .

### Part A

Answer only **five** questions.

1.  $A$  and  $B$  are two financial institutions which provide loans as follows.

#### Institution A

- Charges interest at an annual interest rate of 18%.
- Interest is calculated on the reducing balance.
- The loan amount and interest have to be paid in equal monthly installments.

#### Institution B

- Charges interest at an annual interest rate of 10% and the interest is compounded annually.
- The loan amount and the total interest have to be paid together at the end of the loan period.

Samantha needs to obtain a loan of Rs. 300 000. He intends to settle the loan together with the interest by the end of two years. Samantha's friend says that more interest has to be paid if the loan is taken from Institution A. Find the total interest that has to be paid when this loan amount is taken from each of these two institutions, and with reasons explain the truth / falsehood of the friend's statement.

2. The table below gives the  $y$  values of the function  $y = (x + 1)(x - 3)$  corresponding to several given  $x$  values.

$x$	-1	0	1	2	3	4	5
$y$	0	-3	-4	-3	0	....	12

- (a) (i) Find the value of  $y$  corresponding to  $x = 4$ .  
(ii) Draw the graph of the above function by selecting a suitable scale.
- (b) By using the graph,  
(i) find the minimum value of the function.  
(ii) explain the behavior of the graph on the interval  $-1 < x < 1$ , by indicating whether it is positive or negative and whether it is increasing or decreasing.
- (c) By drawing a suitable straight line graph on the above coordinate plane, obtain a value for  $\sqrt{3}$  to the nearest first decimal place.

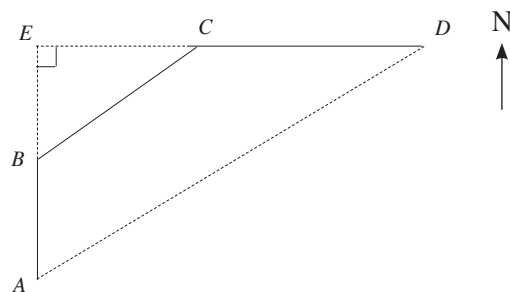


3. The frequency distribution of the number of trips made during a day by 40 vehicles of the same type transporting soil to the construction site of a expressway is given below.

Class Interval (Number of trips)	4 - 8	9 - 13	14 - 18	19 - 23	24 - 28	29 - 33	34 - 38
Frequency (Number of vehicles)	2	3	5	8	15	5	2

- (i) How many vehicles made more than 33 trips transporting soil?
  - (ii) Find the mean number of trips that a vehicle makes during a day transporting soil, to the nearest whole number.
  - (iii) Each vehicle can carry 4 cubes of soil per trip. If the price of one cube of soil is Rs. 2000, what is the expected cost of the soil that is transported during a day by one vehicle?
  - (iv) The engineer states that since all the above vehicles would have to be used for 2 days to transport the soil required for a length of 100 m of the highway, the cost of the soil for this would exceed 14 million rupees. Give reasons to show that his statement could be true.
4. The government provides aid of Rs. 100 per child and Rs. 150 per adult daily for those sheltering in a certain camp due to being displaced by floods. On a certain day, a group of students brought 31 apples, cut into 6 pieces each, and 29 guavas cut into 4 pieces each to be distributed among the displaced in the camp. When the children were provided with 3 pieces of apple each and the adults with 2 pieces each, and the children were provided with 2 pieces of guava each and the adults with 1 piece each, only 2 pieces of guava were left over. Find the number of children and the number of adults among the displaced in this camp and hence find the total amount of money the government needs to give for a week to the displaced in this camp.
5. The length of the hypotenuse of a right angled triangle is 1 cm less than twice the length of its shortest side.
- (i) If the length of the shortest side is  $x$  cm, express the length of the hypotenuse in terms of  $x$ .
  - (ii) If the remaining side is 3 cm more than the length of the shortest side, using Pythagoras' theorem, write down a relationship between the squares of the lengths of the sides of the triangle.
  - (iii) Show that by simplifying the above relationship the equation  $x^2 - 5x = 4$  is obtained.
  - (iv) Find the length of the hypotenuse by solving the above equation using the method of completing the square or by using the formula. (Take  $\sqrt{41} = 6.4$ )

6. Starting from the location  $A$ , Kamal travels 12 metres to the North and arrives at the location  $B$  as shown in the figure. From  $B$  he travels 20 metres on a bearing of  $040^\circ$  and arrives at the location  $C$ , and from  $C$  he travels 25 metres to the East to reach the location  $D$ .



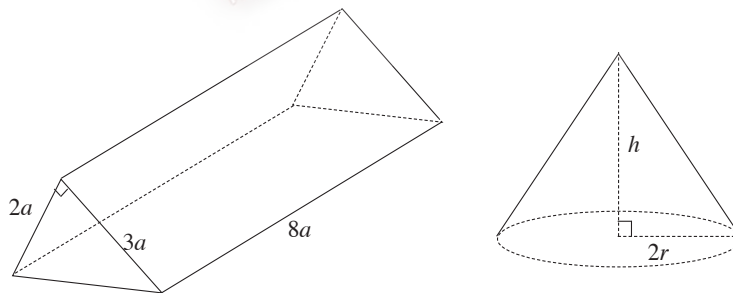
- (i) Copy the above figure, and mark the given information in it.
- (ii) Find the distance  $CE$  to the nearest metre using the trigonometric tables.
- (iii) The distance  $BE$  rounded off to the nearest metre is 15 m. Accordingly, calculate  $\hat{EAD}$  and the distance  $AD$ .

**Part B**

Answer only **five** questions.

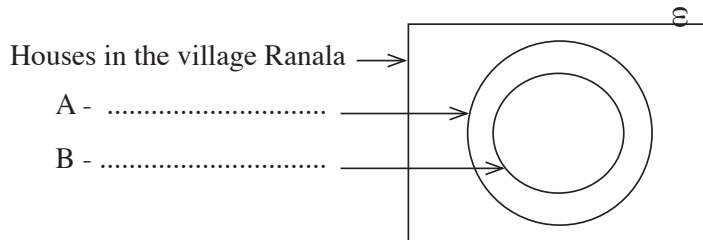
7. A competitor participating in the programme “Wasanavanthaya” organized by a certain television channel, wins by answering 15 questions correctly. The prize money of Rs. 50 for the first question, Rs. 75 for the second question, Rs. 100 for the third question etc., given for the correct answers, are in an arithmetic progression.
- (i) Find the prize money allocated for the 12<sup>th</sup> question.
  - (ii) If Rs. 425 has been allocated as prize money for 2 consecutive questions, which 2 questions are they?
  - (iii) A competitor has to leave the program if a wrong answer is given. In this case, the competitor’s prize money is half the amount allocated for all the questions he has answered correctly thus far. If a certain competitor had to leave the competition with Rs. 1300 due to not giving the correct answer to a certain question, show that the number of questions he answered is 14.
8. Using only a straight edge with a cm/mm scale and showing the construction lines clearly, do the following constructions.
- (i) Construct a straight line segment  $PQ$  of length 8 cm.
  - (ii) Mark the point  $R$  such that  $\hat{QPR} = 45^\circ$  and  $PR = QR$ , and complete the triangle  $PQR$ .
  - (iii) Construct the circle that passes through the points  $P, Q$  and  $R$  and name its centre  $O$ .
  - (iv) Mark the point  $S$  on  $PR$  produced such that  $PR = RS$ .
  - (v) Mark the point  $T$  on the circle such that  $SQ = ST$  and show that  $SQ$  and  $ST$  are two tangents to the circle.

9.



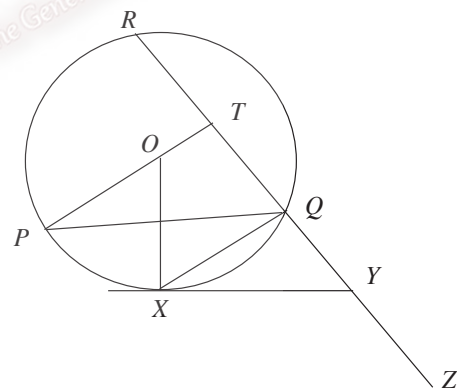
The above figure shows a solid metal right prism with a right triangular cross section, and a solid right circular cone. Their measurements have been given in the figure in the same units. The solid cone with base radius  $2r$  and height  $h$  is made with the metal that is obtained by melting the prism, such that there is no waste. Show that when the height of the cone is 15 cm,  $r^2 = \frac{6a^3}{5\pi}$ . By taking  $\pi = 3.14$  and using the logarithms table, find the value of  $r^2$  when  $a = 4.55$  cm, and hence find the radius of the cone.

10. There are 120 houses in the village Ranala. 113 of these houses have television sets. Of the houses which have television sets, 78 do not have radios. Every house that has a radio has a television set too.



- (i) Identify the sets A and B and complete the Venn diagram using the given information.
- (ii) How many houses have both a television set and a radio?
- (iii) What is the probability of a house picked at random from the houses in the village Ranala being one which does not have a radio?
- (iv) None of the houses in the village Ranala have computers. A private institution decides to provide computers to 25 selected houses in this village. If they select 4 houses which do not have television sets and 9 houses which have radios to be among the houses which receive the computers, draw the Venn diagram again to represent this information, taking C to be the set of houses which are provided with computers.
- (v) Now, shade the region in the Venn diagram which denotes the set of houses which do not have radios but have television sets and computers.

11.  $PQ$  and  $QR$  are two chords of the circle with centre  $O$  shown in the figure.  $PO$  produced bisects  $RQ$  at  $T$ . The tangent drawn to the circle at  $X$  is  $XY$ . The chord  $RQ$  has been produced to  $Z$  and  $\hat{POX} = 40^\circ$ .



- (i) Giving reasons, write the magnitude of  $\hat{PQX}$ .
- (ii) Show that  $OTYX$  is a cyclic quadrilateral.
- (iii) Find the magnitude of  $\hat{XYZ}$ .
- (iv) Name a diameter of the circle passing through the points  $O, T, Y$  and  $X$ , and write the theorem which is the reason for it.

12. In the quadrilateral  $ABCD$  shown in the figure, the points  $E$  and  $F$  lie on the side  $CD$  such that  $CE = EF = FD$ .  $AF$  and  $BE$  produced meet at  $G$  such that  $AF = FG$  and  $BE = EG$ . Sketch a figure with this information marked in it. Show that  $BCGF$  and  $ABCF$  are parallelograms and that they are equal in area.

